Answers

Chapter 1 Sequences and Series

1.1 Arithmetic Sequences, pages 16 to 21

1. a) arithmetic sequence: $t_1 = 16$, $d = 16$; next three terms: 96, 112, 128
b) not arithmetic
c) arithmetic sequence: $t_1 = -4$, $d = -3$; next three terms: $-19, -22, -25$
d) arithmetic sequence: $t_1 = 3$, $d = -3$; next three terms: $-12, -15, -18$

2. a) 5, 8, 11, 14 **b)** $-1, -5, -9, -13$
c) $4, \frac{21}{5}, \frac{22}{5}, \frac{23}{5}$ **d)** 1.25, 1.00, 0.75, 0.50

3. a) $t_1 = 11$ **b)** $t_7 = 29$ **c)** $t_{14} = 50$

4. a) 7, 11, 15, 19, 23; $t_1 = 7$, $d = 4$
b) $6, \frac{9}{2}, 3, \frac{3}{2}$; $t_1 = 6$, $d = -\frac{3}{2}$
c) 2, 4, 6, 8, 10; $t_1 = 2$, $d = 2$

5. a) 30 **b)** 82 **c)** 26 **d)** 17

6. a) $t_2 = 15$, $t_3 = 24$ **b)** $t_2 = 19$, $t_3 = 30$
c) $t_2 = 37$, $t_3 = 32$

7. a) 5, 8, 11, 14, 17 **b)** $t_n = 3n + 2$
c) $t_{50} = 152$, $t_{200} = 602$
d) The general term is a linear equation of the form $y = mx + b$, where $t_n = y$ and $n = x$. Therefore, $t_n = 3n + 2$ has a slope of 3.
e) The constant value of 2 in the general term is the y-intercept of 2.

8. A and C; both sequences have a natural-number value for n.

9. 5

10. $t_n = -3yn + 8y$; $t_{15} = -37y$

11. $x = -16$; first three terms: $-78, -116, -154$

12. $z = 2y - x$

13. a) $t_n = 6n + 4$ **b)** 58
c) 12

14. a) 0, 8, 16, 24
b) 32 players
c) $t_n = 8n - 8$
d) 12:16
e) Example: weather, all foursomes starting on time, etc.

15. 21 square inches

16. a) $t_n = 2n - 1$ **b)** 51st day
c) Susan continues the program until she accomplishes her goal.

17. a)

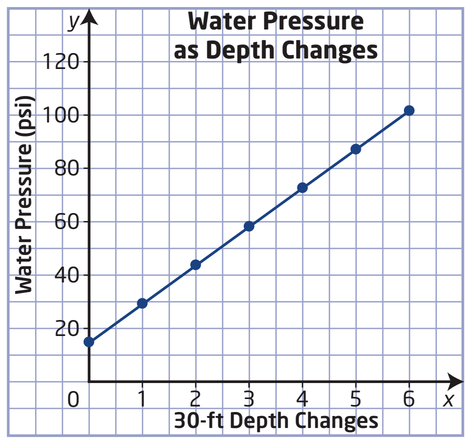
Carbon Atoms	1	2	3	4
Hydrogen Atoms	4	6	8	10

b) $t_n = 2n + 2$ or $H = 2C + 2$
c) 100 carbon atoms

18.

Multiples of	28	7	15
Between	1 and 1000	500 and 600	50 and 500
First Term, t_1	28	504	60
Common Difference, d	28	7	15
nth Term, t_n	980	595	495
General Term	$t_n = 28n$	$t_n = 7n + 497$	$t_n = 15n + 45$
Number of Terms	35	14	30

19. a) 14.7, 29.4, 44.1, 58.8; $t_n = 14.7n$, where n represents every increment of 30 ft in depth.
b) 490 psi at 1000 ft and 980 psi at 2000 ft
c)



d) 14.7 psi
e) 14.7
f) The y-intercept represents the first term of the sequence and the slope represents the common difference.

20. Other lengths are 6 cm, 12 cm, and 18 cm. Add the four terms to find the perimeter. Replace t_2 with $t_1 + d$, t_3 with $t_1 + 2d$, and t_4 with $t_1 + 3d$. Solve for d.

21. a) 4, 8, 12, 16, 20 **b)** $t_n = 4n$
c) 320 min

22. -29 beekeepers

23. 5.8 million carats. This value represents the increase of diamond carats mined each year.

24. 1696.5 m

25. a) 13:54, 13:59, 14:04, 14:09, 14:14; $t_1 = 13{:}54$, $d = 0{:}05$
b) $t_n = 0{:}05n + 13{:}49$
c) Assume that the arithmetic sequence of times continues.
d) 15:49

26. a) $d > 0$ b) $d < 0$
c) $d = 0$ d) t_1
e) t_n

27. Definition: An ordered list of terms in which the difference between consecutive terms is constant.
Common Difference: The difference between successive terms, $d = t_n - t_{n-1}$
Example: 12, 19, 26, …
Formula: $t_n = 7n + 5$

28. **Step 1** The graph of an arithmetic sequence is always a straight line. The common difference is described by the slope of the graph. Since the common difference is always constant, the graph will be a straight line.
Step 2
a) Changing the value of the first term changes the y-intercept of the graph. The y-intercept increases as the value of the first term increases. The y-intercept decreases as the value of the first term decreases.
b) Yes, the graph keeps it shape. The slope stays the same.

Step 3
a) Changing the value of the common difference changes the slope of the graph.
b) As the common difference increases, the slope increases. As the common difference decreases, the slope decreases.

Step 4 The common difference is the slope.
Step 5 The slope of the graph represents the common difference of the general term of the sequence. The slope is the coefficient of the variable n in the general term of the sequence.

1.2 Arithmetic Series, pages 27 to 31

1. a) 493 b) 735
c) −1081 d) $\frac{301}{3} = 100.\overline{3}$

2. a) $t_1 = 1, d = 2, S_8 = 64$
b) $t_1 = 40, d = -5, S_{11} = 165$
c) $t_1 = \frac{1}{2}, d = 1, S_7 = 24.5$
d) $t_1 = -3.5, d = 2.25, S_6 = 12.75$

3. a) 344 b) 663
c) 195 d) 396
e) 133

4. a) 2 b) $\frac{500}{13} \approx 38.46$
c) 4 d) 41

5. a) 16 b) 10

6. a) $t_{10} = 50, S_{10} = 275$
b) $t_{10} = -17, S_{10} = -35$
c) $t_{10} = -46, S_{10} = -280$
d) $t_{10} = 7, S_{10} = 47.5$

7. a) 124 500 b) 82 665

8. 156 times

9. a) 2 b) 40 c) $\frac{n}{2}(1 + 3n)$

10. 8425

11. 3 + 10 + 17 + 24

12. a)
$$S_n = \frac{n}{2}[2t_1 + (n - 1)d]$$
$$S_n = \frac{n}{2}[2(5) + (n - 1)10]$$
$$S_n = \frac{n}{2}[10 + 10n - 10]$$
$$S_n = \frac{n(10n)}{2}$$
$$S_n = \frac{10n^2}{2}$$
$$S_n = 5n^2$$
b)
$$S_{100} = \frac{100}{2}[2(5) + (100 - 1)10]$$
$$S_{100} = \frac{100}{2}[10 + 990]$$
$$S_{100} = \frac{100}{2}(1000)$$
$$S_{100} = 50\ 000$$
$$d(100) = 5(100)^2$$
$$d(100) = 5(10\ 000)$$
$$d(100) = 50\ 000$$

13. 171

14. a) the number of handshakes between six people if they each shake hands once
b) 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9
c) 435
d) Example: The number of games played in a home and away series league for n teams.

15. a) $t_1 = 6.2, d = 1.2$
b) $t_{20} = 29$
c) $S_{20} = 352$

16. 173 cm

17. a) True. Example: 2 + 4 + 6 + 8 = 20, 4 + 8 + 12 + 16 = 40, 40 = 2 × 20
b) False. Example: 2 + 4 + 6 + 8 = 20, 2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 = 72, 72 ≠ 2 × 20
c) True. Example: Given the sequence 2, 4, 6, 8, multiplying each term by 5 gives 10, 20, 30, 40. Both sequences are arithmetic sequences.

18. a) 7 + 11 + 15 b) 250 c) 250
d)
$$S_n = \frac{n}{2}[2t_1 + (n - 1)d]$$
$$S_n = \frac{n}{2}[2(7) + (n - 1)4]$$
$$S_n = \frac{n}{2}[14 + 4n - 4]$$
$$S_n = \frac{n}{2}[4n + 10]$$
$$S_n = n(2n + 5)$$
$$S_n = 2n^2 + 5n$$

19. a) $240 + 250 + 260 + \cdots + 300$
b) $S_n = 235n + 5n^2$
c) 1890
d) Nathan will continue to remove an extra 10 bushels per hour.

20. $(-27) + (-22) + (-17)$

21. Jeanette and Pierre have used two different forms of the same formula. Jeanette has replaced t_n with $t_1 + (n - 1)d$.

22. a) 100
b)
$S_{green} = 1 + 2 + 3 + \cdots + 10$
$S_{blue} = 0 + 1 + 2 + 3 + \cdots + 9$
$S_{total} = S_{green} + S_{blue}$
$S_{total} = \frac{10}{2}(1 + 10) + \frac{10}{2}(0 + 9)$
$S_{total} = 5(11) + 5(9)$
$S_{total} = 55 + 45$
$S_{total} = 100$

23. a) 55
b) The nth triangular number is represented by S_n.
$S_n = \frac{n}{2}[2t_1 + (n - 1)d]$
$S_n = \frac{n}{2}[2(1) + (n - 1)(1)]$
$S_n = \frac{n}{2}[2 + (n - 1)]$
$S_n = \frac{n}{2}(1 + n)$

1.3 Geometric Sequences, pages 39 to 45

1. a) geometric; $r = 2$; $t_n = 2^{n-1}$
b) not geometric
c) geometric; $r = -3$; $t_n = 3(-3)^{n-1}$
d) not geometric
e) geometric; $r = 1.5$; $t_n = 10(1.5)^{n-1}$
f) geometric; $r = 5$; $t_n = -1(5)^{n-1}$

2.

	Geometric Sequence	Common Ratio	6th Term	10th Term
a)	6, 18, 54, ...	3	1458	118 098
b)	1.28, 0.64, 0.32, ...	0.5	0.04	0.0025
c)	$\frac{1}{5}, \frac{3}{5}, \frac{9}{5}, \ldots$	3	$\frac{243}{5}$	$\frac{19\,683}{5}$

3. a) 2, 6, 18, 54 **b)** −3, 12, −48, 192
c) 4, −12, 36, −108 **d)** $2, 1, \frac{1}{2}, \frac{1}{4}$

4. 18.9, 44.1, 102.9

5. a) $t_n = 3(2)^{n-1}$ **b)** $t_n = 192\left(-\frac{1}{4}\right)^{n-1}$
c) $t_n = \frac{5}{9}(3)^{n-1}$ **d)** $t_n = 4(2)^{n-1}$

6. a) 4 **b)** 7 **c)** 5
d) 6 **e)** 9 **f)** 8

7. 37

8. 16, 12, 9; $t_n = 16\left(\frac{3}{4}\right)^{n-1}$

9. a) $t_1 = 3$; $r = 0.75$
b) $t_n = 3(0.75)^{n-1}$
c) approximately 53.39 cm
d) 7

10. a) 95%
b) 100, 95, 90.25, 85.7375
c) 0.95
d) about 59.87%
e) After 27 washings, 25% of the original colour would remain in the jeans. Example: The geometric sequence continues for each washing.

11. 1.77

12. a) 1, 2, 4, 8, 16 **b)** $t_n = 1(2)^{n-1}$
c) 2^{29} or 536 870 912

13. a) 1.031 **b)** 216.3 cm
c) 56 jumps

14. a) 1, 2, 4, 8, 16, 32 **b)** $t_n = 1(2)^{n-1}$
c) 2^{25} or 33 554 432
d) All cells continue to double and all cells live.

15. 2.9%

16. 8 weeks

17. 65.2 m

18. 0.920

19. a) 76.0 mL **b)** 26 h

20. a)

Time, *d* (days)	Charge Level, *C* (%)
0	100
1	98
2	96.04
3	94.12

b) $t_n = 100(0.98)^{n-1}$
c) The formula in part b) includes the first term at $d = 0$ in the sequence. The formula $C = 100(0.98)^n$ does not consider the first term of the sequence.
d) 81.7%

21. a) 24.14 mm **b)** 1107.77 mm

22. Example: If a, b, c are terms of an arithmetic sequence, then $b - a = c - b$. If 6^a, 6^b, 6^c are terms of a geometric series, then $\frac{6^b}{6^a} = \frac{6^c}{6^b}$ and $6^{b-a} = 6^{c-b}$. Therefore, $b - a = c - b$. So, when 6^a, 6^b, 6^c form a geometric sequence, then a, b, c form an arithmetic sequence.

23. $\frac{5}{3}$; 9, 15, 25

24. a) 23.96 cm **b)** 19.02 cm
c) 2.13 cm **d)** 2.01 cm
e) 2.01, 1.90, 1.79; arithmetic; $d = -0.11$ cm

25. Mala's solution is correct. Since the aquarium loses 8% of the water every day, it maintains 92% of the water every day.

26.

	$\frac{1}{500}$			$\frac{50}{3}$				
	$\frac{1}{100}$	$\frac{1}{10}$	1	10				
	$\frac{1}{20}$		2	6	18	54		
$\frac{1}{16}$	$\frac{1}{4}$	1	4			9		
	$\frac{5}{4}$		8			$\frac{3}{2}$		
	$\frac{25}{4}$		16	4	1	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{64}$
	$\frac{125}{4}$		32					
	$\frac{625}{4}$	100	64					

27. a) 0.86 cm^2 **b)** 1.72 cm^2
c) 3.43 cm^2 **d)** 109.88 cm^2

1.4 Geometric Series, pages 53 to 57

1. a) geometric; $r = 6$ **b)** geometric; $r = -\frac{1}{2}$
c) not geometric **d)** geometric; $r = 1.1$

2. a) $t_1 = 6$, $r = 1.5$, $S_{10} = \frac{174\,075}{256}$, $S_{10} \approx 679.98$

b) $t_1 = 18$, $r = -0.5$, $S_{12} = \frac{12\,285}{1024}$, $S_{12} \approx 12.00$

c) $t_1 = 2.1$, $r = 2$, $S_9 = \frac{10\,731}{10}$, $S_9 = 1073.10$

d) $t_1 = 0.3$, $r = 0.01$, $S_{12} = \frac{10}{33}$, $S_{12} \approx 0.30$

3. a) 12 276 **b)** $\frac{3280}{81}$
c) $-\frac{209\,715}{256}$ **d)** $\frac{36\,855}{256}$

4. a) 40.50 **b)** 0.96
c) 109 225 **d)** 39 063

5. a) 3 **b)** 295.7

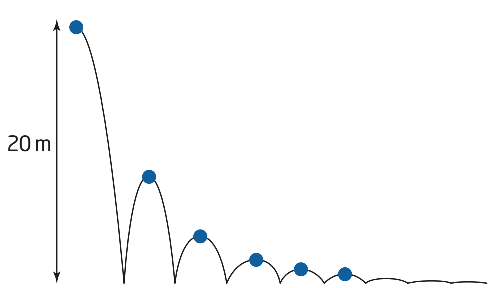
6. 7

7. a) 81 **b)** $81 + 27 + 9 + 3 + 1$

8. $t_2 = -\frac{81}{16}$; $S_6 = 7.8$

9. a) If the person in charge is included, the series is $1 + 4 + 16 + 64 + \cdots$. If the person in charge is not included, the series is $4 + 16 + 64 + \cdots$.
b) If the person in charge is included, the sum is 349 525. If the person in charge is not included, the sum is 1 398 100.

10. 46.4 m



11. 794.3 km

12. b)

Stage Number	Length of Each Line Segment	Number of Line Segments	Perimeter of Snowflake
1	1	3	3
2	$\frac{1}{3}$	12	4
3	$\frac{1}{9}$	48	$\frac{16}{3}$
4	$\frac{1}{27}$	192	$\frac{64}{9}$
5	$\frac{1}{81}$	768	$\frac{256}{27}$

c) length, $t_n = \left(\frac{1}{3}\right)^{n-1}$;
number of line segments, $t_n = 3(4)^{n-1}$;
perimeter, $t_n = 3\left(\frac{4}{3}\right)^{n-1}$

d) $\frac{1024}{81} \approx 12.64$

13. 98 739

14. 91 mm

15. a) 226.9 mg **b)** 227.3 mg

16. 8

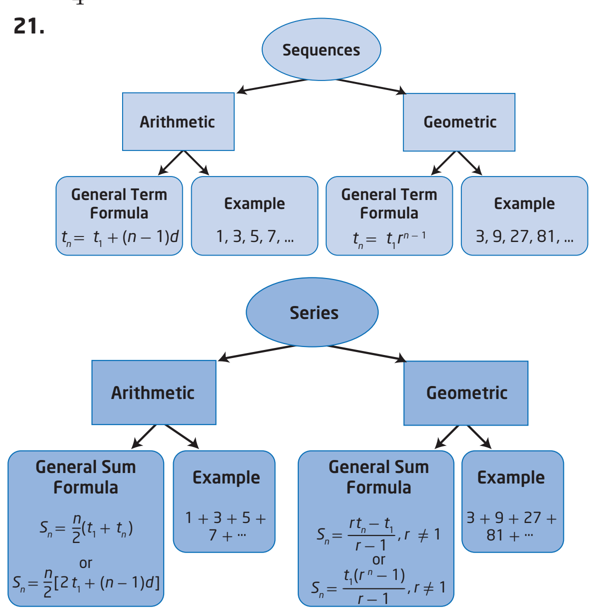
17. $\frac{58\,025}{48}$

18. $a = 5$, $b = 10$, $c = 20$ or $a = 20$, $b = 10$, $c = 5$

19. 15

20. $\frac{341}{4}\pi$

21.



22. Examples:
a) All butterflies produce the same number of eggs and all eggs hatch.
b) No. Tom determined the total number of butterflies from the first to fifth generations. He should have found the fifth term, which would determine the total number of butterflies in the fifth generation only.

c) This is a reasonable estimate, but it does include all butterflies up to the fifth generation, which is 6.42×10^7 more butterflies than those produced in the fifth generation.

d) Determine $t_5 = 1(400)^4$ or 2.56×10^{10}.

1.5 Infinite Geometric Series, pages 63 to 65

1. a) divergent **b)** convergent

c) convergent **d)** divergent

e) divergent

2. a) $\frac{32}{5}$ **b)** no sum

c) no sum **d)** 2

e) 2.5

3. a) $0.87 + 0.0087 + 0.000\,087 + \cdots$; $S_\infty = \frac{87}{99}$ or $\frac{29}{33}$

b) $0.437 + 0.000\,437 + \cdots$; $S_\infty = \frac{437}{999}$

4. Yes. The sum of the infinite series representing 0.999… is equal to 1.

5. a) 15 **b)** $\frac{4}{5}$ or 0.8

c) 14

6. $t_1 = 27$; $27 + 18 + 12 + \cdots$

7. $r = \frac{2}{5}$; $-8 - \frac{16}{5} - \frac{32}{25} - \frac{64}{125} - \cdots$

8. a) 400 000 barrels of oil

b) Determining the lifetime production assumes the oil well continues to produce at the same rate for many months. This is an unreasonable assumption because 94% is a high rate to maintain.

9. $x = \frac{1}{4}$; $1 + \frac{3}{4} + \frac{9}{16} + \frac{27}{64} + \cdots$

10. $r = \frac{1}{2}$

11. a) $-1 < x < 1$ **b)** $-3 < x < 3$

c) $-\frac{1}{2} < x < \frac{1}{2}$

12. 6 cm

13. 250 cm

14. No sum, since $r = 1.1 > 1$. Therefore, the series is divergent.

15. 48 m

16. a) approximately 170.86 cm

b) 300 cm

17. a) Rita

b) $r = -\frac{4}{3}$; therefore, $r < -1$, and the series is divergent.

18. 125 m

19. 72 cm

20. a) Example: $\frac{4}{5} + \left(\frac{4}{5}\right)^2 + \left(\frac{4}{5}\right)^3 + \cdots + \left(\frac{4}{5}\right)^n$ and $\frac{1}{5} + \left(\frac{1}{5}\right)^2 + \left(\frac{1}{5}\right)^3 + \cdots + \left(\frac{1}{5}\right)^n$

b) $S_\infty = \frac{t_1}{1 - r} = \frac{\frac{4}{5}}{1 - \frac{4}{5}} = \frac{\frac{4}{5}}{\frac{1}{5}} = 4$ and

$S_\infty = \frac{t_1}{1 - r} = \frac{\frac{1}{5}}{1 - \frac{1}{5}} = \frac{\frac{1}{5}}{\frac{4}{5}} = \frac{1}{4}$

21. Geometric series converge only when $-1 < r < 1$.

22. a) $S_n = -\frac{3}{8}n^2 + \frac{11}{8}n$

b) $S_n = \frac{\left(\frac{1}{4}\right)^n - 1}{-\frac{3}{4}}$

$S_n = -\frac{4}{3}\left(\frac{1}{4}\right)^n + \frac{4}{3}$

c) $S_\infty = \frac{1}{1 - \frac{1}{4}}$

$S_\infty = \frac{4}{3}$

23. Step 3

n	1	2	3	4
Fraction of Paper	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{64}$	$\frac{1}{256}$

Step 4 $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256}$, Example: $S_\infty = \frac{1}{3}$

Chapter 1 Review, pages 66 to 68

1. a) arithmetic, $d = 4$ **b)** arithmetic, $d = -5$

c) not arithmetic **d)** not arithmetic

2. a) C **b)** D

c) E **d)** B

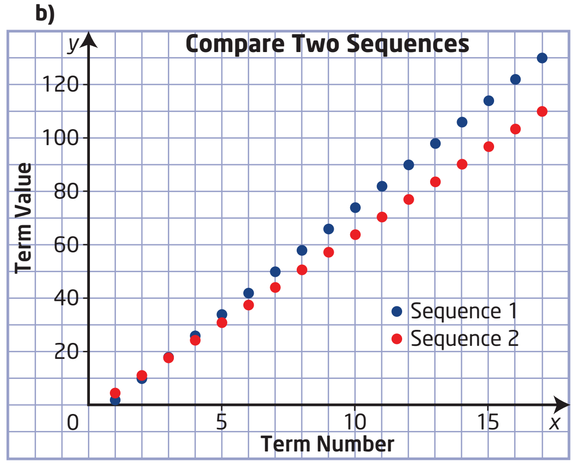
e) A

3. a) term, $n = 14$ **b)** not a term

c) term, $n = 54$ **d)** not a term

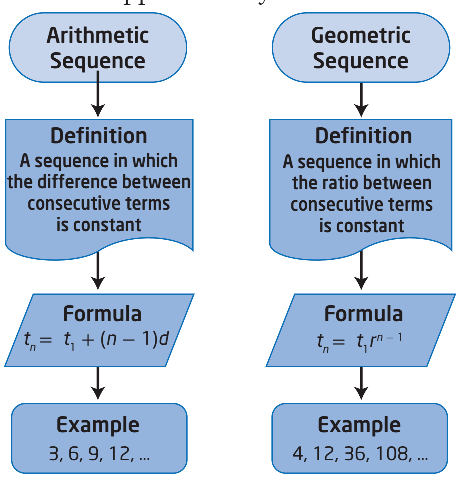
4. a) A

b)



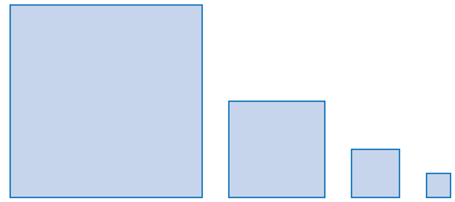
In the graph, sequence 1 has a larger positive slope than sequence 2. The value of term 17 is greater in sequence 1 than in sequence 2.

5. $t_{10} = 41$
6. 306 cm
7. a) $S_{10} = 195$ b) $S_{12} = 285$
c) $S_{10} = -75$ d) $S_{20} = 3100$
8. $S_{40} = 3420$
9. a) 29 b) 225
c) 25 days
10. a) 61 b) 495
11. 1170
12. a) not geometric
b) geometric, $r = -2$, $t_1 = 1$, $t_n = (-2)^{n-1}$
c) geometric, $r = \frac{1}{2}$, $t_1 = 1$, $t_n = \left(\frac{1}{2}\right)^{n-1}$
d) not geometric
13. a) 7346 bacteria
b) $t_n = 5000(1.08)^n$
14. 2π cm or approximately 6.28 cm
15.



16. a) arithmetic b) geometric
c) geometric d) arithmetic
e) arithmetic f) geometric
17. a) $S_{10} = \frac{174\,075}{256}$, $S_{10} \approx 679.98$
b) $S_{12} = \frac{36\,855}{1024}$, $S_{12} \approx 35.99$
c) $S_{20} = \frac{20\,000}{3}$, $S_{20} \approx 6666.67$
d) $S_9 = \frac{436\,905}{4096}$, $S_9 \approx 106.67$
18. a) 19.1 mm b) 1.37 m
19. a) $S_\infty = 15$ b) $S_\infty = \frac{3}{4}$
20. a) convergent, $S_\infty = 16$
b) divergent
c) convergent, $S_\infty = -28$
d) convergent, $S_\infty = \frac{3}{2}$
21. a) $r = -0.4$
b) $S_1 = 7$, $S_2 = 4.2$, $S_3 = 5.32$, $S_4 = 4.872$, $S_5 = 5.0512$
c) 5
d) $S_\infty = 5$

22.



a) $1, \frac{1}{4}, \frac{1}{16}, \frac{1}{64}$. Yes. The areas form a geometric sequence. The common ratio is $\frac{1}{4}$.
b) $1\frac{21}{64}$ or 1.328 125 square units
c) $\frac{4}{3}$ square units
23. a) A series is geometric if there is a common ratio r such that $r \neq 1$.
An infinite geometric series converges if $-1 < r < 1$.
An infinite geometric series diverges if $r < -1$ or $r > 1$.
b) Example:
$4 + 2 + 1 + 0.5 + \cdots$; $S_\infty = 8$
$21 - 10.5 + 5.25 - 2.625 + \cdots$; $S_\infty = 14$

Chapter 1 Practice Test, pages 69 to 70

1. D
2. B
3. B
4. B
5. C
6. 11.62 cm
7. Arithmetic sequences form straight-line graphs, where the slope is the common difference of the sequence. Geometric sequences form curved graphs.
8. $A = 15$, $B = 9$
9. 0.7 km
10. a) 5, 36, 67, 98, 129, 160
b) $t_n = 31n - 26$
c) 5, 10, 20, 40, 80, 160
d) $t_n = 5(2)^{n-1}$
11. a) 17, 34, 51, 68, 85
b) $t_n = 17n$
c) 353 million years
d) Assume that the continents continue to separate at the same rate every year.
12. a) 30 s, 60 s, 90 s, 120 s, 150 s
b) arithmetic
c) 60 days
d) 915 min

Chapter 2 Trigonometry

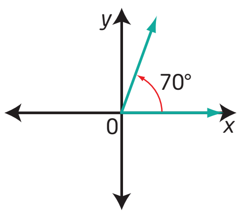
2.1 Angles in Standard Position, pages 83 to 87

1. a) No; the vertex is not at the origin.
b) Yes; the vertex is at the origin and the initial arm is on the *x*-axis.
c) No; the initial arm is not on the *x*-axis.
d) Yes; the vertex is at the origin and the initial arm is on the *x*-axis.

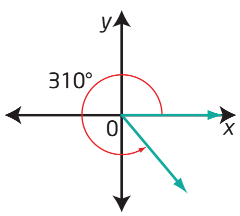
2. a) F **b)** C **c)** A
d) D **e)** B **f)** E

3. a) I **b)** IV **c)** III
d) I **e)** III **f)** II

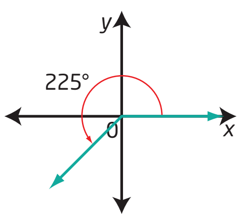
4. a)



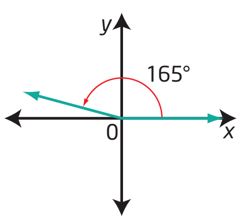
b)



c)



d)



5. a) 10° **b)** 15° **c)** 72° **d)** 35°

6. a) 135°, 225°, 315° **b)** 120°, 240°, 300°
c) 150°, 210°, 330° **d)** 105°, 255°, 285°

7. a) 288° **b)** 124° **c)** 198° **d)** 325°

8.

θ	sin θ	cos θ	tan θ
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$
45°	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

9. 159.6°

10. a) dogwood (−3.5, 2), white pine (3.5, −2), river birch (−3.5, −2)
b) red maple 30°, flowering dogwood 150°, river birch 210°, white pine 330°
c) 40 m

11. $50\sqrt{3}$ cm

12. a) $A'(x, -y)$, $A''(-x, y)$, $A'''(-x, -y)$
b) $\angle A'OC = 360° - \theta$, $\angle A''OC = 180° - \theta$, $\angle A'''OB = 180° + \theta$

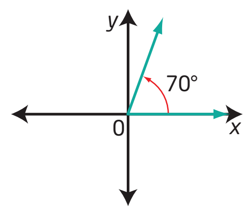
13. $(5\sqrt{3} - 5)$ m or $5(\sqrt{3} - 1)$ m

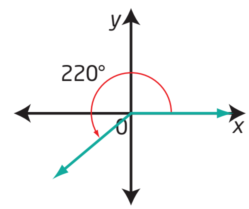
14. 252°

15. Cu (copper), Ag (silver), Au (gold), Uuu (unununium)

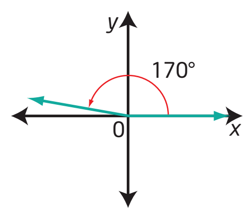
16. a) 216° **b)** 8 days **c)** 18 days

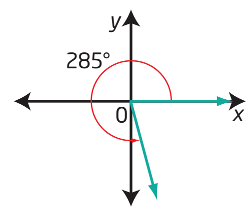
17. a) 70° **b)** 220°





c) 170° **d)** 285°





18. a)

Angle	Height (cm)
0°	12.0
15°	23.6
30°	34.5
45°	43.8
60°	51.0
75°	55.5
90°	57.0

b) A constant increase in the angle does not produce a constant increase in the height. There is no common difference between heights for each pair of angles; for example, 23.6 cm − 12 cm = 11.6 cm, 34.5 cm − 23.6 cm = 10.9 cm.
c) When θ extends beyond 90°, the heights decrease, with the height for 105° equal to the height for 75° and so on.

19. 45° and 135°

20. a) 19.56 m
b) i) 192° **ii)** 9.13 m

21. a) B **b)** D

22. $x^2 + y^2 = r^2$

23. a)

θ	20°	40°	60°	80°
sin θ	0.3420	0.6428	0.8660	0.9848
sin (180° − θ)	0.3420	0.6428	0.8660	0.9848
sin (180° + θ)	−0.3420	−0.6428	−0.8660	−0.9848
sin (360° − θ)	−0.3420	−0.6428	−0.8660	−0.9848

b) Each angle in standard position has the same reference angle, but the sine ratio differs in sign based on the quadrant location. The sine ratio is positive in quadrants I and II and negative in quadrants III and IV.

c) The ratios would be the same as those for the reference angle for cos θ and tan θ in quadrant I but may have different signs than sin θ in each of the other quadrants.

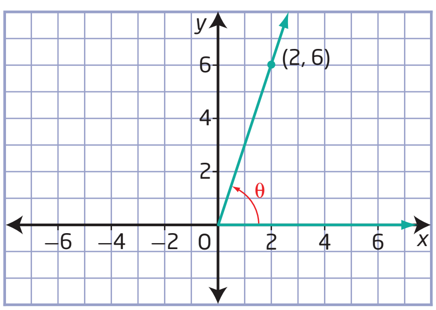
24. a) $\frac{3025\sqrt{3}}{16}$ ft

b) As the angle increases to 45° the distance increases and then decreases after 45°.

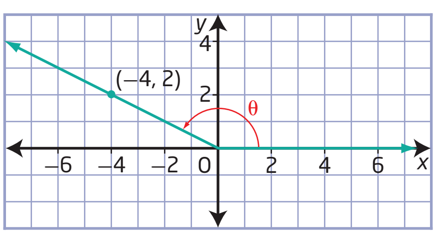
c) The greatest distance occurs with an angle of 45°. The product of cos θ and sin θ has a maximum value when θ = 45°.

2.2 Trigonometric Ratios of Any Angle, pages 96 to 99

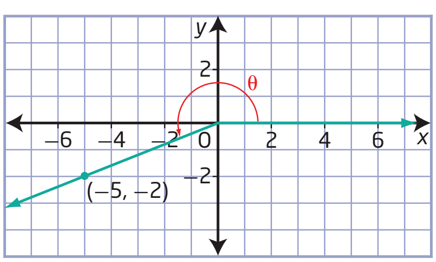
1. a)



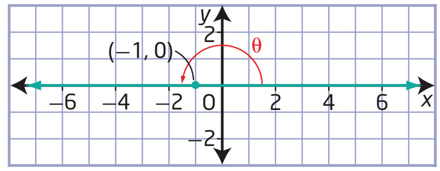
b)



c)



d)



2. a) $\sin 60° = \frac{\sqrt{3}}{2}$, $\cos 60° = \frac{1}{2}$, $\tan 60° = \sqrt{3}$

b) $\sin 225° = -\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$,

$\cos 225° = -\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$, $\tan 225° = 1$

c) $\sin 150° = \frac{1}{2}$, $\cos 150° = -\frac{\sqrt{3}}{2}$,

$\tan 150° = -\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$

d) $\sin 90° = 1$, $\cos 90° = 0$, $\tan 90°$ is undefined

3. a) $\sin \theta = \frac{4}{5}$, $\cos \theta = \frac{3}{5}$, $\tan \theta = \frac{4}{3}$

b) $\sin \theta = -\frac{5}{13}$, $\cos \theta = -\frac{12}{13}$, $\tan \theta = \frac{5}{12}$

c) $\sin \theta = -\frac{15}{17}$, $\cos \theta = \frac{8}{17}$, $\tan \theta = -\frac{15}{8}$

d) $\sin \theta = -\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$, $\cos \theta = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$,

$\tan \theta = -1$

4. a) II **b)** I **c)** III **d)** IV

5. a) $\sin \theta = \frac{12}{13}$, $\cos \theta = -\frac{5}{13}$, $\tan \theta = -\frac{12}{5}$

b) $\sin \theta = -\frac{3}{\sqrt{34}}$ or $-\frac{3\sqrt{34}}{34}$,

$\cos \theta = \frac{5}{\sqrt{34}}$ or $\frac{5\sqrt{34}}{34}$, $\tan \theta = -\frac{3}{5}$

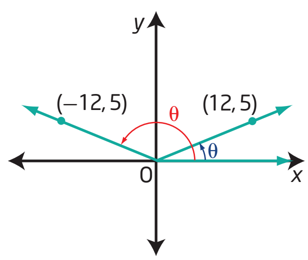
c) $\sin \theta = \frac{3}{\sqrt{45}}$ or $\frac{1}{\sqrt{5}}$, $\cos \theta = \frac{6}{\sqrt{45}}$ or $\frac{2}{\sqrt{5}}$,

$\tan \theta = \frac{1}{2}$

d) $\sin \theta = -\frac{5}{13}$, $\cos \theta = -\frac{12}{13}$, $\tan \theta = \frac{5}{12}$

6. a) positive **b)** positive
c) negative **d)** negative

7. a)



b) 23° or 157°

8. a) $\sin \theta = \frac{\sqrt{5}}{3}$, $\tan \theta = -\frac{\sqrt{5}}{2}$

b) $\cos \theta = \frac{4}{5}$, $\tan \theta = \frac{3}{4}$

c) $\sin \theta = -\frac{4}{\sqrt{41}}$ or $-\frac{4\sqrt{41}}{41}$,

$\cos \theta = \frac{5}{\sqrt{41}}$ or $\frac{5\sqrt{41}}{41}$

d) $\cos\theta = -\frac{2\sqrt{2}}{3}$, $\tan\theta = \frac{\sqrt{2}}{4}$

e) $\sin\theta = -\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$,

$\cos\theta = -\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$

9. a) 60° and 300° **b)** 135° and 225°
c) 150° and 330° **d)** 240° and 300°
e) 60° and 240° **f)** 135° and 315°

10.

θ	sin θ	cos θ	tan θ
0°	0	1	0
90°	1	0	undefined
180°	0	−1	0
270°	−1	0	undefined
360°	0	1	0

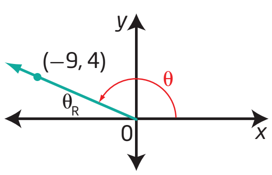
11. a) $x = -8$, $y = 6$, $r = 10$, $\sin\theta = \frac{3}{5}$,

$\cos\theta = -\frac{4}{5}$, $\tan\theta = -\frac{3}{4}$

b) $x = 5$, $y = -12$, $r = 13$, $\sin\theta = -\frac{12}{13}$,

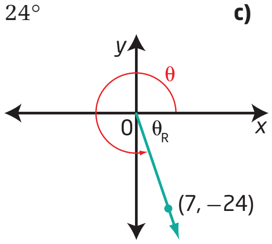
$\cos\theta = \frac{5}{13}$, $\tan\theta = -\frac{12}{5}$

12. a)



b) 24° **c)** 156°

13. a)



b) 74° **c)** 286°

14. a) $\sin\theta = \frac{2}{\sqrt{5}}$ or $\frac{2\sqrt{5}}{5}$

b) $\sin\theta = \frac{2}{\sqrt{5}}$ or $\frac{2\sqrt{5}}{5}$

c) $\sin\theta = \frac{2}{\sqrt{5}}$ or $\frac{2\sqrt{5}}{5}$

d) They all have the same sine ratio. This happens because the points P, Q, and R are collinear. They are on the same terminal arm.

15. a) 74° and 106°

b) $\sin\theta = \frac{24}{25}$, $\cos\theta = \pm\frac{7}{25}$, $\tan\theta = \pm\frac{24}{7}$

16. $\sin\theta = \frac{2\sqrt{6}}{5}$

17. $\sin 0° = 0$, $\cos 0° = 1$, $\tan 0° = 0$, $\sin 90° = 1$, $\cos 90° = 0$, $\tan 90°$ is undefined

18. a) True. θ_R for 151° is 29° and is in quadrant II. The sine ratio is positive in quadrants I and II.

b) True; both sin 225° and cos 135° have a reference angle of 45° and $\sin 45° = \cos 45° = \frac{1}{\sqrt{2}}$.

c) False; tan 135° is in quadrant II, where $\tan\theta < 0$, and tan 225° is in quadrant III, where $\tan\theta > 0$.

d) True; from the reference angles in a 30°-60°-90° triangle,

$\sin 60° = \cos 330° = \frac{\sqrt{3}}{2}$.

e) True; the terminal arms lie on the axes, passing through P(0, −1) and P(−1, 0), respectively, so $\sin 270° = \cos 180° = -1$.

19.

θ	sin θ	cos θ	tan θ
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$
45°	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	undefined
120°	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$
135°	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	$-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$	−1
150°	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$
180°	0	−1	0
210°	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$
225°	$-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$	$-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$	1
240°	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$
270°	−1	0	undefined
300°	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$
315°	$-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	−1
330°	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$
360°	0	1	0

20. a) $\angle A = 45°$, $\angle B = 135°$, $\angle C = 225°$, $\angle D = 315°$

b) $A\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, $B\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, $C\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$, $D\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

21. a)

Angle	Sine	Cosine	Tangent
0°	0	1	0
15°	0.2588	0.9659	0.2679
30°	0.5	0.8660	0.5774
45°	0.7071	0.7071	1
60°	0.8660	0.5	1.7321
75°	0.9659	0.2588	3.7321
90°	1	0	undefined
105°	0.9659	−0.2588	−3.7321
120°	0.8660	−0.5	−1.7321
135°	0.7071	−0.7071	−1
150°	0.5	−0.8660	−0.5774
165°	0.2588	−0.9659	−0.2679
180°	0	−1	0

b) As θ increases from 0° to 180°, sin θ increases from a minimum of 0 to a maximum of 1 at 90° and then decreases to 0 again at 180°. $\sin \theta = \sin (180° - \theta)$. Cos θ decreases from a maximum of 1 at 0° and continues to decrease to a minimum value of −1 at 180°. $\cos \theta = -\cos (180° - \theta)$. Tan θ increases from 0 to being undefined at 90° then back to 0 again at 180°.

c) For $0° \le \theta \le 90°$, $\cos \theta = \sin (90° - \theta)$. For $90° \le \theta \le 180°$, $\cos \theta = -\sin (\theta - 90°)$.

d) Sine ratios are positive in quadrants I and II, and both the cosine and tangent ratios are positive in quadrant I and negative in quadrant II.

e) In quadrant III, the sine and cosine ratios are negative and the tangent ratios are positive. In quadrant IV, the cosine ratios are positive and the sine and tangent ratios are negative.

22. a) $\sin \theta = \dfrac{6}{\sqrt{37}}$ or $\dfrac{6\sqrt{37}}{37}$, $\cos \theta = \dfrac{1}{\sqrt{37}}$ or $\dfrac{\sqrt{37}}{37}$, $\tan \theta = 6$

b) $\dfrac{1}{20}$

23. As θ increases from 0° to 90°, x decreases from 12 to 0, y increases from 0 to 12, sin θ increases from 0 to 1, cos θ decreases from 1 to 0, and tan θ increases from 0 to undefined.

24. $\tan \theta = \dfrac{\sqrt{1 - a^2}}{a}$

25. Since ∠BOA is 60°, the coordinates of point A are $\left(\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right)$. The coordinates of point B are (1, 0) and of point C are (−1, 0). Using the Pythagorean theorem $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$, $d_{AB} = 1$, $d_{BC} = 2$, and $d_{AC} = \sqrt{3}$.
Then, $AB^2 = 1$, $AC^2 = 3$, and $BC^2 = 4$.
So, $AB^2 + AC^2 = BC^2$.
The measures satisfy the Pythagorean Theorem, so △ABC is a right triangle and ∠CAB = 90°.
Alternatively, ∠CAB is inscribed in a semicircle and must be a right angle. Hence, △CAB is a right triangle and the Pythagorean Theorem must hold true.

26. Reference angles can determine the trigonometric ratio of any angle in quadrant I. Adjust the signs of the trigonometric ratios for quadrants II, III, and IV, considering that the sine ratio is positive in quadrant II and negative in quadrants III and IV, the cosine ratio is positive in the quadrant IV but negative in quadrants II and III, and the tangent ratio is positive in quadrant III but negative in quadrants II and IV.

27. Use the reference triangle to identify the measure of the reference angle, and then adjust for the fact that P is in quadrant III. Since $\tan \theta_R = \dfrac{9}{5}$, you can find the reference angle to be 61°. Since the angle is in quadrant III, the angle is 180° + 61° or 241°.

28. Sine is the ratio of the opposite side to the hypotenuse. The hypotenuse is the same value, r, in all four quadrants. The opposite side, y, is positive in quadrants I and II and negative in quadrants III and IV. So, there will be exactly two sine ratios with the same positive values in quadrants I and II and two sine ratios with the same negative values in quadrants III and IV.

29. θ = 240°. Both the sine ratio and the cosine ratio are negative, so the terminal arm must be in quadrant III. The value of the reference angle when $\sin \theta_R = \dfrac{\sqrt{3}}{2}$ is 60°. The angle in quadrant III is 180° + 60° or 240°.

30. **Step 4**

a) As point A moves around the circle, the sine ratio increases from 0 to 1 in quadrant I, decreases from 1 to 0 in quadrant II, decreases from 0 to −1 in quadrant III, and increases from −1 to 0 in quadrant IV. The cosine ratio decreases from 1 to 0 in quadrant I, decreases from 0 to −1 in quadrant II, increases from −1 to 0 in quadrant III, and increases from 0 to 1 in quadrant IV. The tangent ratio increases from 0 to infinity in quadrant I, is undefined for an angle of 90°, increases from negative infinity to 0 in the second quadrant, increases from 0 to positive infinity in the third quadrant, is undefined for an angle of 270°, and increases from negative infinity to 0 in quadrant IV.

b) The sine and cosine ratios are the same when A is at approximately (3.5355, 3.5355) and (−3.5355, −3.5355). This corresponds to 45° and 225°.

c) The sine ratio is positive in quadrants I and II and negative in quadrants III and IV. The cosine ratio is positive in quadrant I, negative in quadrants II and III, and positive in quadrant IV. The tangent ratio is positive in quadrant I, negative in quadrant II, positive in quadrant III, and negative in quadrant IV.

d) When the sine ratio is divided by the cosine ratio, the result is the tangent ratio. This is true for all angles as A moves around the circle.

2.3 The Sine Law, pages 108 to 113

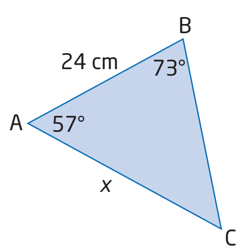
1. a) 8.9 **b)** 50.0
c) 8° **d)** 44°

2. a) 36.9 mm **b)** 50.4 m

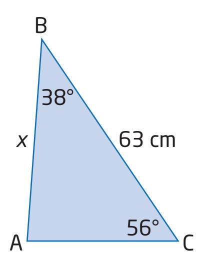
3. a) 53° **b)** 58°

4. a) $\angle C = 86°$, $\angle A = 27°$, $a = 6.0$ m or $\angle C = 94°$, $\angle A = 19°$, $a = 4.2$ m
b) $\angle C = 54°$, $c = 40.7$ m, $a = 33.6$ m
c) $\angle B = 119°$, $c = 20.9$ mm, $a = 12.4$ mm
d) $\angle B = 71°$, $c = 19.4$ cm, $a = 16.5$ cm

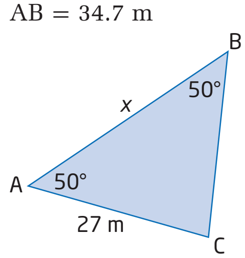
5. a) AC = 30.0 cm



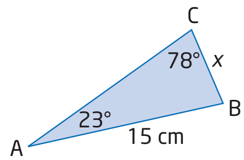
b) AB = 52.4 cm



c) AB = 34.7 m



d) BC = 6.0 cm



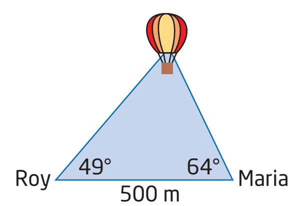
6. a) two solutions **b)** one solution
c) one solution **d)** no solutions

7. a) $a > b \sin A$, $a > h$, $b > h$
b) $a > b \sin A$, $a > h$, $a < b$
c) $a = b \sin A$, $a = h$
d) $a > b \sin A$, $a > h$, $a \geq b$

8. a) $\angle A = 48°$, $\angle B = 101°$, $b = 7.4$ cm or $\angle A = 132°$, $\angle B = 17°$, $b = 2.2$ cm
b) $\angle P = 65°$, $\angle R = 72°$, $r = 20.9$ cm or $\angle P = 115°$, $\angle R = 22°$, $r = 8.2$ cm
c) no solutions

9. a) $a \geq 120$ cm **b)** $a = 52.6$ cm
c) 52.6 cm $< a <$ 120 cm
d) $a < 52.6$ cm

10. a)



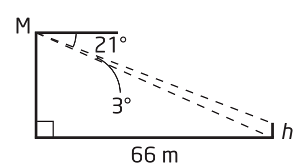
b) 409.9 m

11. 364.7 m

12. 41°

13. 4.5 m

14. a)

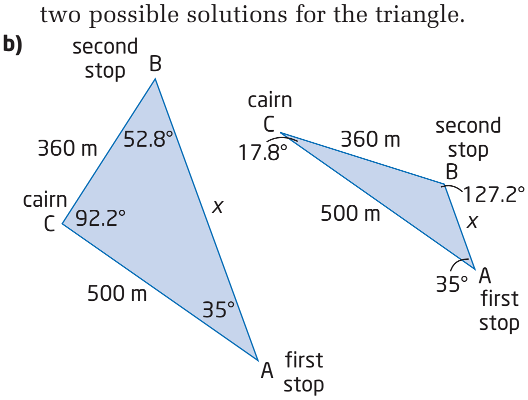


b) 4.1 m **c)** 72.2 m

15. a) 1.51 Å **b)** 0.0151 mm

16. least wingspan 9.1 m, greatest wingspan 9.3 m

17. a) Since $a < b$ (360 < 500) and $a > b \sin A$ (360 > 500 sin 35°), there are two possible solutions for the triangle.
b)



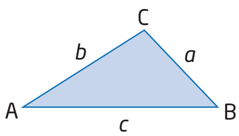
c) Armand's second stop could be either 191.9 m or 627.2 m from his first stop.

18. 911.6 m

19.

Statements	Reasons
$\sin C = \frac{h}{b}$ $\quad \sin B = \frac{h}{c}$	sin B ratio in △ABD sin C ratio in △ACD
$h = b \sin C$ $h = c \sin B$	Solve each ratio for h.
$b \sin C = c \sin B$	Equivalence property or substitution
$\frac{\sin C}{c} = \frac{\sin B}{b}$	Divide both sides by bc.

20.



Given ∠A = ∠B, prove that side AC = BC, or $a = b$.

Using the sine law,

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

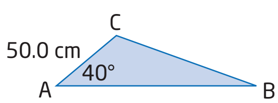
But ∠A = ∠B, so sin A = sin B.

Then, $\frac{a}{\sin A} = \frac{b}{\sin A}$.

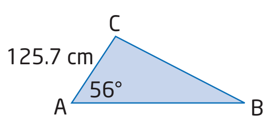
So, $a = b$.

21. 14.1 km^2

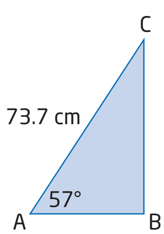
22. a) 32.1 cm $< a <$ 50.0 cm



b) $a <$ 104.2 cm



c) a = 61.8 cm



23. 166.7 m

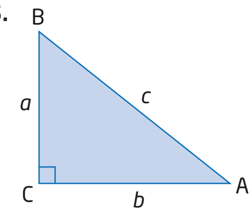
24. a) There is no known side opposite a known angle.

b) There is no known angle opposite a known side.

c) There is no known side opposite a known angle.

d) There is no known angle and only one known side.

25.



In △ABC,

$\sin A = \frac{a}{c}$ and $\sin B = \frac{b}{c}$

Thus, $c = \frac{a}{\sin A}$ and $c = \frac{b}{\sin B}$.

Then, $\frac{a}{\sin A} = \frac{b}{\sin B}$.

This is only true for a right triangle and does not show a proof for oblique triangles.

26. a) 12.9 cm

b) $(4\sqrt{5} + 4)$ cm or $4(\sqrt{5} + 1)$ cm

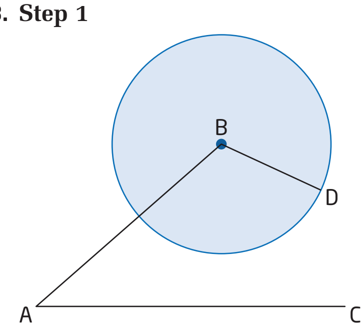
c) 4.9 cm

d) 3.1 cm

e) The spiral is created by connecting the 36° angle vertices for the reducing golden triangles.

27. Concept maps will vary.

28. Step 1

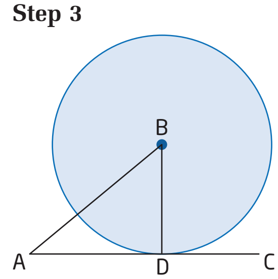


Step 2

a) No.

b) There are no triangles formed when BD is less than the distance from B to the line AC.

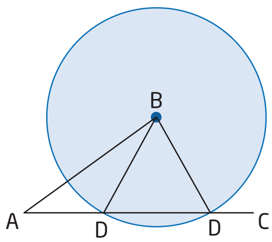
Step 3



a) Yes.

b) One triangle can be formed when BD equals the distance from B to the line AC.

Step 4



a) Yes.
b) Two triangles can be formed when BD is greater than the distance from B to the line AC.

Step 5
a) Yes.
b) One triangle is formed when BD is greater than the length AB.

Step 6 The conjectures will work so long as $\angle A$ is an acute angle. The relationship changes when $\angle A > 90°$.

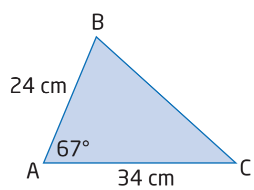
2.4 The Cosine Law, pages 119 to 125

1. a) 6.0 cm **b)** 21.0 mm **c)** 45.0 m

2. a) $\angle J = 34°$ **b)** $\angle L = 55°$
c) $\angle P = 137°$ **d)** $\angle C = 139°$

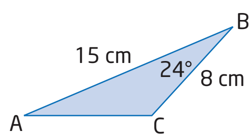
3. a) $\angle Q = 62°$, $\angle R = 66°$, $p = 25.0$ km
b) $\angle S = 100°$, $\angle R = 33°$, $\angle T = 47°$

4. a)



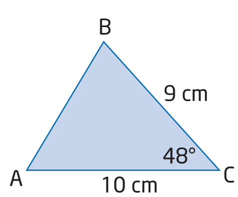
BC = 33.1 cm

b)



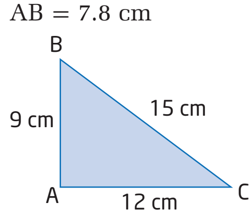
AC = 8.4 m

c)



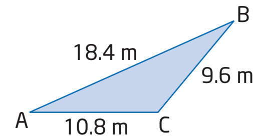
AB = 7.8 cm

d)



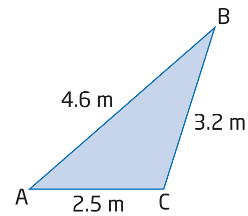
$\angle B = 53°$

e)



$\angle A = 24°$

f)



$\angle C = 107°$

5. a) Use the cosine law because three sides are given (SSS). There is no given angle and opposite side to be able to use the sine law.
b) Use the sine law because two angles and an opposite side are given.
c) Use the cosine law to find the missing side length. Then, use the sine law to find the indicated angle.

6. a) 22.6 cm
b) 7.2 m

7. 53.4 cm

8. 2906 m

9. The angles between the buoys are 35°, 88°, and 57°.

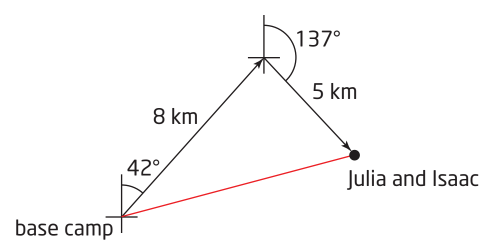
10. 4.2°

11. 22.4 km

12. 54.4 km

13. 458.5 cm

14. a)



b) 9.1 km
c) 255°

15. 9.7 m

16. Use the cosine law in each oblique triangle to find the measure of each obtuse angle. These three angles meet at a point and should sum to 360°. The three angles are 118°, 143°, and 99°. Since $118° + 143° + 99° = 360°$, the side measures are accurate.

17. The interior angles of the bike frame are 73°, 62°, and 45°.

18. 98.48 m

19. 1546 km

20. 438.1 m

21. The interior angles of the building are 65°, 32°, and 83°.

22.

Statement	Reason
$c^2 = (a - x)^2 + h^2$	Use the Pythagorean Theorem in △ABD.
$c^2 = a^2 - 2ax + x^2 + h^2$	Expand the square of a binomial.
$b^2 = x^2 + h^2$	Use the Pythagorean Theorem in △ACD.
$c^2 = a^2 - 2ax + b^2$	Substitute b^2 for $x^2 + h^2$.
$\cos C = \frac{x}{b}$	Use the cosine ratio in △ACD.
$x = b \cos C$	Multiply both sides by b.
$c^2 = a^2 - 2ab \cos C + b^2$	Substitute $b \cos C$ for x in step 4.
$c^2 = a^2 + b^2 - 2ab \cos C$	Rearrange.

23. 36.2 km

24. No. The three given lengths cannot be arranged to form a triangle ($a^2 + b^2 < c^2$). When using the cosine law, the cosines of the angles are either greater than 1 or less than −1, which is impossible.

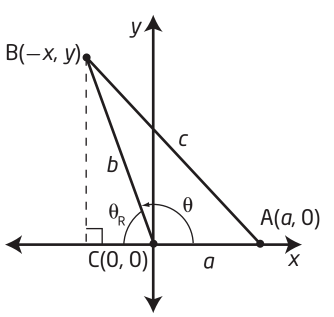
25. 21.2 cm

26. ∠ABC = 65°, ∠ACD = 97°

27. 596 km²

28. 2.1 m

29.



$$\cos \theta_R = -\cos \theta = -\frac{x}{\sqrt{x^2 + y^2}}$$

$$b = \sqrt{x^2 + y^2}$$

$$c = \sqrt{(a + x)^2 + y^2}$$

Prove that $c^2 = a^2 + b^2 - 2ab \cos C$:

$$\begin{aligned}\text{Left Side} &= \left(\sqrt{(a + x)^2 + y^2}\right)^2 \\ &= (a + x)^2 + y^2 \\ &= a^2 + 2ax + x^2 + y^2\end{aligned}$$

$$\begin{aligned}\text{Right Side} &= a^2 + \left(\sqrt{x^2 + y^2}\right)^2 \\ &\quad - 2a\left(\sqrt{x^2 + y^2}\right)\left(-\frac{x}{\sqrt{x^2 + y^2}}\right) \\ &= a^2 + x^2 + y^2 + 2ax \\ &= a^2 + 2ax + x^2 + y^2\end{aligned}$$

Left Side = Right Side

Therefore, the cosine law is true.

30. 115.5 m

31. **a)** 228.05 cm²

b) 228.05 cm²

c) These methods give the same measure when ∠C = 90°.

d) Since cos 90° = 0, $2ab \cos 90° = 0$, so $a^2 + b^2 - 2ab \cos 90° = a^2 + b^2$. Therefore, c^2 can be found using the cosine law or the Pythagorean Theorem when there is a right triangle.

32.

Concept Summary for Solving a Triangle	
Given	**Begin by Using the Method of**
Right triangle	A
Two angles and any side	B
Three sides	C
Three angles	D
Two sides and the included angle	C
Two sides and the angle opposite one of them	B

33. **Step 2**

a) ∠A = 29°, ∠B = 104°, ∠C = 47°

b) The angles at each vertex of a square are 90°. Therefore,
360° = ∠ABC + 90° + ∠GBF + 90°
180° = ∠ABC + ∠GBF
∠GBF = 76°, ∠HCI = 133°, ∠DAE = 151°

c) GF = 6.4 cm, ED = 13.6 cm, HI = 11.1 cm

Step 3

a) For △HCI, the altitude from C to HI is 2.1 cm. For △AED, the altitude from A to DE is 1.6 cm. For △BGF, the altitude from B to GF is 3.6 cm. For △ABC, the altitude from B to AC is 2.9 cm.

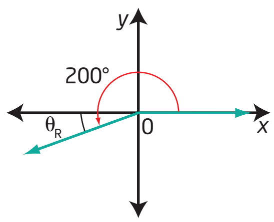
b) area of △ABC is 11.7 cm², area of △BGF is 11.7 cm², area of △AED is 11.7 cm², area of △HCI is 11.7 cm²

Step 4 All four triangles have the same area. Since you use reference angles to determine the altitudes, the product of $\frac{1}{2}bh$ will determine the same area for all triangles. This works for any triangle.

Chapter 2 Review, pages 126 to 128

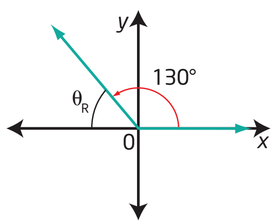
1. a) E **b)** D **c)** B **d)** A
e) F **f)** C **g)** G

2. a)



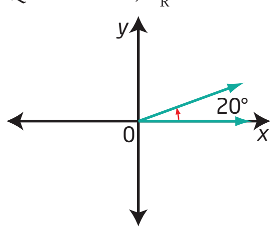
Quadrant III, $\theta_R = 20°$

b)



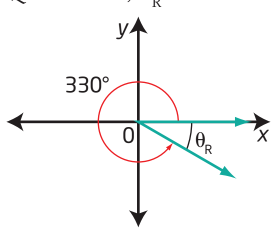
Quadrant II, $\theta_R = 50°$

c)



Quadrant I, $\theta_R = 20°$

d)



Quadrant IV, $\theta_R = 30°$

3. No. Reference angles are measured from the x-axis. The reference angle is 60°.

4. quadrant I: $\theta = 35°$, quadrant II: $\theta = 180° - 35°$ or 145°, quadrant III: $\theta = 180° + 35°$ or 215°, quadrant IV: $\theta = 360° - 35°$ or 325°

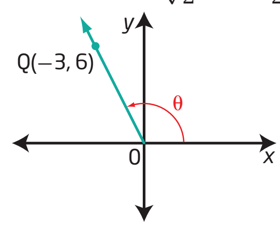
5. a) $\sin 225° = -\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$,
$\cos 225° = -\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$, $\tan 225° = 1$

b) $\sin 120° = \frac{\sqrt{3}}{2}$, $\cos 120° = -\frac{1}{2}$,
$\tan 120° = -\sqrt{3}$

c) $\sin 330° = -\frac{1}{2}$, $\cos 330° = \frac{\sqrt{3}}{2}$,
$\tan 330° = -\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$

d) $\sin 135° = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$,
$\cos 135° = -\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$, $\tan 135° = -1$

6. a)



b) $\sqrt{45}$ or $3\sqrt{5}$

c) $\sin\theta = \frac{6}{\sqrt{45}}$ or $\frac{2\sqrt{5}}{5}$,
$\cos\theta = -\frac{3}{\sqrt{45}}$ or $-\frac{\sqrt{5}}{5}$, $\tan\theta = -2$

d) 117°

7. (2, 5), (−2, 5), (−2, −5)

8. a) $\sin 90° = 1$, $\cos 90° = 0$, $\tan 90°$ is undefined

b) $\sin 180° = 0$, $\cos 180° = -1$, $\tan 180° = 0$

9. a) $\cos\theta = -\frac{4}{5}$, $\tan\theta = \frac{3}{4}$

b) $\sin\theta = -\frac{\sqrt{8}}{3}$ or $-\frac{2\sqrt{2}}{3}$,
$\tan\theta = -\sqrt{8}$ or $-2\sqrt{2}$

c) $\sin\theta = \frac{12}{13}$, $\cos\theta = \frac{5}{13}$

10. a) 130° or 310° **b)** 200° or 340°
c) 70° or 290°

11. a) Yes; there is a known angle (180° − 18° − 114° = 48°) and a known opposite side (3 cm), plus another known angle.

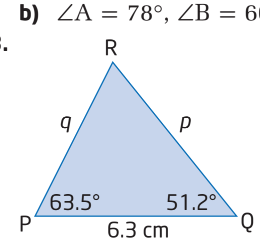
b) Yes; there is a known angle (90°) and opposite side (32 cm), plus one other known side.

c) No; there is no known angle or opposite side.

12. a) $\angle C = 57°$, $c = 36.9$ mm

b) $\angle A = 78°$, $\angle B = 60°$

13.

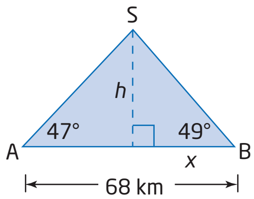


$\angle R = 65.3°$, $q = 5.4$ cm, $p = 6.2$ cm

14. 2.8 km

15. a) Ship B, 50.0 km

b)



Use $\tan 49° = \frac{h}{x}$ and $\tan 47° = \frac{h}{68 - x}$.

Solve $x \tan 49° = (68 - x) \tan 47°$.

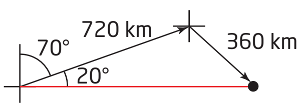
$x = 32.8$ km

Then, use $\cos 49° = \frac{32.8}{BS}$ and $\cos 47° = \frac{35.2}{AS}$ to find BS and AS.

AS = 51.6 km, BS = 50.0 km

16. no solutions if $a < b \sin A$, one solution if $a = b \sin A$ or if $a \geq b$, and two solutions if $b > a > b \sin A$

17. a)



b) 47° E of S

c) 939.2 km

18. a) The three sides do not meet to form a triangle since $4 + 2 < 7$.

b) $\angle A + \angle C > 180°$

c) Sides a and c lie on top of side b, so no triangle is formed.

d) $\angle A + \angle B + \angle C < 180°$

19. a) sine law; there is a known angle and a known opposite side plus another known angle

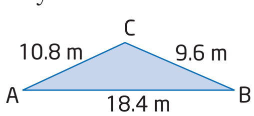
b) cosine law; there is a known SAS (side-angle-side)

20. a) $a = 29.1$ cm

b) $\angle B = 57°$

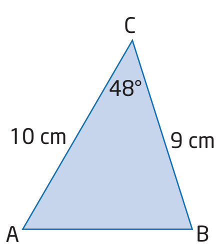
21. 170.5 yd

22. a)



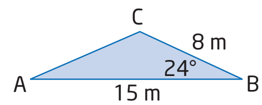
$\angle A = 24°$

b)



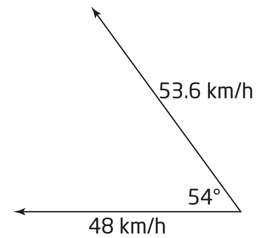
AB = 7.8 cm

c)



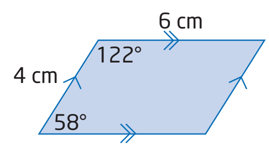
$\angle A = 23°$, $\angle C = 133°$, AC = 8.4 m

23. a)



b) 185.6 km

24. a)



b) 8.8 cm and 5.2 cm

Chapter 2 Practice Test, pages 129 to 130

1. A

2. A

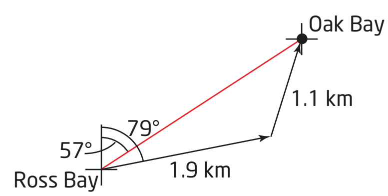
3. C

4. B

5. C

6. −6

7. a)



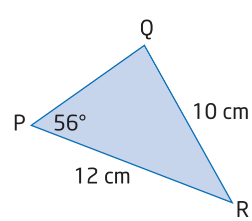
b) 2.6 km

8. a) two

b) $\angle B = 53°$, $\angle C = 97°$, $c = 19.9$ or $\angle B = 127°$, $\angle C = 23°$, $c = 7.8$

9. $\angle R = 17°$

10. a)



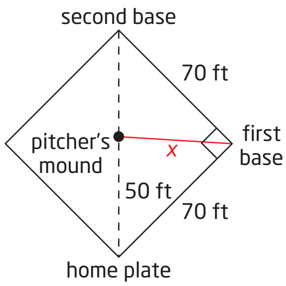
b) $\angle R = 40°$, $\angle Q = 84°$, $r = 7.8$ cm or $\angle R = 28°$, $\angle Q = 96°$, $r = 5.7$ cm

11. 5.2 cm

12. a) 44° **b)** 56° **c)** 1.7 m

13. quadrant I: $\theta = \theta_R$, quadrant II: $\theta = 180° - \theta_R$, quadrant III: $\theta = 180° + \theta_R$, quadrant IV: $\theta = 360° - \theta_R$

14. a)



b)
$$a^2 + b^2 = c^2$$
$$70^2 + 70^2 = c^2$$
$$c = 99$$
Second base to pitcher's mound is 99 − 50 or 49 ft.
Distance from first base to pitcher's mound is $x^2 = 50^2 + 70^2 - 2(50)(70)\cos 45°$ or 49.5 ft.

15. Use the sine law when the given information includes a known angle and a known opposite side, plus one other known side or angle. Use the cosine law when given oblique triangles with known SSS or SAS.

16. patio triangle: 38°, 25°, 2.5 m; shrubs triangle: 55°, 2.7 m, 3.0 m

17. 3.1 km

Cumulative Review, Chapters 1–2, pages 133 to 135

1. a) A **b)** D **c)** E **d)** C **e)** B

2. a) geometric, $r = \frac{2}{3}$; $\frac{16}{3}, \frac{32}{9}, \frac{64}{27}$

b) arithmetic, $d = -3$; 5, 2, −1

c) arithmetic, $d = 5$; −1, 4, 9

d) geometric, $r = -2$; 48, −96, 192

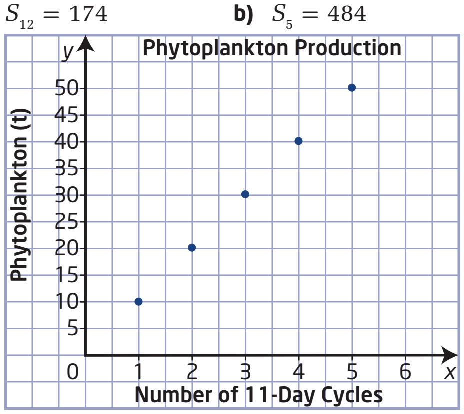
3. a) $t_n = -3n + 21$

b) $t_n = \frac{3}{2}n - \frac{1}{2}$

4. $t_n = 2(-2)^{n-1} \Rightarrow t_{20} = -2^{20}$ or −1 048 576

5. a) $S_{12} = 174$ **b)** $S_5 = 484$

6. a)



b) $t_n = 10n$

c) The general term is a linear equation with a slope of 10.

7. 201 m

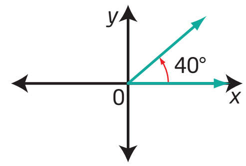
8. a) $r = 0.1$, $S_\infty = 1$

b) Answers will vary.

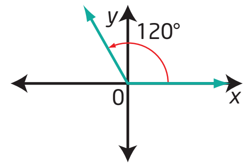
9. $2\sqrt{5}$

10. $\sin\theta = \frac{8}{17}$, $\cos\theta = \frac{15}{17}$, $\tan\theta = \frac{8}{15}$

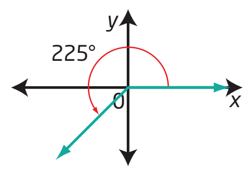
11. a) 40°



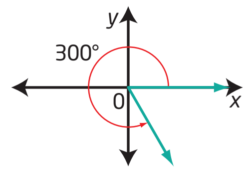
b) 60°



c) 45°

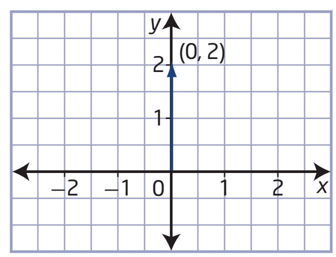


d) 60°



12. a) 90°

b)



c) $\sin\theta = 1$, $\cos\theta = 0$, $\tan\theta$ is undefined

13. a) $\sin 405° = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$

b) $\cos 330° = \frac{\sqrt{3}}{2}$

c) $\tan 225° = 1$

d) $\cos 180° = -1$

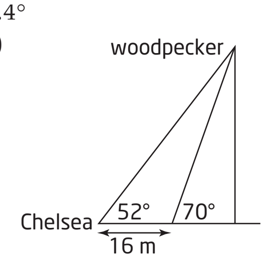
e) $\tan 150° = = -\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$

f) $\sin 270° = -1$

14. The bear is 8.9 km from station A and 7.4 km from station B.

15. 9.4°

16. a)



b) 40.8 m

17. 134.4°

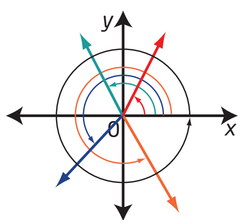
Unit 1 Test, pages 136 to 137

1. B

2. C

3. D

4. C
5. D
6. \$0.15 per cup
7. 45°
8. 300°
9. 2775
10. a) 5 b) −6
c) $t_n = 5n - 11$ d) $S_{10} = 165$
11. \$14 880.35
12. 4 km
13. a) 64, 32, 16, 8, … b) $t_n = 64\left(\frac{1}{2}\right)^{n-1}$
c) 63 games
14. a)



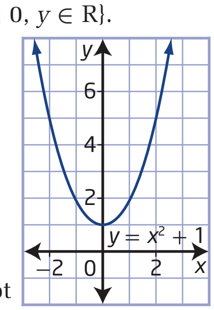
b) 60, 120, 180, 240, 300, 360
c) $t_n = 60n$
15. a) 58° b) 5.3 m
16. 38°

Chapter 3 Quadratic Functions

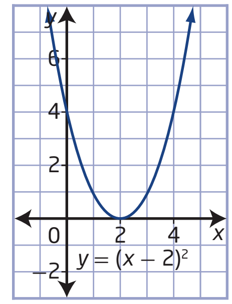
3.1 Investigating Quadratic Functions in Vertex Form, pages 157 to 162

1. a) Since $a > 0$ in $f(x) = 7x^2$, the graph opens upward, has a minimum value, and has a range of $\{y \mid y \geq 0, y \in R\}$.
b) Since $a > 0$ in $f(x) = \frac{1}{6}x^2$, the graph opens upward, has a minimum value, and has a range of $\{y \mid y \geq 0, y \in R\}$.
c) Since $a < 0$ in $f(x) = -4x^2$, the graph opens downward, has a maximum value, and has a range of $\{y \mid y \leq 0, y \in R\}$.
d) Since $a < 0$ in $f(x) = -0.2x^2$, the graph opens downward, has a maximum value, and has a range of $\{y \mid y \leq 0, y \in R\}$.

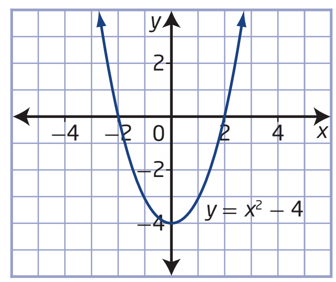
2. a) The shapes of the graphs are the same with the parabola of $y = x^2 + 1$ being one unit higher. vertex: (0, 1), axis of symmetry: $x = 0$, domain: $\{x \mid x \in R\}$, range: $\{y \mid y \geq 1, y \in R\}$, no x-intercepts, y-intercept occurs at (0, 1)



b) The shapes of the graphs are the same with the parabola of $y = (x - 2)^2$ being two units to the right. vertex: (2, 0), axis of symmetry: $x = 2$, domain: $\{x \mid x \in R\}$, range: $\{y \mid y \geq 0, y \in R\}$, x-intercept occurs at (2, 0), y-intercept occurs at (0, 4)

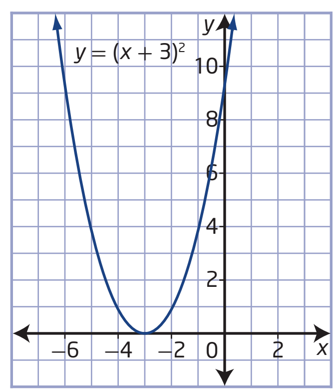


c) The shapes of the graphs are the same with the parabola of $y = x^2 - 4$ being four units lower.



vertex: (0, −4), axis of symmetry: $x = 0$, domain: $\{x \mid x \in R\}$, range: $\{y \mid y \geq -4, y \in R\}$, x-intercepts occur at (−2, 0) and (2, 0), y-intercept occurs at (0, −4)

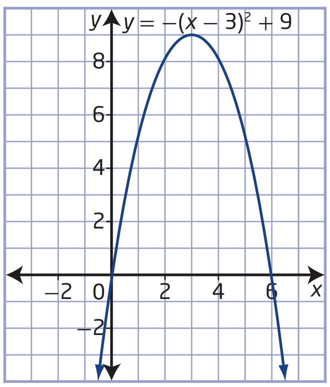
d) The shapes of the graphs are the same with the parabola of $y = (x + 3)^2$ being three units to the left.



vertex: (−3, 0), axis of symmetry: $x = -3$, domain: $\{x \mid x \in R\}$, range: $\{y \mid y \geq 0, y \in R\}$, x-intercept occurs at (−3, 0), y-intercept occurs at (0, 9)

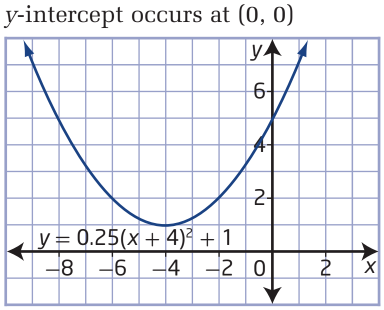
3. **a)** Given the graph of $y = x^2$, move the entire graph 5 units to the left and 11 units up.
b) Given the graph of $y = x^2$, apply the change in width, which is a multiplication of the y-values by a factor of 3, making it narrower, reflect it in the x-axis so it opens downward, and move the entire new graph down 10 units.
c) Given the graph of $y = x^2$, apply the change in width, which is a multiplication of the y-values by a factor of 5, making it narrower. Move the entire new graph 20 units to the left and 21 units down.
d) Given the graph of $y = x^2$, apply the change in width, which is a multiplication of the y-values by a factor of $\frac{1}{8}$, making it wider, reflect it in the x-axis so it opens downward, and move the entire new graph 5.6 units to the right and 13.8 units up.

4. **a)**



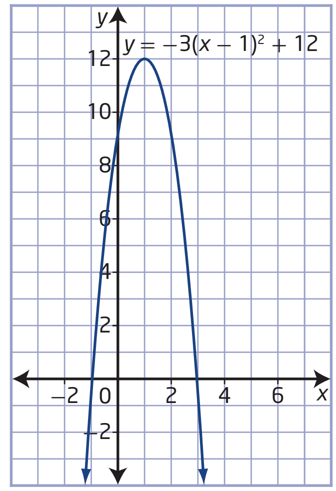
vertex: (3, 9), axis of symmetry: $x = 3$, opens downward, maximum value of 9, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \leq 9, y \in \mathrm{R}\}$, x-intercepts occur at (0, 0) and (6, 0), y-intercept occurs at (0, 0)

b)



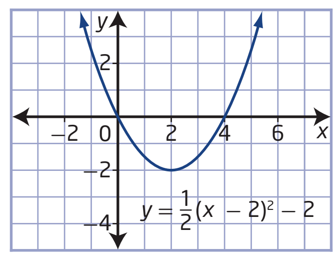
vertex: $(-4, 1)$, axis of symmetry: $x = -4$, opens upward, minimum value of 1, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \geq 1, y \in \mathrm{R}\}$, no x-intercepts, y-intercept occurs at (0, 5)

c)



vertex: (1, 12), axis of symmetry: $x = 1$, opens downward, maximum value of 12, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \leq 12, y \in \mathrm{R}\}$, x-intercepts occur at $(-1, 0)$ and (3, 0), y-intercept occurs at (0, 9)

d)



vertex: $(2, -2)$, axis of symmetry: $x = 2$, opens upward, minimum value of -2, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \geq -2, y \in \mathrm{R}\}$, x-intercepts occur at (0, 0) and (4, 0), y-intercept occurs at (0, 0)

5. **a)** $y_1 = x^2$, $y_2 = 4x^2 + 2$, $y_3 = \frac{1}{2}x^2 - 2$, $y_4 = \frac{1}{4}x^2 - 4$
b) $y_1 = -x^2$, $y_2 = -4x^2 + 2$, $y_3 = -\frac{1}{2}x^2 - 2$, $y_4 = -\frac{1}{4}x^2 - 4$
c) $y_1 = (x + 4)^2$, $y_2 = 4(x + 4)^2 + 2$, $y_3 = \frac{1}{2}(x + 4)^2 - 2$, $y_4 = \frac{1}{4}(x + 4)^2 - 4$
d) $y_1 = x^2 - 2$, $y_2 = 4x^2$, $y_3 = \frac{1}{2}x^2 - 4$, $y_4 = \frac{1}{4}x^2 - 6$

6. For the function $f(x) = 5(x - 15)^2 - 100$, $a = 5$, $p = 15$, and $q = -100$.
a) The vertex is located at (p, q), or $(15, -100)$.
b) The equation of the axis of symmetry is $x = p$, or $x = 15$.
c) Since $a > 0$, the graph opens upward.

d) Since $a > 0$, the graph has a minimum value of q, or -100.

e) The domain is $\{x \mid x \in \mathrm{R}\}$. Since the function has a minimum value of -100, the range is $\{y \mid y \geq -100, y \in \mathrm{R}\}$.

f) Since the graph has a minimum value of -100 and opens upward, there are two x-intercepts.

7. a) vertex: $(0, 14)$, axis of symmetry: $x = 0$, opens downward, maximum value of 14, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \leq 14, y \in \mathrm{R}\}$, two x-intercepts

b) vertex: $(-18, -8)$, axis of symmetry: $x = -18$, opens upward, minimum value of -8, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \geq -8, y \in \mathrm{R}\}$, two x-intercepts

c) vertex: $(7, 0)$, axis of symmetry: $x = 7$, opens upward, minimum value of 0, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \geq 0, y \in \mathrm{R}\}$, one x-intercept

d) vertex: $(-4, -36)$, axis of symmetry: $x = -4$, opens downward, maximum value of -36, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \leq -36, y \in \mathrm{R}\}$, no x-intercepts

8. a) $y = (x + 3)^2 - 4$ **b)** $y = -2(x - 1)^2 + 12$

c) $y = \frac{1}{2}(x - 3)^2 + 1$ **d)** $y = -\frac{1}{4}(x + 3)^2 + 4$

9. a) $y = -\frac{1}{4}x^2$ **b)** $y = 3x^2 - 6$

c) $y = -4(x - 2)^2 + 5$ **d)** $y = \frac{1}{5}(x + 3)^2 - 10$

10. a) $(4, 16) \rightarrow (-1, 16) \rightarrow (-1, 24)$

b) $(4, 16) \rightarrow (4, 4) \rightarrow (4, -4)$

c) $(4, 16) \rightarrow (4, -16) \rightarrow (14, -16)$

d) $(4, 16) \rightarrow (4, 48) \rightarrow (4, 40)$

11. Starting with the graph of $y = x^2$, apply the change in width, which is a multiplication of the y-values by a factor of 5, reflect the graph in the x-axis, and then move the entire graph up 20 units.

12. Example: Quadratic functions will always have one y-intercept. Since the graphs always open upward or downward and have a domain of $\{x \mid x \in \mathrm{R}\}$, the parabola will always cross the y-axis. The graphs must always have a value at $x = 0$ and therefore have one y-intercept.

13. a) $y = \frac{1}{30}x^2$

b) The new function could be $y = \frac{1}{30}(x - 30)^2 - 30$ or $y = \frac{1}{30}(x + 30)^2 - 30$. Both graphs have the same size and shape, but the new function has been transformed by a horizontal translation of 30 units to the right or to the left and a vertical translation of 30 units down to represent a point on the edge as the origin.

14. a) The vertex is located at (36, 20 000), it opens downward, and it has a change in width by a multiplication of the y-values by a factor of 2.5 of the graph $y = x^2$. The equation of the axis of symmetry is $x = 36$, and the graph has a maximum value of 20 000.

b) 36 times

c) 20 000 people

15. Examples: If the vertex is at the origin, the quadratic function will be $y = 0.03x^2$. If the edge of the rim is at the origin, the quadratic function will be $y = 0.03(x - 20)^2 - 12$.

16. a) Example: Placing the vertex at the origin, the quadratic function is $y = \frac{1}{294}x^2$ or $y \approx 0.0034x^2$.

b) Example: If the origin is at the top of the left tower, the quadratic function is $y = \frac{1}{294}(x - 84)^2 - 24$ or $y \approx 0.0034(x - 84)^2 - 24$. If the origin is at the top of the right tower, the quadratic function is $y = \frac{1}{294}(x + 84)^2 - 24$ or $y \approx 0.0034(x + 84)^2 - 24$.

c) 8.17 m; this is the same no matter which function is used.

17. $y = -\frac{9}{121}(x - 11)^2 + 9$

18. $y = -\frac{1}{40}(x - 60)^2 + 90$

19. Example: Adding q is done after squaring the x-value, so the transformation applies directly to the parabola $y = x^2$. The value of p is added or subtracted before squaring, so the shift is opposite to the sign in the bracket to get back to the original y-value for the graph of $y = x^2$.

20. a) $y = -\frac{7}{160\,000}(x - 8000)^2 + 10\,000$

b) domain: $\{x \mid 0 \leq x \leq 16\,000, x \in \mathrm{R}\}$, range: $\{y \mid 7200 \leq y \leq 10\,000, y \in \mathrm{R}\}$

21. a) Since the vertex is located at (6, 30), $p = 6$ and $q = 30$. Substituting these values into the vertex form of a quadratic function and using the coordinates of the given point, the function is $y = -1.5(x - 6)^2 + 30$.

b) Knowing that the x-intercepts are -21 and -5, the equation of the axis of symmetry must be $x = -13$. Then, the vertex is located at $(-13, -24)$. Substituting the coordinates of the vertex and one of the x-intercepts into the vertex form, the quadratic function is $y = 0.375(x + 13)^2 - 24$.

22. a) Examples: I chose $x = 8$ as the axis of symmetry, I choose the position of the hoop to be (1, 10), and I allowed the basketball to be released at various heights (6 ft, 7 ft, and 8 ft) from a distance of 16 ft from the hoop. For each scenario, substitute the coordinates of the release point into the function $y = a(x - 8)^2 + q$ to get an expression for q. Then, substitute the expression for q and the coordinates of the hoop into the function. My three functions are
$y = -\frac{4}{15}(x - 8)^2 + \frac{346}{15}$,
$y = -\frac{3}{15}(x - 8)^2 + \frac{297}{15}$, and
$y = -\frac{2}{15}(x - 8)^2 + \frac{248}{15}$.

b) Example: $y = -\frac{4}{15}(x - 8)^2 + \frac{346}{15}$ ensures that the ball passes easily through the hoop.

c) domain: $\{x \mid 0 \le x \le 16, x \in \mathrm{R}\}$, range: $\left\{y \mid 0 \le y \le \frac{346}{15}, y \in \mathrm{R}\right\}$

23. $(m + p, an + q)$

24. Examples:

a) $f(x) = -2(x - 1)^2 + 3$

b) Plot the vertex (1, 3). Determine a point on the curve, say the y-intercept, which occurs at (0, 1). Determine that the corresponding point of (0, 1) is (2, 1). Plot these two additional points and complete the sketch of the parabola.

25. Example: You can determine the number of x-intercepts if you know the location of the vertex and the direction of opening. Visualize the general position and shape of the graph based on the values of a and q. Consider $f(x) = 0.5(x + 1)^2 - 3$, $g(x) = 2(x - 3)^2$, and $h(x) = -2(x + 3)^2 - 4$. For $f(x)$, the parabola opens upward and the vertex is below the x-axis, so the graph has two x-intercepts. For $g(x)$, the parabola opens upward and the vertex is on the x-axis, so the graph has one x-intercept. For $h(x)$, the parabola opens downward and the vertex is below the x-axis, so the graph has no x-intercepts.

26. Answers may vary.

3.2 Investigating Quadratic Functions in Standard Form, pages 174 to 179

1. a) This is a quadratic function, since it is a polynomial of degree two.

b) This is not a quadratic function, since it is a polynomial of degree one.

c) This is not a quadratic function. Once the expression is expanded, it is a polynomial of degree three.

d) This is a quadratic function. Once the expression is expanded, it is a polynomial of degree two.

2. a) The coordinates of the vertex are (−2, 2). The equation of the axis of symmetry is $x = -2$. The x-intercepts occur at (−3, 0) and (−1, 0), and the y-intercept occurs at (0, −6). The graph opens downward, so the graph has a maximum of 2 of when $x = -2$. The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \le 2, y \in \mathrm{R}\}$.

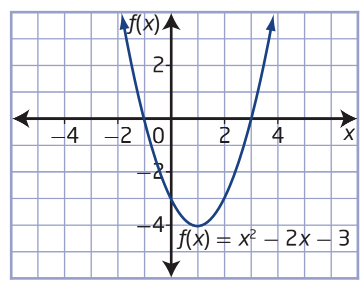
b) The coordinates of the vertex are (6, −4). The equation of the axis of symmetry is $x = 6$. The x-intercepts occur at (2, 0) and (10, 0), and the y-intercept occurs at (0, 5). The graph opens upward, so the graph has a minimum of −4 when $x = 6$. The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \ge -4, y \in \mathrm{R}\}$.

c) The coordinates of the vertex are (3, 0). The equation of the axis of symmetry is $x = 3$. The x-intercept occurs at (3, 0), and the y-intercept occurs at (0, 8). The graph opens upward, so the graph has a minimum of 0 when $x = 3$. The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \ge 0, y \in \mathrm{R}\}$.

3. a) $f(x) = -10x^2 + 50x$

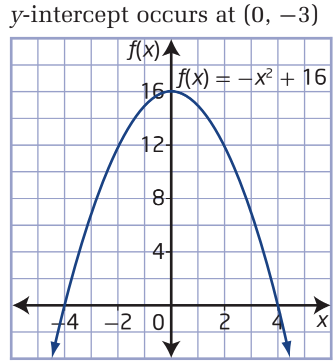
b) $f(x) = 15x^2 - 62x + 40$

4. a)

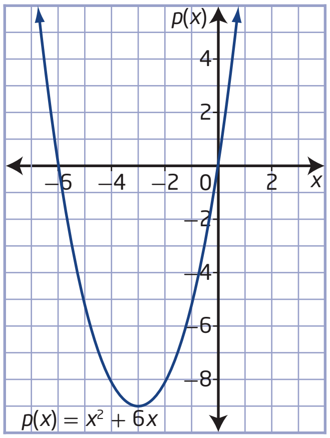


vertex is (1, −4); axis of symmetry is $x = 1$; opens upward; minimum value of −4 when $x = 1$; domain is $\{x \mid x \in \mathrm{R}\}$, range is $\{y \mid y \ge -4, y \in \mathrm{R}\}$; x-intercepts occur at (−1, 0) and (3, 0), y-intercept occurs at (0, −3)

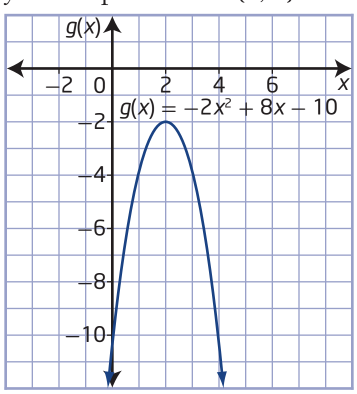
b)



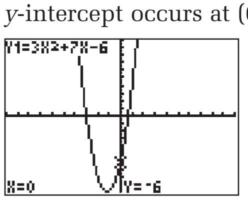
vertex is (0, 16); axis of symmetry is $x = 0$; opens downward; maximum value of 16 when $x = 0$; domain is $\{x \mid x \in \mathrm{R}\}$, range is $\{y \mid y \leq 16, y \in \mathrm{R}\}$; x-intercepts occur at $(-4, 0)$ and $(4, 0)$, y-intercept occurs at $(0, 16)$

c) 

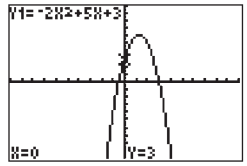
vertex is $(-3, -9)$; axis of symmetry is $x = -3$; opens upward; minimum value of -9 when $x = -3$; domain is $\{x \mid x \in \mathrm{R}\}$, range is $\{y \mid y \geq -9, y \in \mathrm{R}\}$; x-intercepts occur at $(-6, 0)$ and $(0, 0)$, y-intercept occurs at $(0, 0)$

d) 

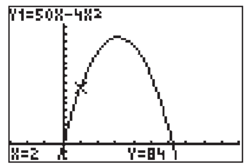
vertex is $(2, -2)$; axis of symmetry is $x = 2$; opens downward; maximum value of -2 when $x = 2$; domain is $\{x \mid x \in \mathrm{R}\}$, range is $\{y \mid y \leq -2, y \in \mathrm{R}\}$; no x-intercepts, y-intercept occurs at $(0, -10)$

5. a) 

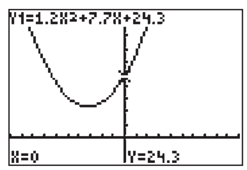
vertex is $(-1.2, -10.1)$; axis of symmetry is $x = -1.2$; opens upward; minimum value of -10.1 when $x = -1.2$; domain is $\{x \mid x \in \mathrm{R}\}$, range is $\{y \mid y \geq -10.1, y \in \mathrm{R}\}$; x-intercepts occur at $(-3, 0)$ and $(0.7, 0)$, y-intercept occurs at $(0, -6)$

b) 

vertex is (1.3, 6.1); axis of symmetry is $x = 1.3$; opens downward; maximum value of 6.1 when $x = 1.3$; domain is $\{x \mid x \in \mathrm{R}\}$, range is $\{y \mid y \leq 6.1, y \in \mathrm{R}\}$; x-intercepts occur at $(-0.5, 0)$ and $(3, 0)$, y-intercept occurs at $(0, 3)$

c) 

vertex is (6.3, 156.3); axis of symmetry is $x = 6.3$; opens downward; maximum value of 156.3 when $x = 6.3$; domain is $\{x \mid x \in \mathrm{R}\}$, range is $\{y \mid y \leq 156.3, y \in \mathrm{R}\}$; x-intercepts occur at $(0, 0)$ and $(12.5, 0)$, y-intercept occurs at $(0, 0)$

d) 

vertex is $(-3.2, 11.9)$; axis of symmetry is $x = -3.2$; opens upward; minimum value of 11.9 when $x = -3.2$; domain is $\{x \mid x \in \mathrm{R}\}$, range is $\{y \mid y \geq 11.9, y \in \mathrm{R}\}$; no x-intercepts, y-intercept occurs at $(0, 24.3)$

6. a) $(-3, -7)$ **b)** $(2, -7)$ **c)** $(4, 5)$

7. a) 10 cm, h-intercept of the graph
b) 30 cm after 2 s, vertex of the parabola
c) approximately 4.4 s, t-intercept of the graph
d) domain: $\{t \mid 0 \leq t \leq 4.4, t \in \mathrm{R}\}$, range: $\{h \mid 0 \leq h \leq 30, h \in \mathrm{R}\}$
e) Example: No, siksik cannot stay in the air for 4.4 s in real life.

8. Examples:
a) Two; since the graph has a maximum value, it opens downward and would cross the x-axis at two different points. One x-intercept is negative and the other is positive.
b) Two; since the vertex is at (3, 1) and the graph passes through the point $(1, -3)$, it opens downward and crosses the x-axis at two different points. Both x-intercepts are positive.

c) Zero; since the graph has a minimum of 1 and opens upward, it will not cross the x-axis.

d) Two; since the graph has an axis of symmetry of $x = -1$ and passes through the x- and y-axes at (0, 0), the graph could open upward or downward and has another x-intercept at $(-2, 0)$. One x-intercept is zero and the other is negative.

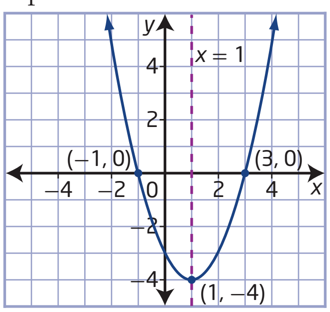
9. a) domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \leq 68, y \in \mathrm{R}\}$

b) domain: $\{x \mid 0 \leq x \leq 4.06, x \in \mathrm{R}\}$, range: $\{y \mid 0 \leq y \leq 68, y \in \mathrm{R}\}$

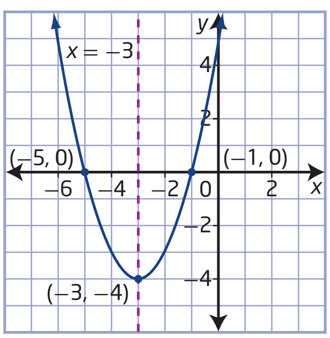
c) Example: The domain and range of algebraic functions may include all real values. For given real-world situations, the domain and range are determined by physical constraints such as time must be greater than or equal to zero and the height must be above ground, or greater than or equal to zero.

10. Examples:

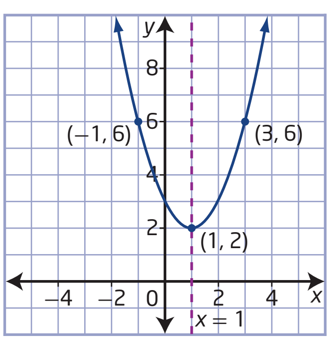
a)



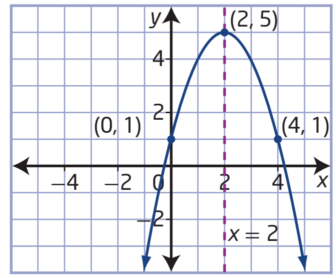
b)



c)

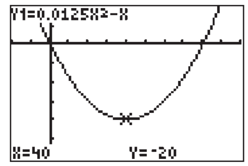


d)



11. a) $\{x \mid 0 \leq x \leq 80, x \in \mathrm{R}\}$

b)

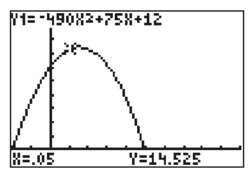


c) The maximum depth of the dish is 20 cm, which is the y-coordinate of the vertex $(40, -20)$. This is not the maximum value of the function. Since the parabola opens upward, this the minimum value of the function.

d) $\{d \mid -20 \leq d \leq 0, d \in \mathrm{R}\}$

e) The depth is approximately 17.19 cm, 25 cm from the edge of the dish.

12. a)



b) The h-intercept represents the height of the log.

c) 0.1 s; 14.9 cm

d) 0.3 s

e) domain: $\{t \mid 0 \leq t \leq 0.3, t \in \mathrm{R}\}$, range: $\{h \mid 0 \leq h \leq 14.9, h \in \mathrm{R}\}$

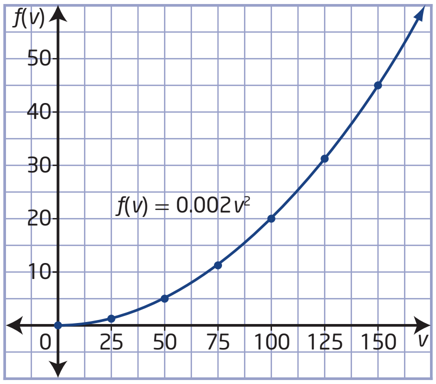
f) 14.5 cm

13. Examples:

a) $\{v \mid 0 \leq v \leq 150, v \in \mathrm{R}\}$

b)

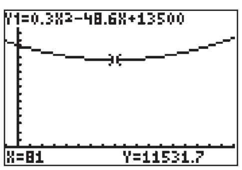
v	f
0	0
25	1.25
50	5
75	11.25
100	20
125	31.25
150	45



c) The graph is a smooth curve instead of a straight line. The table of values shows that the values of f are not increasing at a constant rate for equal increments in the value of v.

d) The values of the drag force increase by a value other than 2. When the speed of the vehicle doubles, the drag force quadruples.

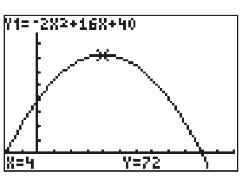
e) The driver can use this information to improve gas consumption and fuel economy.

14. a) 

The coordinates of the vertex are (81, 11 532). The equation of the axis of symmetry is $x = 81$. There are no x-intercepts. The y-intercept occurs at (0, 13 500). The graph opens upward, so the graph has a minimum value of 11 532 when $x = 81$. The domain is $\{n \mid n \geq 0, n \in \mathrm{R}\}$. The range is $\{C \mid C \geq 11\ 532, C \in \mathrm{R}\}$.

b) Example: The vertex represents the minimum cost of \$11 532 to produce 81 000 units. Since the vertex is above the n-axis, there are no n-intercepts, which means the cost of production is always greater than zero. The C-intercept represents the base production cost. The domain represents thousands of units produced, and the range represents the cost to produce those units.

15. a) $A = -2x^2 + 16x + 40$

b) 

c) The values between the x-intercepts will produce a rectangle. The rectangle will have a width that is 2 greater than the value of x and a length that is 20 less 2 times the value of x.

d) The vertex indicates the maximum area of the rectangle.

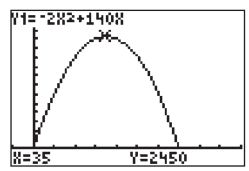
e) domain: $\{x \mid -2 \leq x \leq 10, x \in \mathrm{R}\}$, range: $\{A \mid 0 \leq A \leq 72, A \in \mathrm{R}\}$; the domain represents the values for x that will produce dimensions of a rectangle. The range represents the possible values of the area of the rectangle.

f) The function has both a maximum value and a minimum value for the area of the rectangle.

g) Example: No; the function will open downward and therefore will not have a minimum value for a domain of real numbers.

16. Example: No; the simplified version of the function is $f(x) = 3x + 1$. Since this is not a polynomial of degree two, it does not represent a quadratic function. The graph of the function $f(x) = 4x^2 - 3x + 2x(3 - 2x) + 1$ is a straight line.

17. a) $A = -2x^2 + 140x$; this is a quadratic function since it is a polynomial of degree two.

b) 

c) (35, 2450); The vertex represents the maximum area of 2450 m² when the width is 35 m.

d) domain: $\{x \mid 0 \leq x \leq 70, x \in \mathrm{R}\}$, range: $\{A \mid 0 \leq A \leq 2450, A \in \mathrm{R}\}$
The domain represents the possible values of the width, and the range represents the possible values of the area.

e) The function has a maximum area (value) of 2450 m² and a minimum value of 0 m². Areas cannot have negative values.

f) Example: The quadratic function assumes that Maria will use all of the fencing to make the enclosure. It also assumes that any width from 0 m to 70 m is possible.

18. a) Diagram 4 Diagram 5 Diagram 6

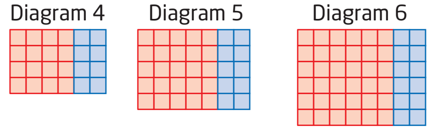


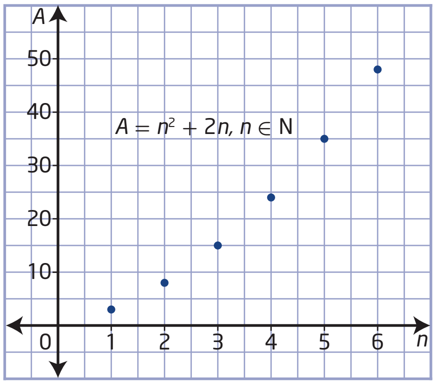
Diagram 4: 24 square units
Diagram 5: 35 square units
Diagram 6: 48 square units

b) $A = n^2 + 2n$

c) Quadratic; the function is a polynomial of degree two.

d) $\{n \mid n \geq 1, n \in \mathrm{N}\}$; The values of n are natural numbers. So, the function is discrete. Since the numbers of both diagrams and small squares are countable, the function is discrete.

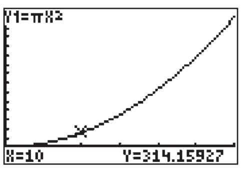
e)



19. a) $A = \pi r^2$

b) domain: $\{r \mid r \geq 0, r \in \mathrm{R}\}$, range: $\{A \mid A \geq 0, A \in \mathrm{R}\}$

c)



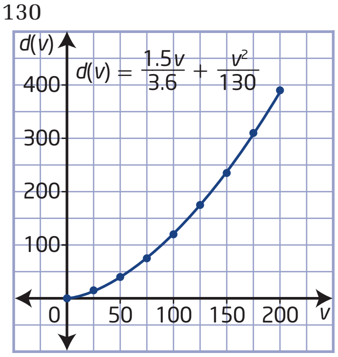
d) The x-intercept and the y-intercept occur at (0, 0). They represent the minimum values of the radius and the area.

e) Example: There is no axis of symmetry within the given domain and range.

20. a) $d(v) = \dfrac{1.5v}{3.6} + \dfrac{v^2}{130}$

b)

v	d
0	0
25	15
50	40
75	75
100	119
125	172
150	236
175	308
200	391

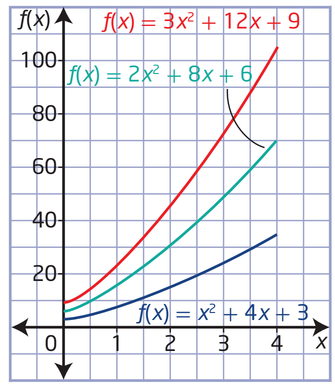


c) No; when v doubles from 25 km/h to 50 km/h, the stopping distance increases by a factor of $\dfrac{40}{15} = 2.67$, and when the velocity doubles from 50 km/h to 100 km/h, the stopping distance increases by a factor of $\dfrac{119}{40} = 2.98$. Therefore, the stopping distance increases by a factor greater than two.

d) Example: Using the graph or table, notice that as the speed increases the stopping distances increase by a factor greater than the increase in speed. Therefore, it is important for drivers to maintain greater distances between vehicles as the speed increases to allow for increasing stopping distances.

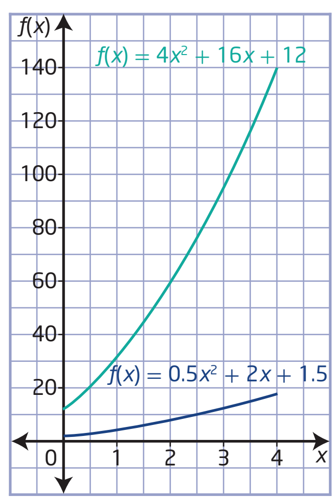
21. a) $f(x) = x^2 + 4x + 3$, $f(x) = 2x^2 + 8x + 6$, and $f(x) = 3x^2 + 12x + 9$

b)

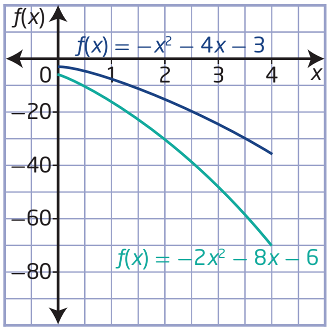


c) Example: The graphs have similar shapes, curving upward at a rate that is a multiple of the first graph. The values of y for each value of x are multiples of each other.

d) Example: If $k = 4$, the graph would start with a y-intercept 4 times as great as the first graph and increase with values of y that are 4 times as great as the values of y of the first function. If $k = 0.5$, the graph would start with a y-intercept $\frac{1}{2}$ of the original y-intercept and increase with values of y that are $\frac{1}{2}$ of the original values of y for each value of x.



e) Example: For negative values of k, the graph would be reflected in the x-axis, with a smooth decreasing curve. Each value of y would be a negative multiple of the original value of y for each value of x.



f) The graph is a line on the x-axis.

g) Example: Each member of the family of functions for $f(x) = k(x^2 + 4x + 3)$ has values of y that are multiples of the original function for each value of x.

22. Example: The value of a in the function $f(x) = ax^2 + bx + c$ indicates the steepness of the curved section of a function in that when $a > 0$, the curve will move up more steeply as a increases and when $-1 < a < 1$, the curve will move up more slowly the closer a is to 0. The sign of a is also similar in that if $a > 0$, then the graph curves up and when $a < 0$, the graph will curve down from the vertex. The value of a in the function $f(x) = ax + b$ indicates the exact steepness or slope of the line determined by the function, whereas the slope of the function $f(x) = ax^2 + bx + c$ changes as the value of x changes and is not a direct relationship for the entire graph.

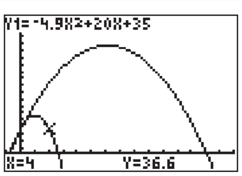
23. a) $b = 3$

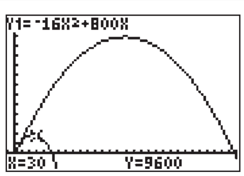
b) $b = -3$ and $c = 1$

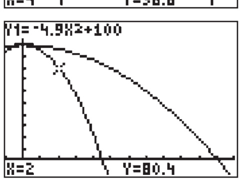
24. a)

Earth	Moon
$h(t) = -4.9t^2 + 20t + 35$	$h(t) = -0.815t^2 + 20t + 35$
$h(t) = -16t^2 + 800t$	$h(t) = -2.69t^2 + 800t$
$h(t) = -4.9t^2 + 100$	$h(t) = -0.815t^2 + 100$

b)







c) Example: The first two graphs have the same y-intercept at (0, 35). The second two graphs pass through the origin (0, 0). The last two graphs share the same y-intercept at (0, 100). Each pair of graphs share the same y-intercept and share the same constant term.

d) Example: Every projectile on the moon had a higher trajectory and stayed in the air for a longer period of time.

25. Examples:

a) $(2m, r)$; apply the definition of the axis of symmetry. The horizontal distance from the y-intercept to the x-coordinate of the vertex is $m - 0$, or m. So, one other point on the graph is $(m + m, r)$, or $(2m, r)$.

b) $(-2j, k)$; apply the definition of the axis of symmetry. The horizontal distance from the given point to the axis of symmetry is $4j - j$, or $3j$. So, one other point on the graph is $(j - 3j, k)$, or $(-2j, k)$.

c) $\left(\frac{s + t}{2}, d\right)$; apply the definitions of the axis of symmetry and the minimum value of a function. The x-coordinate of the vertex is halfway between the x-intercepts, or $\frac{s + t}{2}$. The y-coordinate of the vertex is the least value of the range, or d.

26. Example: The range and direction of opening are connected and help determine the location of the vertex. If $y \geq q$, then the graph will open upward. If $y \leq q$, then the graph will open downward. The range also determines the maximum or minimum value of the function and the y-coordinate of the vertex. The equation of the axis of symmetry determines the x-coordinate of the vertex. If the vertex is above the x-axis and the graph opens upward, there will be no x-intercepts. However, if it opens downward, there will be two x-intercepts. If the vertex is on the x-axis, there will be only one x-intercept.

27. Step 2 The y-intercept is determined by the value of c. The values of a and b do not affect its location.

Step 3 The axis of symmetry is affected by the values of a and b. As the value of a increases, the value of the axis of symmetry decreases. As the value of b increases, the value of the axis of symmetry increases.

Step 4 Increasing the value of a increases the steepness of the graph.

Step 5 Changing the values of a, b, and c affects the position of the vertex, the steepness of the graph, and whether the graph opens upward ($a > 0$) or downward ($a < 0$). a affects the steepness and determines the direction of opening. b and a affect the value of the axis of symmetry, with b having a greater effect. c determines the value of the y-intercept.

3.3 Completing the Square, pages 192 to 197

1. a) $x^2 + 6x + 9$; $(x + 3)^2$
b) $x^2 - 4x + 4$; $(x - 2)^2$
c) $x^2 + 14x + 49$; $(x + 7)^2$
d) $x^2 - 2x + 1$; $(x - 1)^2$

2. a) $y = (x + 4)^2 - 16$; $(-4, -16)$
b) $y = (x - 9)^2 - 140$; $(9, -140)$
c) $y = (x - 5)^2 + 6$; $(5, 6)$
d) $y = (x + 16)^2 - 376$; $(-16, -376)$

3. a) $y = 2(x - 3)^2 - 18$; working backward, $y = 2(x - 3)^2 - 18$ results in the original function, $y = 2x^2 - 12x$.
b) $y = 6(x + 2)^2 - 7$; working backward, $y = 6(x + 2)^2 - 7$ results in the original function, $y = 6x^2 + 24x + 17$.
c) $y = 10(x - 8)^2 - 560$; working backward, $y = 10(x - 8)^2 - 560$ results in the original function, $y = 10x^2 - 160x + 80$.
d) $y = 3(x + 7)^2 - 243$; working backward, $y = 3(x + 7)^2 - 243$ results in the original function, $y = 3x^2 + 42x - 96$.

4. a) $f(x) = -4(x - 2)^2 + 16$; working backward, $f(x) = -4(x - 2)^2 + 16$ results in the original function, $f(x) = -4x^2 + 16x$.
b) $f(x) = -20(x + 10)^2 + 1757$; working backward, $f(x) = -20(x + 10)^2 + 1757$ results in the original function, $f(x) = -20x^2 - 400x - 243$.
c) $f(x) = -(x + 21)^2 + 941$; working backward, $f(x) = -(x + 21)^2 + 941$ results in the original function, $f(x) = -x^2 - 42x + 500$.
d) $f(x) = -7(x - 13)^2 + 1113$; working backward, $f(x) = -7(x - 13)^2 + 1113$ results in the original function, $f(x) = -7x^2 + 182x - 70$.

5. Verify each part by expanding the vertex form of the function and comparing with the standard form and by graphing both forms of the function.

6. a) minimum value of -11 when $x = -3$
b) minimum value of -11 when $x = 2$
c) maximum value of 25 when $x = -5$
d) maximum value of 5 when $x = 2$

7. a) minimum value of $-\frac{13}{4}$
b) minimum value of $\frac{1}{2}$
c) maximum value of 47
d) minimum value of -1.92
e) maximum value of 18.95
f) maximum value of 1.205

8. a) $y = \left(x + \frac{3}{4}\right)^2 - \frac{121}{16}$
b) $y = -\left(x + \frac{3}{16}\right)^2 + \frac{9}{256}$
c) $y = 2\left(x - \frac{5}{24}\right)^2 + \frac{263}{288}$

9. a) $f(x) = -2(x - 3)^2 + 8$
b) Example: The vertex of the graph is $(3, 8)$. From the function $f(x) = -2(x - 3)^2 + 8$, $p = 3$ and $q = 8$. So, the vertex is $(3, 8)$.

10. a) maximum value of 62; domain: $\{x \mid x \in R\}$, range: $\{y \mid y \leq 62, y \in R\}$
b) Example: By changing the function to vertex form, it is possible to find the maximum value since the function opens down and $p = 62$. This also helps to determine the range of the function. The domain is all real numbers for non-restricted quadratic functions.

11. Example: By changing the function to vertex form, the vertex is $\left(\frac{13}{4}, -\frac{3}{4}\right)$ or $(3.25, -0.75)$.

12. a) There is an error in the second line of the solution. You need to add and subtract the square of half the coefficient of the x-term.
$y = x^2 + 8x + 30$
$y = (x^2 + 8x + 16 - 16) + 30$
$y = (x + 4)^2 + 14$
b) There is an error in the second line of the solution. You need to add and subtract the square of half the coefficient of the x-term. There is also an error in the last line. The factor of 2 disappeared.
$f(x) = 2x^2 - 9x - 55$
$f(x) = 2[x^2 - 4.5x + 5.0625 - 5.0625] - 55$
$f(x) = 2[(x^2 - 4.5x + 5.0625) - 5.0625] - 55$
$f(x) = 2[(x - 2.25)^2 - 5.0625] - 55$
$f(x) = 2(x - 2.25)^2 - 10.125 - 55$
$f(x) = 2(x - 2.25)^2 - 65.125$
c) There is an error in the third line of the solution. You need to add and subtract the square of half the coefficient of the x-term.
$y = 8x^2 + 16x - 13$
$y = 8[x^2 + 2x] - 13$
$y = 8[x^2 + 2x + 1 - 1] - 13$
$y = 8[(x^2 + 2x + 1) - 1] - 13$
$y = 8[(x + 1)^2 - 1] - 13$
$y = 8(x + 1)^2 - 8 - 13$
$y = 8(x + 1)^2 - 21$

d) There are two errors in the second line of the solution. You need to factor the leading coefficient from the first two terms and add and subtract the square of half the coefficient of the x-term. There is also an error in the last line. The -3 factor was not distributed correctly.
$f(x) = -3x^2 - 6x$
$f(x) = -3[x^2 + 2x + 1 - 1]$
$f(x) = -3[(x^2 + 2x + 1) - 1]$
$f(x) = -3[(x + 1)^2 - 1]$
$f(x) = -3(x + 1)^2 + 3$

13. 12 000 items

14. 9 m

15. a) 5.56 ft; 0.31 s after being shot
b) Example: Verify by graphing and finding the vertex or by changing the function to vertex form and using the values of p and q to find the maximum value and when it occurs.

16. a) Austin got $+12x$ when dividing $72x$ by -6 and should have gotten $-12x$. He also forgot to square the quantity $(x + 6)$. Otherwise his work was correct and his answer should be $y = -6(x - 6)^2 + 196$. Yuri got an answer of -216 when he multiplied -6 by -36. He should have gotten 216 to get the correct answer of $y = -6(x - 6)^2 + 196$.
b) Example: To verify an answer, either work backward to show the functions are equivalent or use technology to show the graphs of the functions are identical.

17. 18 cm

18. a) The maximum revenue is \$151 250 when the ticket price is \$55.
b) 2750 tickets
c) Example: Assume that the decrease in ticket prices determines the same increase in ticket sales as indicated by the survey.

19. a) $R(n) = -50n^2 + 1000n + 100\,800$, where R is the revenue of the sales and n is the number of \$10 increases in price.
b) The maximum revenue is \$105 800 when the bikes are sold for \$460.
c) Example: Assume that the predictions of a decrease in sales for every increase in price holds true.

20. a) $P(n) = -0.1n^2 + n + 120$, where P is the production of peas, in kilograms, and n is the increase in plant rows.
b) The maximum production is 122.5 kg of peas when the farmer plants 35 rows of peas.
c) Example: Assume that the prediction holds true.

21. a) Answers may vary.
b) $A = -2w^2 + 90w$, where A is the area and w is the width.
c) 1012.5 m^2
d) Example: Verify the solution by graphing or changing the function to vertex form, where the vertex is (22.5, 1012.5).
e) Example: Assume that the measurements can be any real number.

22. The dimensions of the large field are 75 m by 150 m, and the dimensions of the small fields are 75 m by 50 m.

23. a) The two numbers are 14.5 and 14.5, and the maximum product is 210.25.
b) The two numbers are 6.5 and -6.5, and the minimum product is -42.25.

24. 8437.5 cm^2

25. $f(x) = -\frac{3}{4}\left(x - \frac{3}{4}\right)^2 + \frac{47}{64}$

26. a)
$y = ax^2 + bx + c$
$y = a\left(x^2 + \frac{b}{a}x\right) + c$
$y = a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b^2}{4a^2}\right)\right) + c$
$y = a\left(x + \frac{b}{2a}\right)^2 - \frac{ab^2}{4a^2} + c$
$y = a\left(x + \frac{b}{2a}\right)^2 + \frac{4a^2c - ab^2}{4a^2}$
$y = a\left(x + \frac{b}{2a}\right)^2 + \frac{a(4ac - b^2)}{4a^2}$
$y = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$
b) $\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$
c) Example: This formula can be used to find the vertex of any quadratic function without using an algebraic method to change the function to vertex form.

27. a) (3, 4)
b) $f(x) = 2(x - 3)^2 + 4$, so the vertex is (3, 4).
c) $a = a$, $p = -\frac{b}{2a}$, and $q = \frac{4ac - b^2}{4a}$

28. a) $A = -\left(\frac{4 + \pi}{8}\right)w^2 + 3w$
b) maximum area of $\frac{18}{4 + \pi}$, or approximately 2.52 m^2, when the width is $\frac{12}{4 + \pi}$, or approximately 1.68 m
c) Verify by graphing and comparing the vertex values, $\left(\frac{12}{4 + \pi}, \frac{18}{4 + \pi}\right)$, or approximately (1.68, 2.52).
d) width: $\frac{12}{4 + \pi}$ or approximately 1.68 m, length: $\frac{6}{4 + \pi}$ or approximately 0.84 m, radius: $\frac{6}{4 + \pi}$ or approximately 0.84 m; Answers may vary.

29. Examples:

a) The function is written in both forms; standard form is $f(x) = 4x^2 + 24$ and vertex form is $f(x) = 4(x + 0)^2 + 24$.

b) No, since it is already in completed square form.

30. Martine's first error was that she did not correctly factor -4 from $-4x^2 + 24x$. Instead of $y = -4(x^2 + 6x) + 5$, it should have been $y = -4(x^2 - 6x) + 5$. Her second error occurred when she completed the square. Instead of $y = -4(x^2 + 6x + 36 - 36) + 5$, it should have been $y = -4(x^2 - 6x + 9 - 9) + 5$. Her third error occurred when she factored $(x^2 + 6x + 36)$. This is not a perfect square trinomial and is not factorable. Her last error occurred when she expanded the expression $-4[(x + 6)^2 - 36] + 5$. It should be $-4(x - 3)^2 + 36 + 5$ not $-4(x + 6)^2 - 216 + 5$. The final answer is $y = -4(x - 3)^2 + 41$.

31. a) $R = -5x^2 + 50x + 1000$

b) By completing the square, you can determine the maximum revenue and price to charge to produce the maximum revenue, as well as predict the number of T-shirts that will sell.

c) Example: Assume that the market research holds true for all sales of T-shirts.

Chapter 3 Review, pages 198 to 200

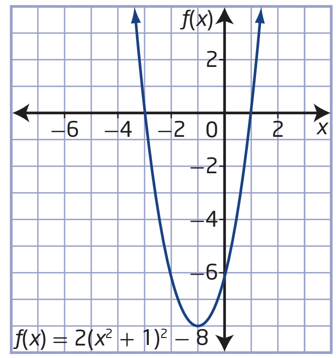
1. a) Given the graph of $f(x) = x^2$, move it 6 units to the left and 14 units down.
vertex: $(-6, -14)$, axis of symmetry: $x = -6$, opens upward, minimum value of -14, domain: $\{x \mid x \in R\}$, range: $\{y \mid y \geq -14, y \in R\}$

b) Given the graph of $f(x) = x^2$, change the width by multiplying the y-values by a factor of 2, reflect it in the x-axis, and move the entire graph up 19 units.
vertex: $(0, 19)$, axis of symmetry: $x = 0$, opens downward, maximum value of 19, domain: $\{x \mid x \in R\}$, range: $\{y \mid y \leq 19, y \in R\}$

c) Given the graph of $f(x) = x^2$, change the width by multiplying the y-values by a factor of $\frac{1}{5}$, move the entire graph 10 units to the right and 100 units up.
vertex: $(10, 100)$, axis of symmetry: $x = 10$, opens upward, minimum value of 100, domain: $\{x \mid x \in R\}$, range: $\{y \mid y \geq 100, y \in R\}$

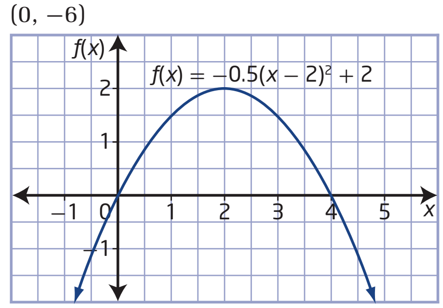
d) Given the graph of $f(x) = x^2$, change the width by multiplying the y-values by a factor of 6, reflect it in the x-axis, and move the entire graph 4 units to the right.
vertex: $(4, 0)$, axis of symmetry: $x = 4$, opens downward, maximum value of 0, domain: $\{x \mid x \in R\}$, range: $\{y \mid y \leq 0, y \in R\}$

2. a)



vertex: $(-1, -8)$, axis of symmetry: $x = -1$, minimum value of -8, domain: $\{x \mid x \in R\}$, range: $\{y \mid y \geq -8, y \in R\}$, x-intercepts occur at $(-3, 0)$ and $(1, 0)$, y-intercept occurs at $(0, -6)$

b)



vertex: $(2, 2)$, axis of symmetry: $x = 2$, maximum value of 2, domain: $\{x \mid x \in R\}$, range: $\{y \mid y \leq 2, y \in R\}$, x-intercepts occur at $(0, 0)$ and $(4, 0)$, y-intercept occurs at $(0, 0)$

3. Examples:

a) Yes. The vertex is $(5, 20)$, which is above the x-axis, and the parabola opens downward to produce two x-intercepts.

b) Yes. Since $y \geq 0$, the graph touches the x-axis at only one point and has one x-intercept.

c) Yes. The vertex of $(0, 9)$ is above the x-axis and the parabola opens upward, so the graph does not cross or touch the x-axis and has no x-intercepts.

d) No. It is not possible to determine if the graph opens upward to produce two x-intercepts or downward to produce no x-intercepts.

4. a) $y = -0.375x^2$ **b)** $y = 1.5(x - 8)^2$

c) $y = 3(x + 4)^2 + 12$

d) $y = -4(x - 4.5)^2 + 25$

5. a) $y = \frac{1}{4}(x + 3)^2 - 6$ **b)** $y = -2(x - 1)^2 + 5$

6. Example: Two possible functions for the mirror are $y = 0.0069(x - 90)^2 - 56$ and $y = 0.0069x^2$.

7. a) **i)** $y = \frac{22}{18\,769}x^2$ **ii)** $y = \frac{22}{18\,769}x^2 + 30$

iii) $y = \frac{22}{18\,769}(x - 137)^2 + 30$

b) Example: The function will change as the seasons change with the heat or cold changing the length of the cable and therefore the function.

8. $y = -\frac{8}{15}(x - 7.5)^2 + 30$ or $y \approx -0.53(x - 7.5)^2 + 30$

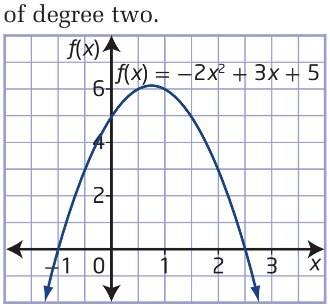
9. a) vertex: (2, 4), axis of symmetry: $x = 2$, maximum value of 4, opens downward, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \leq 4, y \in \mathrm{R}\}$, x-intercepts occur at (−2, 0) and (6, 0), y-intercept occurs at (0, 3)

b) vertex: (−4, 2), axis of symmetry: $x = -4$, maximum value of 2, opens upward, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \geq 2, y \in \mathrm{R}\}$, no x-intercepts, y-intercept occurs at (0, 10)

10. a) Expanding $y = 7(x + 3)^2 - 41$ gives $y = 7x^2 + 42x + 22$, which is a polynomial of degree two.

b) Expanding $y = (2x + 7)(10 - 3x)$ gives $y = -6x^2 - x + 70$, which is a polynomial of degree two.

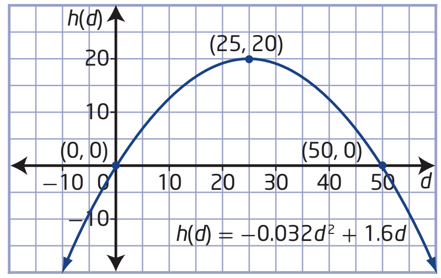
11. a)



vertex: (0.75, 6.125), axis of symmetry: $x = 0.75$, opens downward, maximum value of 6.125, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \leq 6.125, y \in \mathrm{R}\}$, x-intercepts occur at (−1, 0) and (2.5, 0), y-intercept occurs at (0, 5)

b) Example: The vertex is the highest point on the curve. The axis of symmetry divides the graph in half and is defined by the x-coordinate of the vertex. Since $a < 0$, the graph opens downward. The maximum value is the y-coordinate of the vertex. The domain is all real numbers. The range is less than or equal to the maximum value, since the graph opens downward. The x-intercepts are where the graph crosses the x-axis, and the y-intercept is where the graph crosses the y-axis.

12. a)



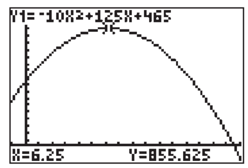
b) The maximum height of the ball is 20 m. The ball is 25 m downfield when it reaches its maximum height.

c) The ball lands downfield 50 m.

d) domain: $\{x \mid 0 \leq x \leq 50, x \in \mathrm{R}\}$, range: $\{y \mid 0 \leq y \leq 20, y \in \mathrm{R}\}$

13. a) $y = (5x + 15)(31 - 2x)$ or $y = -10x^2 + 125x + 465$

b)



c) The values between the x-intercepts will produce a rectangle.

d) Yes; the maximum value is 855.625; the minimum value is 0.

e) The vertex represents the maximum area and the value of x that produces the maximum area.

f) domain: $\{x \mid 0 \leq x \leq 15.5, x \in \mathrm{R}\}$, range: $\{y \mid 0 \leq y \leq 855.625, y \in \mathrm{R}\}$

14. a) $y = (x - 12)^2 - 134$

b) $y = 5(x + 4)^2 - 107$

c) $y = -2(x - 2)^2 + 8$

d) $y = -30(x + 1)^2 + 135$

15. vertex: $\left(\frac{5}{4}, -\frac{13}{4}\right)$, axis of symmetry: $x = \frac{5}{4}$, minimum value of $-\frac{13}{4}$, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\left\{y \mid y \geq -\frac{13}{4}, y \in \mathrm{R}\right\}$

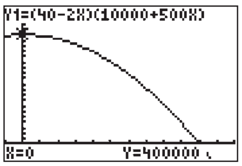
16. a) In the second line, the second term should have been $+3.5x$. In the third line, Amy found the square of half of 3.5 to be 12.25; it should have been 3.0625 and this term should be added and then subtracted. The solution should be

$y = -22x^2 - 77x + 132$
$y = -22(x^2 + 3.5x) + 132$
$y = -22(x^2 + 3.5x + 3.0625 - 3.0625) + 132$
$y = -22(x^2 + 3.5x + 3.0625) + 67.375 + 132$
$y = -22(x + 1.75)^2 + 199.375$

b) Verify by expanding the vertex form to standard form and by graphing both forms to see if they produce the same graph.

17. a) $R = (40 - 2x)(10\ 000 + 500x)$ or $R = -1000x^2 + 400\ 000$ where R is the revenue and x is the number of price decreases.

b) The maximum revenue is \$400 000 and the price is \$40 per coat.

c) 

d) The y-intercept represents the sales before changing the price. The x-intercepts indicate the number of price increases or decreases that will produce revenue.

e) domain: $\{x \mid -20 \leq x \leq 20, x \in R\}$, range: $\{y \mid 0 \leq y \leq 400\ 000, y \in R\}$

f) Example: Assume that a whole number of price increases can be used.

Chapter 3 Practice Test, pages 201 to 203

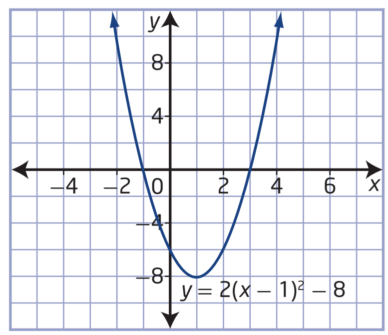
1. D
2. C
3. A
4. D
5. D
6. A

7. a) $y = (x - 9)^2 - 108$
b) $y = 3(x + 6)^2 - 95$
c) $y = -10(x + 2)^2 + 40$

8. a) vertex: $(-6, 4)$, axis of symmetry: $x = -6$, maximum value of 4, domain: $\{x \mid x \in R\}$, range: $\{y \mid y \leq 4, y \in R\}$, x-intercepts occur at $(-8, 0)$ and $(-4, 0)$
b) $y = -(x + 6)^2 + 4$

9. a) **i)** change in width by a multiplication of the y-values by a factor of 5
ii) vertical translation of 20 units down
iii) horizontal translation of 11 units to the left
iv) change in width by a multiplication of the y-values by a factor of $\frac{1}{7}$ and a reflection in the x-axis

b) Examples:
i) The vertex of the functions in part a) ii) and iii) will be different as compared to $f(x) = x^2$ because the entire graph is translated. Instead of a vertex of $(0, 0)$, the graph of the function in part a) ii) will be located at $(0, -20)$ and the vertex of the graph of the function in part a) iii) will be located at $(-11, 0)$.
ii) The axis of symmetry of the function in part a) iii) will be different as compared to $f(x) = x^2$ because the entire graph is translated horizontally. Instead of an axis of symmetry of $x = 0$, the graph of the function in part a) iii) will have an axis of symmetry of $x = -11$.
iii) The range of the functions in part a) ii) and iv) will be different as compared to $f(x) = x^2$ because the entire graph is either translated vertically or reflected in the x-axis. Instead of a range of $\{y \mid y \geq 0, y \in R\}$, the function in part a) ii) will have a range of $\{y \mid y \geq -20, y \in R\}$ and the function in part a) iv) will have a range of $\{y \mid y \leq 0, y \in R\}$.

10. 

Vertex	$(1, -8)$
Axis of Symmetry	$x = 1$
Direction of Opening	upward
Domain	$\{x \mid x \in R\}$
Range	$\{y \mid y \geq -8, y \in R\}$
x-Intercepts	-1 and 3
y-Intercept	-6

11. a) In the second line, the 2 was not factored out of the second term. In the third line, you need to add and subtract the square of half the coefficient of the x-term. The first three steps should be
$y = 2x^2 - 8x + 9$
$y = 2(x^2 - 4x) + 9$
$y = 2(x^2 - 4x + 4 - 4) + 9$

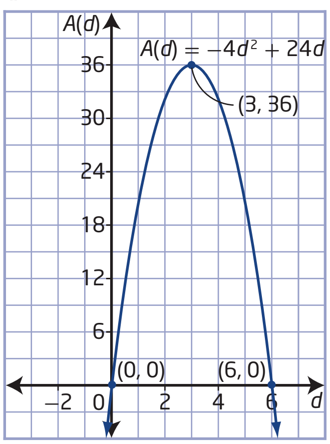
b) The rest of the process is shown.
$y = 2[(x^2 - 4x + 4) - 4] + 9$
$y = 2(x - 2)^2 - 8 + 9$
$y = 2(x - 2)^2 + 1$

c) The solution can be verified by expanding the vertex form to standard form or by graphing both functions to see that they coincide.

12. Examples:
   a) The vertex form of the function $C(v) = 0.004v^2 - 0.62v + 30$ is $C(v) = 0.004(v - 77.5)^2 + 5.975$. The most efficient speed would be 77.5 km/h and will produce a fuel consumption of 5.975 L/100 km.
   b) By completing the square and determining the vertex of the function, you can determine the most efficient fuel consumption and at what speed it occurs.

13. a) The maximum height of the flare is 191.406 25 m, 6.25 s after being shot.
   b) Example: Complete the square to produce the vertex form and use the value of q to determine the maximum height and the value of p to determine when it occurs, or use the fact that the x-coordinate of the vertex of a quadratic function in standard form is $x = -\frac{b}{2a}$ and substitute this value into the function to find the corresponding y-coordinate, or graph the function to find the vertex.

14. a) $A(d) = -4d^2 + 24d$
   b) Since the function is a polynomial of degree two, it satisfies the definition of a quadratic function.
   c)



   Example: By completing the square, determine the vertex, find the y-intercept and its corresponding point, plot the three points, and join them with a smooth curve.
   d) (3, 36); the maximum area of 36 m² happens when the fence is extended to 3 m from the building.
   e) domain: $\{d \mid 0 \leq d \leq 6, d \in \mathrm{R}\}$, range: $\{A \mid 0 \leq A \leq 36, A \in \mathrm{R}\}$; negative distance and area do not have meaning in this situation.
   f) Yes; the maximum value is 36 when d is 3, and the minimum value is 0 when d is 0 or 6.
   g) Example: Assume that any real-number distance can be used to build the fence.

15. a) $f(x) = -0.03x^2$
   b) $f(x) = -0.03x^2 + 12$
   c) $f(x) = -0.03(x + 20)^2 + 12$
   d) $f(x) = -0.03(x - 28)^2 - 3$

16. a) $R = (2.25 - 0.05x)(120 + 8x)$
   b) Expand and complete the square to get the vertex form of the function. A price of \$1.50 gives the maximum revenue of \$360.
   c) Example: Assume that any whole number of price decreases can occur.

Chapter 4 Quadratic Equations

4.1 Graphical Solutions of Quadratic Equations, pages 215 to 217

1. a) 1 b) 2 c) 0 d) 2
2. a) 0 b) −1 and −4
   c) none d) −3 and 8
3. a) $x = -3, x = 8$ b) $r = -3, r = 0$
   c) no real solutions d) $x = 3, x = -2$
   e) $z = 2$ f) no real solutions
4. a) $n \approx -3.2, n \approx 3.2$ b) $x = -4, x = 1$
   c) $w = 1, w = 3$ d) $d = -8, d = -2$
   e) $v \approx -4.7, v \approx -1.3$ f) $m = 3, m = 7$
5. 60 yd
6. a) $-x^2 + 9x - 20 = 0$ or $x^2 - 9x + 20 = 0$
   b) 4 and 5
7. a) $x^2 + 2x - 168 = 0$
   b) $x = 12$ and $x = 14$ or $x = -12$ and $x = -14$
8. a) Example: Solving the equation leads to the distance from the firefighter that the water hits the ground. The negative solution is not part of this situation.
   b) 12.2 m
   c) Example: Assume that aiming the hose higher would not reach farther. Assume that wind does not affect the path of the water.
9. a) Example: Solving the equation leads to the time that the fireworks hit the ground. The negative solution is not part of the situation.
   b) 6.1 s
10. a) $-0.75d^2 + 0.9d + 1.5 = 0$ b) 2.1 m
11. a) $-2d^2 + 3d + 10 = 0$ b) 3.1 m
12. a) first arch: $x = 0$ and $x = 84$, second arch: $x = 84$ and $x = 168$, third arch: $x = 168$ and $x = 252$
   b) The zeros represent where the arches reach down to the bridge deck.
   c) 252 m

13. a) $k = 9$ b) $k < 9$ c) $k > 9$
14. a) 64 ft
b) The relationship between the height, radius, and span of the arch stays the same. Input the measures in metres and solve.
15. about 2.4 s
16. For the value of the function to change from negative to positive, it must cross the x-axis and therefore there must be an x-intercept between the two values of x.
17. The other x-intercept would have to be 4.
18. The x-coordinate of the vertex is halfway between the two roots. So, it is at 2. You can then substitute $x = 2$ into the equation to find the minimum value of -16.

4.2 Factoring Quadratic Equations, pages 229 to 233

1. a) $(x + 2)(x + 5)$ b) $5(z + 2)(z + 6)$
c) $0.2(d - 4)(d - 7)$
2. a) $(y - 1)(3y + 7)$ b) $(4k - 5)(2k + 1)$
c) $0.2(2m - 3)(m + 3)$
3. a) $(x + 5)(x - 4)$ b) $(x - 6)^2$
c) $\frac{1}{4}(x + 2)(x + 6)$ d) $2(x + 3)^2$
4. a) $(2y + 3x)(2y - 3x)$
b) $(0.6p + 0.7q)(0.6p - 0.7q)$
c) $\left(\frac{1}{2}s + \frac{3}{5}t\right)\left(\frac{1}{2}s - \frac{3}{5}t\right)$
d) $(0.4t + 4s)(0.4t - 4s)$
5. a) $(x + 8)(x - 5)$
b) $(2x^2 - 8x + 9)(3x^2 - 12x + 11)$
c) $(-4)(8j)$
6. a) $(10b)(10b - 7)$
b) $16(x^2 - x + 1)(x^2 + x + 1)$
c) $(10y^3 - x)(10y^3 + x)$
7. a) $x = -3, x = -4$ b) $x = 2, x = -\frac{1}{2}$
c) $x = -7, x = 8$ d) $x = 0, x = -5$
e) $x = -\frac{1}{3}, x = \frac{4}{5}$ f) $x = 4, x = \frac{7}{2}$
8. a) $n = -2, n = 2$ b) $x = -4, x = -1$
c) $w = -9, x = -\frac{1}{3}$ d) $y = \frac{5}{4}, y = \frac{3}{2}$
e) $d = -\frac{3}{2}, d = -1$ f) $x = \frac{3}{2}$
9. a) 0 and 5   b) $-\frac{8}{9}$ and 1
c) -5 and -3 d) $-\frac{21}{5}$ and $\frac{21}{5}$
e) -5 and 7   f) $\frac{7}{2}$
10. a) -6 and 7   b) -10 and 3
c) -7 and 3   d) $-\frac{1}{3}$ and $\frac{3}{2}$
e) -5 and 2   f) -3 and $\frac{1}{2}$
11. a) $(x + 10)(2x - 3) = 54$ b) 3.5 cm
12. a) 1 s and 5 s
b) Assume that the mass of the fish does not affect the speed at which the osprey flies after catching the fish. This may not be a reasonable assumption for a large fish.
13. a) $150t - 5t^2 = 0$ b) 30 s
14. 8 and 10 or 0 and -2
15. 15 cm
16. 3 s; this seems a very long time considering the ball went up only 39 ft.
17. a) 1 cm
b) 7 cm by 5 cm
18. a) No; $(x - 5)$ is not a factor of the expression $x^2 - 5x - 36$, since $x = 5$ does not satisfy the equation $x^2 - 5x - 36 = 0$.
b) Yes; $(x + 3)$ is a factor of the expression $x^2 - 2x - 15$, since $x = -3$ satisfies the equation $x^2 - 2x - 15 = 0$.
c) No; $(4x + 1)$ is not a factor of the expression $6x^2 + 11x + 4$, since $x = -\frac{1}{4}$ does not satisfy the equation $6x^2 + 11x + 4 = 0$.
d) Yes; $(2x - 1)$ is a factor of the expression $4x^2 + 4x - 3$, since $x = \frac{1}{2}$ satisfies the equation $4x^2 + 4x - 3 = 0$.
19. a) $-\frac{1}{2}$ and 2   b) -4 and 3
20. 20 cm and 21 cm
21. 8 m and 15 m
22. a) $x(x - 7) = 690$ b) 30 cm by 23 cm
23. 5 m
24. 5 m
25. $P = \frac{1}{2}d(v_1 + v_2)(v_1 - v_2)$
26. No; the factor $6x - 4$ still has a common factor of 2.
27. a) $6(z - 1)(2z + 5)$
b) $4(2m^2 - 8 - 3n)(2m^2 - 8 + 3n)$
c) $\frac{1}{36}(2y - 3x)^2$
d) $7\left(w - \frac{5}{3}\right)(5w + 1)$
28. $4(3x + 5y)$ centimetres
29. The shop will make a profit after 4 years.
30. a) $x^2 - 9 = 0$ b) $x^2 - 4x + 4 = 0$
c) $3x^2 - 14x + 8 = 0$
d) $10x^2 - x - 3 = 0$
31. Example: $x^2 - x + 1 = 0$
32. a) Instead of evaluating $81 - 36$, use the difference of squares pattern to rewrite the expression as $(9 - 6)(9 + 6)$ and then simplify. You can use this method when a question asks you to subtract a square number from a square number.

b) Examples:

$$\begin{aligned}144 - 25 &= (12 - 5)(12 + 5)\\ &= (7)(17)\\ &= 119\\ 256 - 49 &= (16 - 7)(16 + 7)\\ &= (9)(23)\\ &= 207\end{aligned}$$

4.3 Solving Quadratic Equations by Completing the Square, pages 240 to 243

1. a) $c = \frac{1}{4}$ **b)** $c = \frac{25}{4}$
c) $c = 0.0625$ **d)** $c = 0.01$
e) $c = \frac{225}{4}$ **f)** $c = \frac{81}{4}$

2. a) $(x + 2)^2 = 2$ **b)** $(x + 2)^2 = \frac{17}{3}$
c) $(x - 3)^2 = -1$

3. a) $(x - 6)^2 - 27 = 0$ **b)** $5(x - 2)^2 - 21 = 0$
c) $-2\left(x - \frac{1}{4}\right)^2 - \frac{7}{8} = 0$
d) $0.5(x + 2.1)^2 + 1.395 = 0$
e) $-1.2(x + 2.125)^2 - 1.981\ 25 = 0$
f) $\frac{1}{2}(x + 3)^2 - \frac{21}{2} = 0$

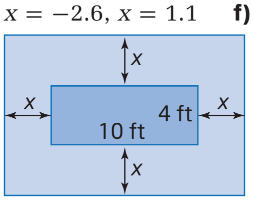
4. a) $x = \pm 8$ **b)** $s = \pm 2$
c) $t = \pm 6$ **d)** $y = \pm\sqrt{11}$

5. a) $x = 1, x = 5$ **b)** $x = -5, x = 1$
c) $d = -\frac{3}{2}, d = \frac{1}{2}$ **d)** $h = \frac{3 \pm \sqrt{7}}{4}$
e) $s = \frac{-12 \pm \sqrt{3}}{2}$ **f)** $x = -4 \pm 3\sqrt{2}$

6. a) $x = -5 \pm \sqrt{21}$ **b)** $x = 4 \pm \sqrt{3}$
c) $x = -1 \pm \sqrt{\frac{2}{3}}$ or $\frac{-3 \pm \sqrt{6}}{3}$
d) $x = 1 \pm \sqrt{\frac{5}{2}}$ or $\frac{2 \pm \sqrt{10}}{2}$
e) $x = -3 \pm \sqrt{13}$ **f)** $x = 4 \pm 2\sqrt{7}$

7. a) $x = 8.5, x = -0.5$ **b)** $x = -0.8, x = 2.1$
c) $x = 12.8, x = -0.8$ **d)** $x = -7.7, x = 7.1$
e) $x = -2.6, x = 1.1$ **f)** $x = -7.8, x = -0.2$

8. a)



b) $4x^2 + 28x - 40 = 0$
c) 12.4 ft by 6.4 ft

9. a) $-0.02d^2 + 0.4d + 1 = 0$
b) 22.2 m

10. 200.5 m

11. 6 in. by 9 in.

12. 53.7 m

13. a) $x^2 - 7 = 0$ **b)** $x^2 - 2x - 2 = 0$
c) $4x^2 - 20x + 14 = 0$ or $2x^2 - 10x + 7 = 0$

14. a) $x = -1 \pm \sqrt{k + 1}$ **b)** $x = \frac{1 \pm \sqrt{k^2 + 1}}{k}$
c) $x = \frac{k \pm \sqrt{k^2 + 4}}{2}$

15. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ No. Some will result in a negative in the radical, which means the solution(s) are not real.

16. a) $n = 43$ **b)** $n = 39$

17. a) $12^2 = 4^2 + x^2 - 2(4)(x) \cos(60°)$
b) 13.5 m

18. Example: In the first equation, you must take the square root to isolate or solve for x. This creates the $\pm$ situation. In the second equation, $\sqrt{9}$ is already present, which means the principle or positive square root only.

19. Example: Allison did all of her work on one side of the equation; Riley worked on both sides. Both end up at the same solution but by different paths.

20. Example:
- Completing the square requires operations with rational numbers, which could lead to arithmetic errors.
- Graphing the corresponding function using technology is very easy. Without technology, the manual graph could take a longer amount of time.
- Factoring should be the quickest of the methods.

All of the methods lead to the same answers.

21. a) Example: $y = 2(x - 1)^2 - 3$, $0 = 2x^2 - 4x - 1$
b) Example: $y = 2(x + 2)^2$, $0 = 2x^2 + 8x + 8$
c) Example: $y = 3(x - 2)^2 + 1$, $0 = 3x^2 - 12x + 13$

4.4 The Quadratic Formula, pages 254 to 257

1. a) two distinct real roots
b) two distinct real roots
c) two distinct real roots
d) one distinct real root
e) no real roots
f) one distinct real root

2. a) 2 **b)** 2 **c)** 1
d) 1 **e)** 0 **f)** 2

3. a) $x = -3, x = -\frac{3}{7}$ **b)** $p = \frac{3 \pm 3\sqrt{2}}{2}$
c) $q = \frac{-5 \pm \sqrt{37}}{6}$ **d)** $m = \frac{-2 \pm 3\sqrt{2}}{2}$
e) $j = \frac{7 \pm \sqrt{17}}{4}$ **f)** $g = -\frac{3}{4}$

4. a) $z = -4.28, z = -0.39$
b) $c = -0.13, c = 1.88$
c) $u = 0.13, u = 3.07$
d) $b = -1.41, b = -0.09$
e) $w = -0.15, w = 4.65$
f) $k = -0.27, k = 3.10$

5. a) $x = \frac{-3 \pm \sqrt{6}}{3}$, -0.18 and -1.82

b) $h = \frac{-1 \pm \sqrt{73}}{12}$, -0.80 and 0.63

c) $m = \frac{-0.3 \pm \sqrt{0.17}}{0.4}$, -1.78 and 0.28

d) $y = \frac{3 \pm \sqrt{2}}{2}$, 0.79 and 2.21

e) $x = \frac{1 \pm \sqrt{57}}{14}$, -0.47 and 0.61

f) $z = \frac{3 \pm \sqrt{7}}{2}$, 0.18 and 2.82

6. Example: Some are easily solved so they do not require the use of the quadratic formula. $x^2 - 9 = 0$

7. a) $n = -1 \pm \sqrt{3}$; complete the square
b) $y = 3$; factor
c) $u = \pm 2\sqrt{2}$; square root
d) $x = \frac{1 \pm \sqrt{19}}{3}$; quadratic formula
e) no real roots; graphing

8. 5 m by 20 m or 10 m by 10 m

9. 0.89 m

10. $1 \pm \sqrt{23}$, -3.80 and 5.80

11. 5 m

12. a) $(30 - 2x)(12 - 2x) = 208$
b) 2 in.
c) 8 in. by 26 in. by 2 in.

13. a) 68.8 km/h **b)** 95.2 km/h
c) 131.2 km/h

14. a) 4.2 ppm **b)** 3.4 years

15. $155, 130 jackets

16. 169.4 m

17. $b = 13$, $x = \frac{3}{2}$

18. 2.2 cm

19. a) $\left(-3 + 3\sqrt{5}\right)$ m **b)** $\left(-45 + 27\sqrt{5}\right)$ m^2

20. 3.5 h

21. Error in Line 1: The $-b$ would make the first number $-(-7) = 7$.
Error in Line 2: $-4(-3)(2) = +24$ not -24.
The correct solution is $x = \frac{-7 \pm \sqrt{73}}{6}$.

22. a) $x = -1$ and $x = 4$
b) Example: The axis of symmetry is halfway between the roots. $\frac{-1 + 4}{2} = \frac{3}{2}$. Therefore, the equation of the axis of symmetry is $x = \frac{3}{2}$.

23. Example: If the quadratic is easily factored, then factoring is faster. If it is not easily factored, then using the quadratic formula will yield exact answers. Graphing with technology is a quick way of finding out if there are real solutions.

24. Answers may vary.

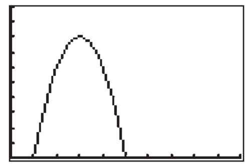
Chapter 4 Review, pages 258 to 260

1. a) $x = -6$, $x = -2$ **b)** $x = -1$, $x = 5$
c) $x = -2$, $x = -\frac{4}{3}$ **d)** $x = -3$, $x = 0$
e) $x = -5$, $x = 5$

2. D

3. Example: The graph cannot cross over or touch the x-axis.

4. a) Example:



b) 1000 key rings or 5000 key rings produce no profit or loss because the value of P is 0 then.

5. a) -1 and 6 **b)** 6 m

6. a) $(x - 1)(4x - 9)$ **b)** $\frac{1}{2}(x + 1)(x - 4)$
c) $(3v + 10)(v + 2)$
d) $(3a^2 - 12 + 35b)(3a^2 - 12 - 35b)$

7. a) $x = -7$, $x = -3$ **b)** $m = -10$, $m = 2$
c) $p = -3$, $p = \frac{2}{5}$ **d)** $z = \frac{1}{2}$, $z = 3$

8. a) $g = 3$, $g = -\frac{1}{2}$ **b)** $y = \frac{1}{2}$, $y = \frac{5}{4}$
c) $k = \frac{3}{5}$ **d)** $x = -\frac{3}{2}$, $x = 6$

9. a) Example: $0 = x^2 - 5x + 6$
b) Example: $0 = x^2 + 6x + 5$
c) Example: $0 = 2x^2 + 5x - 12$

10. 6 s

11. a) $V = 15(x)(x + 2)$ **b)** $2145 = 15x(x + 2)$
c) 11 m by 13 m

12. $x = -4$ and $x = 6$. Example: Factoring is fairly easy and exact.

13. a) $k = 4$ **b)** $k = \frac{9}{4}$

14. a) $x = \pm 7$ **b)** $x = 2$, $x = -8$
c) $x = 5 \pm 2\sqrt{6}$ **d)** $x = \frac{3 \pm \sqrt{5}}{3}$

15. a) $x = 4 \pm \sqrt{\frac{29}{2}}$ or $\frac{8 \pm \sqrt{58}}{2}$
b) $y = -2 \pm \sqrt{\frac{19}{5}}$ or $\frac{-10 \pm \sqrt{95}}{5}$
c) no real solutions

16. 68.5 s

17. a) $0 = -\frac{1}{2}d^2 + 2d + 1$ **b)** 4.4 m

18. a) two distinct real roots
b) one distinct real root
c) no real roots
d) two distinct real roots

19. a) $x = -\frac{5}{3}$, $x = 1$ **b)** $x = \frac{-7 \pm \sqrt{29}}{10}$
c) $x = \frac{2 \pm \sqrt{7}}{3}$ **d)** $x = -\frac{9}{5}$

20. a) $0 = -2x^2 + 6x + 1$ **b)** 3.2 m

21. a) $3.7 - 0.05x$ **b)** $2480 + 40x$

c) $R = -2x^2 + 24x + 9176$

d) 5 or 7

22.

Algebraic Steps	Explanations
$ax^2 + bx = -c$	Subtract c from both sides.
$x^2 + \frac{b}{a}x = -\frac{c}{a}$	Divide both sides by a.
$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$	Complete the square.
$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$	Factor the perfect square trinomial.
$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$	Take the square root of both sides.
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Solve for x.

Chapter 4 Practice Test, pages 261 to 262

1. C

2. B

3. D

4. B

5. B

6. a) $x = 3, x = 1$ **b)** $x = -\frac{3}{2}, x = 5$

c) $x = -3, x = 1$

7. $x = \frac{-5 \pm \sqrt{37}}{6}$

8. $x = -2 \pm \sqrt{11}$

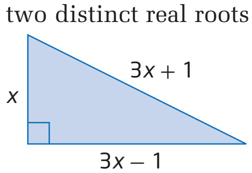
9. a) one distinct real root

b) two distinct real roots

c) no real roots

d) two distinct real roots

10. a)



b) $x^2 + (3x - 1)^2 = (3x + 1)^2$

c) 12 cm, 35 cm, and 37 cm

11. a) 3.8 s

b) 35 m

c) Example: Choose graphing with technology so you can see the path and know which points correspond to the situation.

12. 5 cm

13. 22 cm by 28 cm

14. a) $(9 + 2x)(6 + 2x) = 108$ or $4x^2 + 30x - 54 = 0$

b) $x = 1.5$

Example: Factoring is the most efficient strategy.

c) 42 m

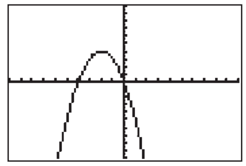
Cumulative Review, Chapters 3–4, pages 264 to 265

1. a) C **b)** A **c)** D **d)** B

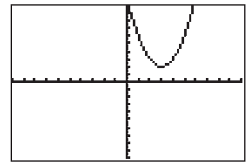
2. a) not quadratic **b)** quadratic

c) not quadratic **d)** quadratic

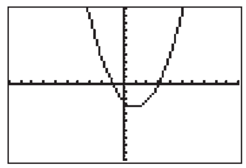
3. a) Example:



b) Example:



c) Example:



4. a) vertex: $(-4, -3)$, domain: $\{x \mid x \in R\}$, range: $\{y \mid y \geq -3, y \in R\}$, axis of symmetry: $x = -4$, x-intercepts occur at approximately $(-5.7, 0)$ and $(-2.3, 0)$, y-intercept occurs at $(0, 13)$

b) vertex: $(2, 1)$, domain: $\{x \mid x \in R\}$, range: $\{y \mid y \leq 1, y \in R\}$, axis of symmetry: $x = 2$, x-intercepts occur at $(1, 0)$ and $(3, 0)$, y-intercept occurs at $(0, -3)$

c) vertex: $(0, -6)$, domain: $\{x \mid x \in R\}$, range: $\{y \mid y \leq -6, y \in R\}$, axis of symmetry: $x = 0$, no x-intercepts, y-intercept occurs at $(0, -6)$

d) vertex: $(-8, 6)$, domain: $\{x \mid x \in R\}$, range: $\{y \mid y \geq 6, y \in R\}$, axis of symmetry: $x = -8$, no x-intercepts, y-intercept occurs at $(0, 38)$

5. a) $y = (x - 5)^2 - 7$; the shapes of the graphs are the same with the parabola of $y = (x - 5)^2 - 7$ being translated 5 units to the right and 7 units down.

b) $y = -(x - 2)^2 - 3$; the shapes of the graphs are the same with the parabola of $y = -(x - 2)^2 - 3$ being reflected in the x-axis and translated 2 units to the right and 3 units down.

c) $y = 3(x - 1)^2 + 2$; the shape of the graph of $y = 3(x - 1)^2 + 2$ is narrower by a multiplication of the y-values by a factor of 3 and translated 1 unit to the right and 2 units up.

d) $y = \frac{1}{4}(x + 8)^2 + 4$; the shape of the graph of $y = \frac{1}{4}(x + 8)^2 + 4$ is wider by a multiplication of the y-values by a factor of $\frac{1}{4}$ and translated 8 units to the left and 4 units up.

6. a) 22 m **b)** 2 m **c)** 4 s

7. In order: roots, zeros, x-intercepts

8. a) $(3x + 4)(3x - 2)$ **b)** $(4r - 9s)(4r + 9s)$
c) $(x + 3)(2x + 9)$ **d)** $(xy + 4)(xy - 9)$
e) $5(a + b)(13a + b)$ **f)** $(11r + 20)(11r - 20)$

9. 7, 8, 9 or -9, -8, -7

10. 15 seats per row, 19 rows

11. 3.5 m

12. Example: Dallas did not divide the 2 out of the -12 in the first line or multiply the 36 by 2 and thus add 72 to the right side instead of 36 in line two. Doug made a sign error on the -12 in the first line. He should have calculated 200 as the value in the radical, not 80. When he simplified, he took $\sqrt{80}$ divided by 4 to get $\sqrt{20}$, which is not correct.
The correct answer is $3 \pm \frac{5}{\sqrt{2}}$ or $\frac{6 \pm 5\sqrt{2}}{2}$.

13. a) Example: square root, $x = \pm\sqrt{2}$
b) Example: factor, $m = 2$ and $m = 13$
c) Example: factor, $s = -5$ and $s = 7$
d) Example: use quadratic formula, $x = -\frac{1}{16}$ and $x = 3$

14. a) two distinct real roots
b) one distinct real root
c) no real roots

15. a) $85 = x^2 + (x + 1)^2$
b) Example: factoring, $x = -7$ and $x = 6$
c) The top is 7-in. by 7-in. and the bottom is 6-in. by 6-in.
d) Example: Negative lengths are not possible.

Unit 2 Test, pages 266 to 267

1. A
2. D
3. D
4. B
5. B
6. 76
7. \$900
8. 0.18
9. a) 53.5 cm **b)** 75.7 cm **c)** No
10. a) 47.5 m **b)** 6.1 s
11. 12 cm by 12 cm
12. a) $3x^2 + 6x - 672 = 0$
b) $x = -16$ and $x = 14$
c) 14 in., 15 in., and 16 in.
d) Negative lengths are not possible.

Chapter 5 Radical Expressions and Equations

5.1 Working With Radicals, pages 278 to 281

1.

Mixed Radical Form	Entire Radical Form
$4\sqrt{7}$	$\sqrt{112}$
$5\sqrt{2}$	$\sqrt{50}$
$-11\sqrt{8}$	$-\sqrt{968}$
$-10\sqrt{2}$	$-\sqrt{200}$

2. a) $2\sqrt{14}$ **b)** $15\sqrt{3}$
c) $2\sqrt[3]{3}$ **d)** $cd\sqrt{c}$

3. a) $6m^2\sqrt{2}$, $m \in R$ **b)** $2q\sqrt[3]{3q^2}$, $q \in R$
c) $-4st\sqrt[5]{5t}$, s, $t \in R$

4.

Mixed Radical Form	Entire Radical Form
$3n\sqrt{5}$	$\sqrt{45n^2}$, $n \geq 0$ or $-\sqrt{45n^2}$, $n < 0$
$-6\sqrt[3]{2}$	$\sqrt[3]{-432}$
$\frac{1}{2a}\sqrt[3]{7a}$	$\sqrt[3]{\frac{7}{8a^2}}$, $a \neq 0$
$4x\sqrt[3]{2x}$	$\sqrt[3]{128x^4}$

5. a) $15\sqrt{5}$ and $40\sqrt{5}$ **b)** $32z^4\sqrt{7}$ and $48z^2\sqrt{7}$
c) $-35\sqrt[4]{w^2}$ and $9w^2(\sqrt[4]{w^2})$
d) $6\sqrt[3]{2}$ and $18\sqrt[3]{2}$

6. a) $3\sqrt{6}$, $7\sqrt{2}$, 10
b) $-3\sqrt{2}$, -4, $-2\sqrt{\frac{7}{2}}$, $-2\sqrt{3}$
c) $\sqrt[3]{21}$, 2.8, $2\sqrt[3]{5}$, $3\sqrt[3]{2}$

7. Example: Technology could be used.

8. a) $4\sqrt{5}$ **b)** $10.4\sqrt{2} - 7$
c) $-4\sqrt[4]{11} + 14$ **d)** $-\frac{2}{3}\sqrt{6} + 2\sqrt{10}$

9. a) $12\sqrt{3}$ **b)** $6\sqrt{2} + 6\sqrt{7}$
c) $-28\sqrt{5} + 22.5$ **d)** $\frac{13}{4}\sqrt[3]{3} - 7\sqrt{11}$

10. a) $8a\sqrt{a}$, $a \geq 0$ **b)** $9\sqrt{2x} - \sqrt{x}$, $x \geq 0$
c) $2(r - 10)\sqrt[3]{5r}$, $r \in R$
d) $\frac{4w}{5} - 6\sqrt{2w}$, $w \geq 0$

11. $25.2\sqrt{3}$ m/s
12. $12\sqrt{2}$ cm
13. $12\sqrt[3]{3025}$ million kilometres
14. $2\sqrt{30}$ m/s $\approx$ 11 m/s
15. a) $2\sqrt{38}$ m **b)** $8\sqrt{19}$ m
16. $\sqrt{1575}$ mm^2, $15\sqrt{7}$ mm^2
17. $7\sqrt{5}$ units
18. $14\sqrt{2}$ m
19. Brady is correct. The answer can be further simplified to $10y^2\sqrt{y}$.
20. $4\sqrt{58}$
Example: Simplify each radical to see which is not a like radical to $12\sqrt{6}$.
21. $\sqrt{2} - \sqrt{3}$ m

22. $12\sqrt{2}$ cm

23. $5\sqrt{3}$ and $7\sqrt{3}$
It is an arithmetic sequence with a common difference of $2\sqrt{3}$.

24. a) $2\sqrt{75}$ and $108^{\frac{1}{2}}$ Example: Write the radicals in simplest form; then, add the two radicals with the greatest coefficients.
b) $2\sqrt{75}$ and $-3\sqrt{12}$ Example: Write the radicals in simplest form; then, subtract the radical with the least coefficient from the radical with the greatest coefficient.

25. a) Example: If $x = 3$,
$(-3)^2 = (-3)(-3)$
$(-3)^2 = 9$
$(-3)^2 = 3^2$
b) Example: If $x = 3$,
$\sqrt{3^2} = \sqrt{9}$
$\sqrt{9} = 3$
$\sqrt{3^2} \neq -3$

5.2 Multiplying and Dividing Radical Expressions, pages 289 to 293

1. a) $14\sqrt{15}$ **b)** -56 **c)** $4\sqrt[4]{15}$
d) $4x\sqrt{38x}$ **e)** $3y^3\left(\sqrt[3]{12y^2}\right)$ **f)** $\frac{3t^3}{2}\sqrt{6}$

2. a) $3\sqrt{11} - 4\sqrt{77}$
b) $-14\sqrt{10} - 6\sqrt{3} + \sqrt{26}$
c) $2y + \sqrt{y}$ **d)** $6z^2 - 5z^2\sqrt{3} + 2z\sqrt{3}$

3. a) $6\sqrt{2} + 12$ **b)** $1 - 9\sqrt{6}$
c) $\sqrt{15j} + 33\sqrt{5}$, $j \geq 0$ **d)** $3 - 16\sqrt[3]{4k}$

4. a) $8\sqrt{14} - 24\sqrt{7} + 2\sqrt{2} - 6$
b) -389
c) $-27 + 3\sqrt{5}$
d) $36\sqrt[3]{4} - 48\sqrt{13}(\sqrt[3]{2}) + 208$
e) $-4\sqrt{3} + 3\sqrt{30} - \sqrt{6} + 4\sqrt{2} - 6\sqrt{5} + 2$

5. a) $15c\sqrt{2} - 90\sqrt{c} + 2\sqrt{2c} - 12$, $c \geq 0$
b) $2 + 7\sqrt{5x} - 40x\sqrt{2x} - 140x^2\sqrt{10}$, $x \geq 0$
c) $258m - 144m\sqrt{3}$, $m \geq 0$
d) $20r\sqrt[3]{6r^2} + 30r\sqrt[3]{12r} - 16r\sqrt[3]{3} - 24\sqrt[3]{6r^2}$

6. a) $2\sqrt{2}$ **b)** -1
c) $3\sqrt{2}$ **d)** $\frac{9m\sqrt{35}}{7}$

7. a) $\frac{87\sqrt{11p}}{11}$ **b)** $\frac{6v^2\sqrt[3]{98}}{7}$

8. a) $2\sqrt{10}$ **b)** $\frac{-\sqrt{3m}}{m}$
c) $\frac{-\sqrt{15u}}{9u}$ **d)** $4\sqrt[3]{150t}$

9. a) $2\sqrt{3} - 1$; 11 **b)** $7 + \sqrt{11}$; 38
c) $8\sqrt{z} + 3\sqrt{7}$; $64z - 63$
d) $19\sqrt{h} - 4\sqrt{2h}$; $329h$

10. a) $10 + 5\sqrt{3}$ **b)** $\frac{-7\sqrt{3} + 28\sqrt{2}}{29}$
c) $\frac{\sqrt{35} + 2\sqrt{14}}{3}$ **d)** $\frac{-8 - \sqrt{39}}{5}$

11. a) $\frac{4r^2\sqrt{6} - 36r}{6r^2 - 81}$, $r \neq \frac{\pm 3\sqrt{6}}{2}$
b) $\frac{9\sqrt{2}}{2}$, $n > 0$
c) $\frac{16 + 4\sqrt{6t}}{8 - 3t}$, $t \neq \frac{8}{3}$, $t \geq 0$
d) $\frac{5\sqrt{30y} - 10\sqrt{3y}}{6}$, $y \geq 0$

12. $c^2 + 7c\sqrt{3c} + c^2\sqrt{c} + 7c^2\sqrt{3}$

13. a) When applying the distributive property, Malcolm distributed the 4 to both the whole number and the root. The 4 should only be distributed to the whole number. The correct answer is $12 + 8\sqrt{2}$.
b) Example: Verify using decimal approximations.
$\frac{4}{3 - 2\sqrt{2}} \approx 23.3137$
$12 + 8\sqrt{2} \approx 23.3137$

14. $\frac{\sqrt{5} + 1}{2}$

15. a) $T = \frac{\pi\sqrt{10L}}{5}$ **b)** $\frac{9\pi\sqrt{30}}{5}$ s

16. $860 + 172\sqrt{5}$ m

17. $-28 - 16\sqrt{3}$

18. a) $4\sqrt[3]{3}$ mm **b)** $2\sqrt[3]{6}$ mm **c)** $2\sqrt[3]{3} : \sqrt[3]{6}$

19. a) Lev forgot to switch the inequality sign when he divided by -5. The correct answer is $x < \frac{3}{5}$.
b) The square root of a negative number is not a real number.
c) Example: The expression cannot have a variable in the denominator or under the radical sign. $\frac{2x\sqrt{14}}{3\sqrt{5}}$

20. Olivia evaluated $\sqrt{25}$ as ± 5 in the third step. The final steps should be as follows:
$$\frac{\sqrt{3}(2c - 5c)}{3} = \frac{\sqrt{3}(-3c)}{3}$$
$$= -c\sqrt{3}$$

21. 735 cm^3

22. 12 m^2

23. $\left(\frac{15\sqrt{3}}{2}, \frac{9\sqrt{2}}{2}\right)$

24. $\frac{25x^2 + 30x\sqrt{x} + 9x}{625x^2 - 450x + 81}$ or $\frac{x(25x + 30\sqrt{x} + 9)}{(25x - 9)^2}$

25. a) $-3 \pm \sqrt{6}$ **b)** -6 **c)** 3
d) Examples: The answer to part b) is the opposite value of the coefficient of the middle term. The answer to part c) is the value of the constant.

26. $\frac{(\sqrt[c]{a})(\sqrt[n-1]{r})}{r}$

27. $(15\sqrt{14} + 42\sqrt{7} + 245\sqrt{2} + 7\sqrt{2702})$ cm^2

28. Example: You cannot multiply or divide radical expressions with different indices, or algebraic expressions with different variables.

29. Examples: To rationalize the denominator you need to multiply the numerator and denominator by a conjugate. To factor a difference of squares, each factor is the conjugate of the other. If you factor $3a - 16$ as a difference of squares, the factors are $\sqrt{3a} - 4$ and $\sqrt{3a} + 4$. The factors form a conjugate pair.

30. a) 3 m

b) $h(t) = -5(t - 1)^2 + 8$; $t = \sqrt{\frac{8 - h}{5}} + 1$

c) $\frac{19 + 4\sqrt{10}}{4}$ m
Example: The snowboarder starts the jump at $t = 0$ and ends the jump at $t = \frac{5 + 2\sqrt{10}}{5}$. The snowboarder will be halfway at $t = \frac{5 + 2\sqrt{10}}{10}$. Substitute this value of t into the original equation to find the height at the halfway point.

31. Yes, they are. Example: using the quadratic formula

32. a) $\frac{\sqrt[3]{6V(V - 1)^2}}{V - 1}$

b) A volume greater than one will result in a real ratio.

33. Step 1

$y = \sqrt{x}$

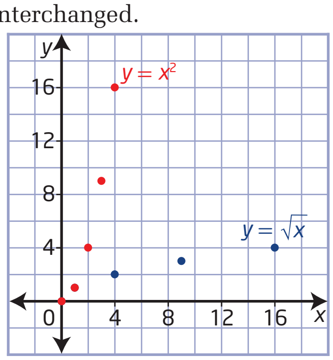
x	y
0	0
1	1
4	2
9	3
16	4

$y = x^2$

x	y
0	0
1	1
2	4
3	9
4	16

Step 2 Example: The values of x and y have been interchanged.

Step 3



Example: The restrictions on the radical function produce the right half of the parabola.

5.3 Radical Equations, pages 300 to 303

1. a) $3z$ **b)** $x - 4$
c) $4(x + 7)$ **d)** $16(9 - 2y)$

2. Example: Isolate the radical and square both sides. $x = 36$

3. a) $x = \frac{9}{2}$ **b)** $x = -2$ **c)** $x = -22$

4. a) $z = 25$ **b)** $y = 36$
c) $x = \frac{4}{3}$ **d)** $m = -\frac{49}{6}$

5. $k = -8$ is an extraneous root because if -8 is substituted for k, the result is a square root that equals a negative number, which cannot be true in the real-number system.

6. a) $n = 50$ **b)** no solution **c)** $x = -1$

7. a) $m = \pm 2\sqrt{7}$ **b)** $x = -16$, $x = 4$
c) $q = 2 + 2\sqrt{6}$ **d)** $n = 4$

8. a) $x = 10$ **b)** $x = -32$, $x = 2$
c) $d = 4$ **d)** $j = -\frac{2}{3}$

9. a) $k = 4$ **b)** $m = 0$
c) $j = 16$ **d)** $n = \frac{50 + 25\sqrt{3}}{2}$

10. a) $z = 6$ **b)** $y = 8$ **c)** $r = 5$ **d)** $x = 6$

11. The equation $\sqrt{x + 8} + 9 = 2$ has an extraneous root because simplifying it further to $\sqrt{x + 8} = -7$ has no solution.

12. Example: Jerry made a mistake when he squared both sides, because he squared each term on the right side rather than squaring $(x - 3)$. The right side should have been $(x - 3)^2 = x^2 - 6x + 9$, which gives $x = 8$ as the correct solution. Jerry should have listed the restriction following the first line: $x \geq -17$.

13. 11.1 m

14. a) $B \approx 6$ **b)** about 13.8 km/h

15. 1200 kg

16. $2 + \sqrt{n} = n$; $n = 4$

17. a) $v = \sqrt{19.6h}$, $h \geq 0$ **b)** 45.9 m
c) 34.3 m/s; A pump at 35 m/s will meet the requirements.

18. 6372.2 km

19. $a = \frac{3x - 4\sqrt{3x} + 4}{x}$

20. a) Example: $\sqrt{4a} = -8$
b) Example: $2 + \sqrt{x + 4} = x$

21. 2.9 m

22. 104 km

23. a) The maximum profit is \$10 000 and it requires 100 employees.
b) $n = 100 \pm \sqrt{10\,000 - P}$
c) $P \leq 10\,000$
d) domain: $n \geq 0$, $n \in W$
range: $P \leq 10\,000$, $P \in W$

24. Example: Both types of equations may involve rearranging. Solving a radical involves squaring both sides; using the quadratic formula involves taking a square root.

25. Example: Extraneous roots may occur because squaring both sides and solving the quadratic equation may result in roots that do not satisfy the original equation.

26. a) 6.8% **b)** $P_f = P_i(r + 1)^3$
c) 320, 342, 365, 390
d) geometric sequence with $r = 1.068...$

27. Step 1

1	$\sqrt{6+\sqrt{6}}$	2.906 800 603
2	$\sqrt{6+\sqrt{6+\sqrt{6}}}$	2.984 426 344
3	$\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}$	2.997 403 267
4	$\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}}$	2.999 567 18
5	$\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}}}$	2.999 927 862
6	$\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}}}}$	2.999 987 977
7	$\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}}}}}$	2.999 997 996
8	$\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}}}}}}$	2.999 999 666
9	$\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}}}}}}}$	2.999 999 944

Step 2 Example: 3.0

Step 3
$$\begin{aligned} x &= \sqrt{6 + x},\ x \geq -6 \\ x^2 &= 6 + x \\ (x - 3)(x + 2) &= 0 \\ x &= 3 \text{ or } x = -2 \end{aligned}$$

Step 4 The value of x must be positive because it is a square root.

Chapter 5 Review, pages 304 to 305

1. a) $\sqrt{320}$ **b)** $\sqrt[5]{-96}$
c) $\sqrt{63y^6}$ **d)** $\sqrt[3]{-108z^4}$
2. a) $6\sqrt{2}$ **b)** $6\sqrt{10}$
c) $3m\sqrt{3}$ **d)** $2xy^2(\sqrt[3]{10x^2})$
3. a) $\sqrt{13}$ **b)** $-4\sqrt{7}$ **c)** $\sqrt[3]{3}$
4. a) $-33x\sqrt{5x} + 14\sqrt{3x},\ x \geq 0$
b) $\frac{3}{10}\sqrt{11a} + 12a\sqrt{a},\ a \geq 0$
5. $3\sqrt{42}$ Example: Simplify each radical to see if it equals $8\sqrt{7}$.
6. $3\sqrt{7}$, 8, $\sqrt{65}$, $2\sqrt{17}$
7. a) $v = 13\sqrt{d}$ **b)** 48 km/h
8. $8\sqrt{6}$ km
9. a) false **b)** true **c)** false
10. a) $2\sqrt{3}$ **b)** $-30f^4\sqrt{3}$ **c)** $6\sqrt[4]{9}$
11. a) -1 **b)** $83 - 20\sqrt{6}$
c) $a^2 + 17a\sqrt{a} + 42a,\ a \geq 0$
12. Yes; they are conjugate pairs and the solutions to the quadratic equation.
13. a) $\frac{\sqrt{2}}{2}$ **b)** $\frac{-(\sqrt[3]{25})^2}{25}$ **c)** $\frac{-4a\sqrt{2}}{3}$
14. a) $\frac{-8 - 2\sqrt{3}}{13}$ **b)** $\frac{2\sqrt{35} + 7}{13}$
c) $\frac{12 - 6\sqrt{3m}}{4 - 3m},\ m \geq 0 \text{ and } m \neq \frac{4}{3}$
d) $\frac{a^2 + 2a\sqrt{b} + b}{a^2 - b},\ b \geq 0 \text{ and } b \neq a^2$
15. $4\sqrt{2} + 8\sqrt{5}$
16. a) $\frac{5\sqrt{6}}{18}$ **b)** $\frac{-2a^2\sqrt{2}}{3}$
17. $\frac{24 + 6\sqrt{2}}{7}$ units
18. a) radical defined for $x \geq 0$; solution: $x = 49$
b) radical defined for $x \leq 4$; no solution
c) radical defined for $x \geq 0$; solution: $x = 18$
d) radical defined for $x \geq 0$; solution: $x = 21$
19. a) restriction: $x \geq \frac{12}{7}$; solution: $x = \frac{9}{2}$
b) restriction: $y \geq 3$; solution: $y = 3$ and $y = 4$
c) restriction: $n \geq \frac{-25}{7}$; solution: $n = 8$
d) restriction: $0 \leq m \leq 24$; solution: $m = 12$
e) no restrictions; solution: $x = -21$
20. Example: Isolate the radical; then, square both sides. Expand and simplify. Solve the quadratic equation. $n = -3$ is an extraneous root because when it is substituted into the original equation a false statement is reached.
21. 33.6 m

Chapter 5 Practice Test, pages 306 to 307

1. B
2. D
3. C
4. D
5. B
6. C
7. $3\sqrt{11}$, $5\sqrt{6}$, $\sqrt{160}$, $9\sqrt{2}$
8. $\frac{-12n\sqrt{10} - 288n\sqrt{5}}{287}$
9. The radical is defined for $x \leq -\sqrt{5}$ and $x \geq \sqrt{5}$. The solution is $x = \frac{7}{3}$.
10. The solution is $\frac{102 + 6\sqrt{214}}{25}$. The extraneous root is $\frac{102 - 6\sqrt{214}}{25}$.
11. $15\sqrt{2}$
12. $9\sqrt{2}$ km
13. a) $\sqrt{6}$ **b)** $\sqrt{y - 3}$ **c)** $\sqrt[3]{49}$
14. $6300
15. She is correct.
16. a) $\sqrt{1 + x^2}$ **b)** $2\sqrt{30}$ units
17. a) $R = \frac{P}{I^2}$ **b)** 400 Ω
18. a) $x = \sqrt{\frac{SA}{6}}$ **b)** $\frac{\sqrt{22}}{2}$ cm **c)** $\sqrt{2}$
19. a) $3713.15 = 3500(1 + i)^2$ **b)** 3%

Chapter 6 Rational Expressions and Equations

6.1 Rational Expressions, pages 317 to 321

1. a) 18 **b)** $14x$ **c)** 7
d) $4x - 12$ **e)** 8 **f)** $y + 2$

2. a) Divide both by pq.
b) Multiply both by $(x - 4)$.
c) Divide both by $(m - 3)$.
d) Multiply both by $(y^2 + y)$.

3. a) 0 **b)** 1 **c)** -5
d) none **e)** ± 1 **f)** none

4. The following values are non-permissible because they would make the denominator zero, and division by zero is not defined.
a) 4 **b)** 0 **c)** $-2, 4$
d) $-3, 1$ **e)** 0 **f)** $\frac{4}{3}, -\frac{5}{2}$

5. a) $r \neq 0$ **b)** $t \neq \pm 1$
c) $x \neq 2$ **d)** $g \neq 0, \pm 3$

6. a) $\frac{2}{3}$; $c \neq 0, 5$ **b)** $\frac{3(2w + 3)}{2(3w + 2)}$; $w \neq -\frac{2}{3}, 0$
c) $\frac{x + 7}{2x - 1}$; $x \neq \frac{1}{2}, 7$ **d)** $-\frac{1}{2}$; $a \neq -2, 3$

7. a) x^2 is not a factor.
b) Factor the denominator. Set each factor equal to zero and solve. $x \neq -3, 1$
c) Factor the numerator and denominator. Determine the non-permissible values. Divide like factors. $\frac{x + 1}{x + 3}$

8. a) $\frac{3r}{2p}$, $r \neq 0, p \neq 0$ **b)** $-\frac{3}{5}$, $x \neq 2$
c) $\frac{b - 4}{2(b - 6)}$, $b \neq \pm 6$ **d)** $\frac{k + 3}{2(k - 3)}$, $k \neq -\frac{5}{2}, 3$
e) -1, $x \neq 4$ **f)** $\frac{5(x + y)}{x - y}$, $x \neq y$

9. Sometimes true. The statement is not true when $x = 3$.

10. There may have been another factor that divided out. For example: $\frac{y(y + 3)}{(y - 6)(y + 3)}$

11. yes, provided the non-permissible value, $x \neq 5$, is discussed

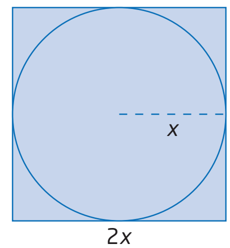
12. Examples: $\frac{x^2 + 2x + 1}{x^2 + 3x + 2}$, $\frac{x^2 + 4x + 4}{x^2 + 5x + 6}$, $\frac{2x^2 + 5x + 2}{3x^2 + 7x + 2}$
Write a rational expression in simplest form, and multiply both the numerator and the denominator by the same factor. For example, the first expression was obtained as follows: $\frac{x + 1}{x + 2} = \frac{(x + 1)(x + 1)}{(x + 2)(x + 1)}$.

13. Shali divided the term 2 in the numerator and the denominator. You may only divide by factors. The correct solution is the second step, $\frac{g + 2}{2}$.

14. Example: $\frac{2p}{p^2 + p - 2}$

15. a) $\frac{2n^2 + 11n + 12}{2n^2 - 32}$ **b)** $\frac{2n + 3}{2(n - 4)}$, $n \neq \pm 4$

16. a)



b) $\frac{\pi x^2}{4x^2}$
c) $x \neq 0$
d) $\frac{\pi}{4}$
e) 79%

17. a) The non-permissible value, -2, does not make sense in the context as the mass cannot be -2 kg.
b) $p = 0$ **c)** 900 kg

18. a) $\frac{50}{q}$, $q \neq 0$ **b)** $\frac{100}{p - 4}$, $p \neq 4$

19. a) \$620 **b)** $\frac{350 + 9n}{n}$
c) \$20.67

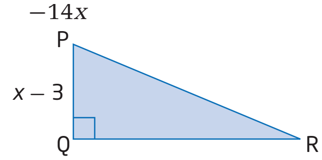
20. a) No; she divided by the term, 5, not a factor.
b) Example: If $m = 5$ then $\frac{5}{10} \neq \frac{1}{6}$.

21. a) Multiply by $\frac{5}{5}$. **b)** Multiply by $\frac{x - 2}{x - 2}$.

22. a) $\frac{4x - 8}{12}$ **b)** $\frac{3x - 6}{9}$
c) $\frac{2x^2 + x - 10}{6x + 15}$

23. a) $\frac{25b}{5b}$ **b)** $\frac{4a^2bx + 4a^2b}{12a^2b}$
c) $\frac{2b - 2a}{-14x}$

24. a)



b) $2(x + 2)$ **c)** $x \neq 3$

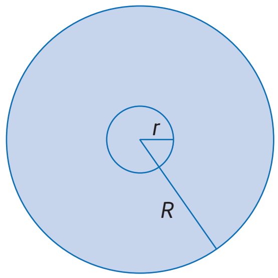
25. a) $\frac{(2x - 1)(3x + 1)}{(3x + 1)(3x - 1)} = \frac{(2x - 1)}{(3x - 1)}$, $x \neq \pm\frac{1}{3}$
b) In the last step: $\frac{n + 3}{-n} = \frac{-n - 3}{n}$, $n \neq 0, \frac{5}{2}$

26. a) $\frac{x + 6}{x + 3}$, $x \neq \pm 3$
b) $(2x - 7)(2x - 5)$, $x \neq -3$
c) $\frac{(x - 3)(x + 2)}{(x + 3)(x - 2)}$, $x \neq -3, -1, 2$
d) $\frac{(x + 5)(x + 3)}{3}$, $x \neq \pm 1$

27. $6x^2 + \frac{19}{2}x + 2$, $x \neq \frac{1}{4}, \frac{3}{2}$

28. a) Lt

b)



$\pi(R - r)(R + r)$

c) $L = \dfrac{\pi(R + r)(R - r)}{t}$, $t > 0$, $R > r$, and t, R, and r should be expressed in the same units.

29. Examples:

a) $\dfrac{2}{(x + 2)(x - 5)}$

b) $\dfrac{x^2 + 3x}{x^2 + 2x - 3}$; the given expression has a non-permissible value of -1. Multiply the numerator and denominator by a factor, $x + 3$, that has a non-permissible value of -3.

30. a) Example: if $y = 7$,

$$\frac{y-3}{4} = \frac{7-3}{4} = 1 \quad \text{and} \quad \frac{2y^2 - 5y - 3}{8y + 4} = \frac{2(7^2) - 5(7) - 3}{8(7) + 4} = \frac{60}{60} = 1$$

b) $\dfrac{2y^2 - 5y - 3}{8y + 4} = \dfrac{(2y + 1)(y - 3)}{4(2y + 1)} = \dfrac{y - 3}{4}$

c) The algebraic approach, in part b), proves that the expressions are equivalent for all values of y, except the non-permissible value.

31. a) $m = \dfrac{p - 8}{p + 1}$

b) Any value $-1 < p < 8$ will give a negative slope. Example: If $p = 0$, $m = \dfrac{-8}{1}$.

c) If $p = -1$, then the expression is undefined, and the line is vertical.

32. Example: $\dfrac{12}{15} = \dfrac{(3)(4)}{(3)(5)} = \dfrac{4}{5}$,

$$\frac{x^2 - 4}{x^2 + 5x + 6} = \frac{(x + 2)(x - 2)}{(x + 3)(x + 2)} = \frac{(x - 2)}{(x + 3)}, \ x \neq -3, -2$$

6.2 Multiplying and Dividing Rational Expressions, pages 327 to 330

1. a) $9m$, $c \neq 0$, $f \neq 0$, $m \neq 0$

b) $\dfrac{a - 5}{5(a - 1)}$, $a \neq -5, 1$, $a \neq b$

c) $\dfrac{4(y - 7)}{(2y - 3)(y - 1)}$, $y \neq -3, 1, \pm\dfrac{3}{2}$

2. a) $\dfrac{d - 10}{4}$, $d \neq -10$ **b)** $\dfrac{a - 1}{a - 3}$, $a \neq \pm 3, -1$

c) $\dfrac{1}{2}$, $z \neq 4, \pm\dfrac{5}{2}$

d) $\dfrac{p + 1}{3}$, $p \neq -3, 1, \dfrac{3}{2}, \dfrac{1}{2}$

3. a) $\dfrac{t}{2}$ **b)** $\dfrac{3}{2x - 1}$

c) $\dfrac{y - 3}{8}$ **d)** $\dfrac{p - 3}{2p - 3}$

4. a) $s \neq 0$, $t \neq 0$ **b)** $r \neq \pm 7, 0$

c) $n \neq \pm 1$

5. $x - 3$, $x \neq -3$

6. $\dfrac{y}{y + 3}$, $y \neq \pm 3, 0$

7. a) $\dfrac{3 - p}{p - 3} = \dfrac{-1(p - 3)}{p - 3} = -1$, $p \neq 3$

b) $\dfrac{7k - 1}{3k} \times \dfrac{1}{1 - 7k}$

$= \dfrac{7k - 1}{3k} \times \dfrac{1}{-1(7k - 1)}$

$= \dfrac{-1}{3k}$ or $-\dfrac{1}{3k}$, $k \neq 0, \dfrac{1}{7}$

8. a) $\dfrac{w - 2}{3}$, $w \neq -2, -\dfrac{3}{2}$

b) $\dfrac{v^2}{v + 3}$, $v \neq 0, -3, 5,$

c) $\dfrac{-1(3x - 1)}{x + 5}$, $x \neq -5, 2, -\dfrac{1}{3}$

d) $\dfrac{-2}{y - 2}$, $y \neq \pm 1, 2, -\dfrac{1}{2}, \dfrac{3}{4}$

9. -3 and -2 are the non-permissible values of the original denominators, and -1 is the non-permissible value when the reciprocal of the divisor is created.

10. $\dfrac{n^2 - 4}{n + 1} \div (n - 2)$; $\dfrac{n + 2}{n + 1}$, $n \neq -1, 2$

11. a) $\dfrac{(x - 3)}{5}(60) = 12x - 36$ metres

b) $900 \div \dfrac{600}{n + 1} = \dfrac{3n + 3}{2}$ kilometres per hour, $n \neq -1$

c) $\dfrac{x^2 + 2x + 1}{(2x - 3)(x + 1)} = \dfrac{x + 1}{2x - 3}$ metres, $x \neq \dfrac{3}{2}, -1$

12. They are reciprocals of each other. This is always true. The divisor and dividend are interchanged.

13. Example:

$$1 \text{ yd}\left(\frac{3 \text{ ft}}{1 \text{ yd}}\right)\left(\frac{12 \text{ in.}}{1 \text{ ft}}\right)\left(\frac{2.54 \text{ cm}}{1 \text{ in.}}\right) = 91.44 \text{ cm}$$

14. a) Tessa took the reciprocal of the dividend, not the divisor.

b) $= \dfrac{(c + 6)(c - 6)}{2c} \times \dfrac{8c^2}{c + 6}$

$= 4c(c - 6)$

$= 4c^2 - 24c$, $c \neq 0, -6$

c) The correct answer is the reciprocal of Tessa's answer. Taking reciprocals of either factor produces reciprocal answers.

15. $(x^2 - 9) \div \dfrac{x^2 - 2x - 3}{x + 1} = x + 3;\ x \neq 3,\ x \neq -1$

16. $\left(\dfrac{1}{2}\right)\left(\dfrac{x+2}{x-8}\right)\left(\dfrac{x^2 - 7x - 8}{x^2 - 4}\right);\ \dfrac{x+1}{2(x-2)},\ x \neq \pm 2,\ 8$

17. a) $K = \dfrac{Pw}{2h},\ m \neq 0,\ w \neq 0,\ h \neq 0$

b) $y = \dfrac{2\pi r}{x},\ d \neq 0,\ x \neq 0,\ r \neq 0$

c) $a = vw,\ w \neq 0$

18. $2(n - 4)$, n $\neq -4, 1, 4$

19. a) Yes; when the two binomial factors are multiplied, you get the expression $x^2 - 5$.

b) $\dfrac{x + \sqrt{7}}{x - \sqrt{3}}$

c) $x + \sqrt{7}$; it is the same.

20. a) approximately 290 m

b) $\dfrac{(x+3)^2}{4g(x-5)^2}$ metres

21. Agree. Example: $\left(\dfrac{2}{3}\right)\left(\dfrac{1}{5}\right) = \dfrac{(2)(1)}{(3)(5)} = \dfrac{2}{15}$,

and $\dfrac{2}{3} \div \dfrac{1}{5} = \left(\dfrac{2}{3}\right)\left(\dfrac{5}{1}\right) = \dfrac{10}{3}$

$$\begin{aligned}\frac{(x+2)}{(x+3)} \times \frac{(x+1)}{(x+3)} &= \frac{(x+2)(x+1)}{(x+3)(x+3)}\\ &= \frac{x^2 + 3x + 2}{x^2 + 6x + 9},\ x \neq -3\end{aligned}$$

$$\begin{aligned}\frac{(x+2)}{(x+3)} \div \frac{(x+1)}{(x+3)} &= \frac{(x+2)}{(x+3)} \times \frac{(x+3)}{(x+1)}\\ &= \frac{(x+2)}{(x+1)},\ x \neq -3,-1\end{aligned}$$

22. a) $\dfrac{p+2}{4-p}$ **b)** $\dfrac{p-4}{p+2}$

23. a) $\tan \mathrm{B} = \dfrac{b}{a}$ **b)** $\dfrac{\frac{b}{c}}{\frac{a}{c}} = \dfrac{b}{a}$

c) They are the same; $\tan \mathrm{B} = \dfrac{\sin \mathrm{B}}{\cos \mathrm{B}}$.

6.3 Adding and Subtracting Rational Expressions, pages 336 to 340

1. a) $\dfrac{7x}{6}$ **b)** $\dfrac{10}{x},\ x \neq 0$

c) $\dfrac{4t+4}{5}$ or $\dfrac{4(t+1)}{5}$ **d)** $m,\ m \neq -1$

e) $a + 3,\ a \neq 4$

2.
$$\begin{aligned}\frac{3x-7}{9} + \frac{6x+7}{9} &= \frac{3x - 7 + 6x + 7}{9}\\ &= \frac{9x}{9}\\ &= x\end{aligned}$$

3. a) $\dfrac{-4x + 13}{(x-3)(x+1)},\ x \neq -1,\ 3$

b) $\dfrac{3x(x+6)}{(x-2)(x+10)(x+2)},\ x \neq -10,\ \pm 2$

4. a) 24, 12; LCD = 12

b) $50a^3y^3,\ 10a^2y^2$; LCD $= 10a^2y^2$

c) $(9 - x^2)(3 + x),\ 9 - x^2$; LCD $= 9 - x^2$ or $(3 - x)(3 + x)$

5. a) $\dfrac{11}{15a},\ a \neq 0$ **b)** $\dfrac{x+9}{6x},\ x \neq 0$

c) $\dfrac{2(10x - 3)}{5x},\ x \neq 0$

d) $\dfrac{(2z - 3x)(2z + 3x)}{xyz},\ x \neq 0,\ y \neq 0,\ z \neq 0$

e) $\dfrac{4st + t^2 - 4}{10t^3},\ t \neq 0$

f) $\dfrac{6bxy^2 - 2ax + a^2b^2y}{a^2b^2y},\ a \neq 0,\ b \neq 0,\ y \neq 0$

6. a) $\dfrac{-5x + 18}{(x+2)(x-2)},\ x \neq \pm 2$

b) $\dfrac{3x - 11}{(x-4)(x+3)},\ x \neq -3,\ 4$

c) $\dfrac{2x(x-4)}{(x-2)(x+2)},\ x \neq \pm 2$

d) $\dfrac{3}{y},\ y \neq -1,\ 0$

e) $\dfrac{-3(5h + 9)}{(h+3)(h+3)(h-3)},\ h \neq \pm 3$

f) $\dfrac{(2x-3)(x+2)}{x(x-2)(x-1)(x+3)},\ x \neq -3,\ 0,\ 1,\ 2$

7. a) $\dfrac{2(x^2 - 3x + 5)}{(x-5)(x+5)},\ x \neq \pm 5,\ \dfrac{1}{2}$

b) $\dfrac{-x + 4}{(x-2)(x+3)},\ x \neq -3,\ 0,\ 2,\ 8$

c) $\dfrac{n + 8}{(n-4)(n-2)},\ n \neq 2,\ 3,\ 4$

d) $\dfrac{w + 9}{(w+3)(w+4)},\ w \neq -2,\ -3,\ -4$

8. In the third line, multiplying by -7 should give $-7x + 14$. Also, she has forgotten to list the non-permissible values.

$$\begin{aligned}&= \frac{6x + 12 + 4 - 7x + 14}{(x-2)(x+2)}\\ &= \frac{-x + 30}{(x-2)(x+2)},\ x \neq \pm 2\end{aligned}$$

9. Yes. Factor -1 from the numerator to create $-1(x - 5)$. Then, the expression simplifies to $\dfrac{-1}{x+5}$.

10. a) $\dfrac{2x}{x+3},\ x \neq 0,\ \pm 3$

b) $\dfrac{3(t+6)}{2(t-3)},\ t \neq -6,\ -2,\ 0,\ 3$

c) $\dfrac{3m}{m+3},\ m \neq 0,\ -\dfrac{3}{2},\ -3$

d) $\dfrac{x}{x-2},\ x \neq \pm 4,\ 2$

11. a) $\dfrac{\frac{AD}{B}+C}{D} = \left(\dfrac{AD+CB}{B}\right) \div D$

$= \left(\dfrac{AD+CB}{B}\right)\left(\dfrac{1}{D}\right)$

$= \dfrac{AD+CB}{BD}$

$= \dfrac{AD}{BD} + \dfrac{CB}{BD}$

$= \dfrac{A}{B} + \dfrac{C}{D}$

b) $\left[\dfrac{\left(\frac{AB}{D}+C\right)D}{F} + E\right]F = \left(\dfrac{AB}{D}+C\right)D + EF$

$= AB + CD + EF$

12. $\dfrac{\sqrt{5x^2-2x+1}}{4}$

13. a) $\dfrac{200}{m}$ tells the expected number of weeks to gain 200 kg; $\dfrac{200}{m+4}$ tells the number of weeks to gain 200 kg when the calf is on the healthy growth program.

b) $\dfrac{200}{m} - \dfrac{200}{m+4}$

c) $\dfrac{800}{m(m+4)}$, $m \neq 0, -4$; yes, the expressions are equivalent.

14. a) $\dfrac{200}{n}$ minutes

b) $\left(\dfrac{200}{n} + \dfrac{500}{n} + \dfrac{1000}{n}\right)$ minutes

c) $\dfrac{1700}{n}$ minutes; the time it would take to type all three assignments

d) $\left(\dfrac{200}{n} + \dfrac{500}{n-5} + \dfrac{1000}{n-10}\right) - \dfrac{1700}{n}$

$= \dfrac{12\,500n - 75\,000}{n(n-5)(n-10)}$

15. a) $\dfrac{2x^2+13}{(x-4)(x+5)}$, $x \neq -5, -2, 0, 3, 4$

b) $\dfrac{-9}{(x-1)(x+2)}$, $x \neq -3, -2, 0, 1, \dfrac{1}{2}$

c) $\dfrac{3(1-4x)}{(x+5)(x-4)}$, $x \neq -5, -2, 0, 3, 4$

d) $\dfrac{15}{(x+6)(x+3)}$, $x \neq 0, -2, -3, -6, -\dfrac{1}{2}$

16. $\left(\dfrac{20}{x} + \dfrac{16}{x-2}\right)$ hours

17. Example: In a three-person relay, Barry ran the first 12 km at a constant rate. Jim ran the second leg of 8 km at a rate 3 km/h faster, and Al ran the last leg of 5 km at a rate 2 km/h slower than Barry. The total time for the relay would be $\left(\dfrac{12}{x} + \dfrac{8}{x+3} + \dfrac{5}{x-2}\right)$ hours.

18. a) Incorrect: $\dfrac{a}{b} - \dfrac{b}{a} = \dfrac{a^2-b^2}{ab}$. Find the LCD first, do not just combine pieces.

b) Incorrect: $\dfrac{ca+cb}{c+cd} = \dfrac{a+b}{1+d}$. Factor c from the numerator and from the denominator, remembering that $c(1) = c$.

c) Incorrect: $\dfrac{a}{4} - \dfrac{6-b}{4} = \dfrac{a-6+b}{4}$. Distribute the subtraction to both terms in the numerator of the second rational expression by first putting the numerator in brackets.

d) Incorrect: $\dfrac{1}{1-\frac{a}{b}} = \dfrac{b}{b-a}$. Simplify the denominator first, and then divide.

e) Incorrect: $\dfrac{1}{a-b} = \dfrac{-1}{b-a}$. Multiplying both numerator and denominator by -1, which is the same as multiplying the whole expression by 1, changes every term to its opposite.

19. a) Agree. Each term in the numerator is divided by the denominator, and then can be simplified.

b) Disagree. If Keander was given the rational expression $\dfrac{3x-7}{x}$, there are multiple original expressions that he could come up with, for example $\dfrac{2x-1}{x} + \dfrac{x-6}{x}$ or $\dfrac{x^2-x+11}{x} - \dfrac{x^2-4x+18}{x}$.

20. a) $\dfrac{12}{13}\ \Omega$ **b)** $\dfrac{R_1R_2R_3}{R_2R_3 + R_1R_3 + R_1R_2}$

c) $\dfrac{12}{13}\ \Omega$

d) the simplified form from part b), because with it you do not need to find the LCD first

21. Example:

Arithmetic:	Algebra:
If $\frac{2}{3} = \frac{6}{9}$, then $\frac{2}{3} = \frac{2-6}{3-9}$ $= \frac{-4}{-6}$ $= \frac{2}{3}$	If $\frac{x}{2} = \frac{3x}{6}$, then $\frac{x}{2} = \frac{x-3x}{2-6}$ $= \frac{-2x}{-4}$ $= \frac{x}{2}$

22. a) $\dfrac{-2p+9}{2(p-3)}$, $p \neq 3$

b) $\dfrac{3}{0}$; the slope is undefined when $p = 3$, so this is a vertical line through A and B.

c) The slope is negative.

d) When $p = 4$, the slope is positive; from $p = 5$ to $p = 10$ the slope is always negative.

23. 3

24. Examples: $\frac{2}{5} + \frac{1}{5} = \frac{2+1}{5} = \frac{3}{5}$ and

$\frac{2}{5} + \frac{1}{3} = \frac{2(3) + 1(5)}{15} = \frac{11}{15}$

$\frac{2}{x} + \frac{1}{x} = \frac{2+1}{x} = \frac{3}{x}$ and

$\frac{2}{x} + \frac{1}{y} = \frac{2(y) + 1(x)}{xy} = \frac{2y + x}{xy}$

25. a) The student's suggestion is correct.

Example: find the average of $\frac{1}{2}$ and $\frac{3}{4}$.

$$\left(\frac{1}{2} + \frac{3}{4}\right) \div 2 = \left(\frac{2+3}{4}\right) \times \left(\frac{1}{2}\right) = \frac{5}{8}$$

Halfway between $\frac{1}{2}$ and $\frac{3}{4}$, or $\frac{4}{8}$ and $\frac{6}{8}$, is $\frac{5}{8}$.

b) $\frac{13}{4a}$, $a \neq 0$

26. Yes. Example: $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$ and $\frac{1}{2} + \frac{1}{3} = \frac{1}{\frac{6}{5}} = \frac{5}{6}$

$\frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy}$ and $\frac{1}{x} + \frac{1}{y} = \frac{1}{\frac{xy}{x+y}} = \frac{x+y}{xy}$

27. a) $\frac{1}{u} + \frac{1}{v} = \frac{u+v}{uv}$ **b)** 5.93 cm

c) $f = \frac{uv}{u+v}$

28. Step 3 Yes

Step 4 a) $A = 2, B = 1$

b) $A = 3, B = 3$

Step 5 Always:

$$\frac{3}{x-4} + \frac{-2}{x-1} = \frac{3(x-1) + -2(x-4)}{(x-4)(x-1)} = \frac{x+5}{(x-4)(x-1)}$$

6.4 Rational Equations, pages 348 to 351

1. a) $4(x-1) - 3(2x-5) = 5 + 2x$

b) $2(2x+3) + 1(x+5) = 7$

c) $4x - 5(x-3) = 2(x+3)(x-3)$

2. a) $f = -1$ **b)** $y = 6, y \neq 0$

c) $w = 12, w \neq 3, 6$

3. a) $t = 2$ or $t = 6, t \neq 0$ **b)** $c = 2, c \neq \pm 3$

c) $d = -2$ or $d = 3, d \neq -4, 1$

d) $x = 3, x \neq \pm 1$

4. No. The solution is not a permissible value.

5. a) $\frac{3-x}{x^2} - \frac{2}{x}, \frac{3-3x}{x^2}, x > 0$

b) $\frac{3-x}{x^2} \times \frac{2}{x}, \frac{6-2x}{x^3}, x > 0$

c) $x = \frac{1}{2}$

6. a) $b = 3.44$ or $b = 16.56$

b) $c = -3.54$ or $c = 2.54$

7. $l = 15(\sqrt{5} + 1)$, 48.5 cm

8. The numbers are 5 and 20.

9. The numbers are 3 and 4.

10. 30 students

11. The integers are 5 and 6.

12. a) Less than 2 min. There is more water going in at once.

b)

	Time to Fill Tub (min)	Fraction Filled in 1 min	Fraction Filled in x minutes
Cold Tap	2	$\frac{1}{2}$	$\frac{x}{2}$
Hot Tap	3	$\frac{1}{3}$	$\frac{x}{3}$
Both Taps	x	$\frac{1}{x}$	1

c) $\frac{x}{2} + \frac{x}{3} = 1$ **d)** 1.2 min

13. 6 h

14. a)

	Distance (km)	Rate (km/h)	Time (h)
Downstream	18	$x + 3$	$\frac{18}{x+3}$
Upstream	8	$x - 3$	$\frac{8}{x-3}$

b) $\frac{18}{x+3} = \frac{8}{x-3}$ **c)** 7.8 km/h

d) $x \neq \pm 3$

15. 28.8 h

16. 5.7 km/h

17. about 50 km/h west of Swift Current, and 60 km/h east of Swift Current

18. about 3.5 km/h

19.

	Reading Rate in Pages per Day	Number of Pages Read	Number of Days
First Half	x	259	$\frac{259}{x}$
Second Half	$x + 12$	259	$\frac{259}{x+12}$

about 20 pages per day for the first half of the book

20. a) 2 L **b)** 4.5 L

21. $a = \pm\frac{1}{3}$

22. a) $\frac{\frac{1}{a} + \frac{1}{b}}{2} = \frac{1}{x}$, $x = \frac{2ab}{a+b}$

b) 4 and 12, or −6 and 2

23. a)

$$\frac{1}{x} - \frac{1}{y} = a \qquad \text{or} \qquad \frac{1}{x} - \frac{1}{y} = a$$

$$\begin{aligned} y - x &= axy \\ y &= axy + x \\ y &= x(ay + 1) \\ \frac{y}{ay+1} &= x \end{aligned} \qquad\qquad \begin{aligned} \frac{y-x}{xy} &= a \\ y - x &= axy \\ y &= axy + x \\ y &= x(ay+1) \\ \frac{y}{ay+1} &= x \end{aligned}$$

In both, $x \neq 0$, $y \neq 0$, $ay \neq -1$.

b) $\frac{2d - gt^2}{2t} = v_0$, $t \neq 0$

c) $n = \frac{Ir}{E - IR}$, $n \neq 0$, $R \neq -\frac{r}{n}$, $E \neq Ir$, $I \neq 0$

24. a) Rational expressions combine operations and variables in one or more terms. Rational equations involve rational expressions and an equal sign.

Example: $\frac{1}{x} + \frac{1}{y}$ is a rational expression, which can be simplified but not solved.

$\frac{1}{x} + \frac{1}{2x} = 5$ is a rational equation that can be solved.

b) Multiply each term by the LCD. Then, divide common factors.

$$\frac{5}{x} - \frac{1}{x-1} = \frac{1}{x-1}$$

$$\cancel{x}(x-1)\left(\frac{5}{\cancel{x}}\right) - x\cancel{(x-1)}\left(\frac{1}{\cancel{x-1}}\right) = x\cancel{(x-1)}\left(\frac{1}{\cancel{x-1}}\right)$$

Simplify the remaining factors by multiplying. Solve the resulting linear equation.

$$(x-1)(5) - x(1) = x(1)$$
$$5x - 5 - x = x$$
$$3x = 5$$
$$x = \frac{5}{3}$$

c) Example: Add the second term on the left to both sides, to give $\frac{5}{x} = \frac{2}{x-1}$.

25. a) 5.5 pages per minute

b) and **c)** Answers may vary.

26. a) 46

b) $\frac{45}{50}$ is 90%, so $\frac{10(40) + 5(x)}{15} = 45$. For this equation to be true, you would need 55 on each of the remaining quizzes, which is not possible.

27. a) The third line should be

$$2x + 2 - 3x^2 + 3 = 5x^2 - 5x$$
$$0 = 8x^2 - 7x - 5$$

b) $\frac{7 \pm \sqrt{209}}{16}$

c) $x = 1.34$ or $x = -0.47$

Chapter 6 Review, pages 352 to 354

1. a) 0. It creates an expression that is undefined.

b) Example: Some rational expressions have non-permissible values.

For $\frac{2}{x-3}$, x may not take on the value 3.

2. Agree. Example: There are an unlimited number of ways of creating equivalent expressions by multiplying the numerator and denominator by the same term; because you are actually multiplying by 1 $\left(\frac{X}{X} = 1\right)$.

3. a) $y \neq 0$ **b)** $x \neq -1$ **c)** none

d) $a \neq -2, 3$ **e)** $m \neq -1, \frac{3}{2}$ **f)** $t \neq \pm 2$

4. a) -6; $s \neq 0$ **b)** -1; $x \neq \frac{3}{5}$ **c)** $-\frac{1}{4}$; $b \neq 2$

5. a) $\frac{2x^2 - 6x}{10x}$ **b)** $\frac{1}{x+3}$

c) $\frac{3c - 6d}{9f}$ **d)** $\frac{m^2 - 3m - 4}{m^2 - 16}$

6. a) Factor the denominator(s), set each factor equal to zero, and solve.

Example: Since $\frac{m-4}{m^2-9} = \frac{m-4}{(m+3)(m-3)}$, the non-permissible values are ± 3.

b) i) $x - 5$, $x \neq -\frac{2}{3}$ **ii)** $\frac{a}{a+3}$, $a \neq \pm 3$

iii) $-\frac{3}{4}$, $x \neq y$ **iv)** $\frac{9x-2}{2}$, $x \neq \frac{2}{9}$

7. a) $x + 1$

b) $x \neq 1$, as this would make a width of 0, and $x \neq -1$, as this would make a length of 0.

8. Example: The same processes are used for rational expressions as for fractions. Multiplying involves finding the product of the numerators and then the product of the denominators. To divide, you multiply by the reciprocal of the divisor. The differences are that rational expressions involve variables and may have non-permissible values.

$\left(\frac{1}{2}\right)\left(\frac{3}{5}\right) = \frac{(1)(3)}{(2)(5)}$ $= \frac{3}{10}$	$\frac{x+2}{2} \times \frac{x+3}{5} = \frac{(x+2)(x+3)}{(2)(5)}$ $= \frac{x^2+5x+6}{10}$
$\frac{3}{4} \div \frac{1}{2} = \left(\frac{3}{4}\right)\left(\frac{2}{1}\right)$ $= \frac{3}{2}$	$\frac{x+2}{4} \div \frac{x+1}{2} = \frac{x+2}{4} \times \frac{2}{x+1}$ $= \frac{x+2}{2(x+1)}$, $x \neq -1$

9. a) $\frac{5q}{2r}$, $r \neq 0$, $p \neq 0$ **b)** $\frac{m^2}{4t^3}$, $m \neq 0$, $t \neq 0$

c) $\frac{3}{2}$, $a \neq -b$

d) $\frac{2(x-2)(x+5)}{(x^2+25)}$, $x \neq -2, 0$

e) 1, $d \neq -3, -2, -1$

f) $\frac{-(y-8)(y+5)}{(y-1)}$, $y \neq \pm 1, 5, 9$

10. a) $8t$ **b)** $\frac{1}{b}$, $a \neq 0$, $b \neq 0$

c) $\frac{-1}{5(x+y)}$, $x \neq \pm y$ **d)** $\frac{3}{a+3}$, $a \neq \pm 3$

e) $\frac{1}{x+1}$, $x \neq -2, -1, 0, \pm\frac{2}{3}$

f) $\frac{-(x+2)}{3}$, $x \neq 2$

11. a) $\frac{m}{2}$, $m \neq 0$ **b)** $\frac{x-1}{x}$, $x \neq -3, -2, 0, 2$

c) $\frac{1}{6}$, $a \neq \pm 3, 4$ **d)** $\frac{1}{5}$, $x \neq 3, -\frac{4}{3}, -4$

12. x centimetres

13. a) $10x$

b) $(x-2)(x+1)$

Example: The advantage is that less simplifying needs to be done.

14. a) $\frac{m+3}{5}$ **b)** $\frac{m}{x}, x \neq 0$
c) $1, x \neq -y$ **d)** -1
e) $\frac{1}{x-y}, x \neq \pm y$

15. a) $\frac{5x}{12}$ **b)** $1, y \neq 0$
c) $\frac{9x+34}{(x+3)(x-3)}, x \neq \pm 3$
d) $\frac{a}{(a+3)(a-2)}, a \neq -3, 2$
e) $1, a \neq \pm b$
f) $\frac{2x^2-6x-3}{(x+1)(2x-3)(2x+3)}, x \neq -1, \pm\frac{3}{2}$

16. a) $\frac{1}{a}+\frac{1}{b}=\frac{a+b}{ab}$
b) $$\begin{aligned}\text{Left Side} &= \frac{1}{a}+\frac{1}{b}\\ &= \frac{b}{ab}+\frac{a}{ab}\\ &= \frac{a+b}{ab}\\ &= \text{Right Side}\end{aligned}$$

17. Exam mark, $d=\frac{a+b+c}{3}$;
$$\begin{aligned}\text{Final mark} &= \left(\frac{1}{2}\right)\left(\frac{a+b+c}{3}\right)+\left(\frac{1}{2}\right)d\\ &= \frac{a+b+c+3d}{6}\end{aligned}$$
Example: $\frac{60+70+80}{3}=d$
$\frac{60+70+80+3(70)}{6}=70$

18. a) **i)** the amount that Beth spends per chair; \$10 more per chair than planned
ii) the amount that Helen spends per chair; \$10 less per chair than planned
iii) the number of chairs Helen bought
iv) the number of chairs Beth bought
v) the total number of chairs purchased by the two sisters
b) $\frac{450c-500}{c^2-100}$ or $\frac{50(9c-10)}{(c-10)(c+10)}, c \neq \pm 10$

19. Example: When solving a rational equation, you multiply all terms by the LCD to eliminate the denominators. In addition and subtraction of rational expressions, you use an LCD to simplify by grouping terms over one denominator.

Add or subtract.	Solve.
$\frac{x}{3}+\frac{x}{2}$ $=\frac{2x}{6}+\frac{3x}{6}$ $=\frac{5x}{6}$	$\frac{x}{3}+\frac{x}{2}=5$ $2x+3x=30$ $5x=30$ $x=6$

20. a) $s=-9, s \neq -3$
b) $x=-4$ or $x=-1, x \neq 1, -\frac{2}{3}$
c) $z=1, z \neq 0$
d) $m=1$ or $m=-\frac{21}{2}, m \neq \pm 3$
e) no solution, $x \neq 3$
f) $x=\frac{\pm\sqrt{6}}{2}, x \neq 0, -\frac{1}{2}$
g) $x=-5$ or $x=1, x \neq -2, 3$

21. The numbers are 4 and 8.

22. Elaine would take 7.5 h.

23. a) $\frac{160}{x}+36+\frac{160}{x+0.7}=150$
b) $570x^2-1201x-560=0, x=2.5$ m/s.
c) The rate of ascent is 9 km/h.

Chapter 6 Practice Test, page 355

1. D
2. B
3. A
4. A
5. D
6. $x \neq -3, -1, 3, \frac{5}{3}$
7. $k=-1$
8. $\frac{5y-2}{6}, y \neq 2$
9. Let x represent the time for the smaller auger to fill the bin.
$\frac{6}{x}+\frac{6}{x-5}=1$
10. Example: For both you use an LCD. When solving, you multiply by the LCD to eliminate the denominators, while in addition and subtraction of rational expressions, you use the LCD to group terms over a single denominator.

Add or subtract.	Solve.
$\frac{x}{4}-\frac{x}{7}$ $=\frac{7x}{28}-\frac{4x}{28}$ $=\frac{3x}{28}$	$\frac{x}{5}+\frac{x}{3}=16$ $15\left(\frac{x}{5}\right)+15\left(\frac{x}{3}\right)=15(16)$ $3x+5x=240$ $8x=240$ $x=30$

11. $x=4; x \neq -2, 3$
12. $\frac{5x+3}{5x}-\frac{2x-1}{2x}=\frac{2x-1}{2x}-\frac{3-x}{x}; x=2.3$
13. The speed in calm air is 372 km/h.

Chapter 7 Absolute Value and Reciprocal Functions

7.1 Absolute Value, pages 363 to 367

1. a) 9 **b)** 0 **c)** 7
d) 4.728 **e)** 6.25 **f)** 5.5
2. $-0.8, -0.4, \left|\frac{3}{5}\right|, |0.8|, 1.1, \left|-1\frac{1}{4}\right|, |-2|$
3. $2.2, \left|-\frac{7}{5}\right|, |1.3|, \left|1\frac{1}{10}\right|, |-0.6|, -1.9, -2.4$

4. a) 7 b) −5 c) 10 d) 13
5. Examples:
   a) $|2.1 - (-6.7)| = 8.8$ b) $|5.8 - (-3.4)| = 9.2$
   c) $|2.1 - (-3.4)| = 5.5$ d) $|-6.7 - 5.8| = 12.5$
6. a) 10 b) −2.8 c) 5.25 d) 9 e) 17
7. Examples:
   a) $|3 - 8| = 5$ b) $|-8 - 12| = 20$
   c) $|9 - 2| = 7$ d) $|15 - (-7)| = 22$
   e) $|a - b|$ f) $|m - n|$
8. $|7 - (-11)| + |-9 - 7|$; 34 °C
9. Example:
   $|24 - 0| + |24 - 10| + |24 - 17| + |24 - 30| + |24 - 42| + |24 - 55| + |24 - 72|$; 148 km
10. 1743 miles
11. a) \$369.37
    b) The net change is the change from the beginning point to the end point. The total change is all the changes in between added up.
12. a) 7.5 b) 90 c) 0.875
13. 4900 m or 4.9 km
14. a) 1649 ft b) 2325 ft
15. \$0.36
16. a) 6 km b) 9 km
17. a) The students get the same result of 90.66.
    b) It does not matter the order in which you square something and take the absolute value of it.
    c) Yes, because the result of squaring a number is the same whether it was positive or negative.
18. a) Michel looks at both cases; the argument is either positive or negative.
    b) i) $|x - 7| = \begin{cases} x - 7 \text{ if } x \geq 7 \\ 7 - x \text{ if } x < 7 \end{cases}$
       ii) $|2x - 1| = \begin{cases} 2x - 1 \text{ if } x \geq \frac{1}{2} \\ 1 - 2x \text{ if } x < \frac{1}{2} \end{cases}$
       iii) $|3 - x| = \begin{cases} 3 - x, \text{ if } x \leq 3 \\ x - 3, \text{ if } x > 3 \end{cases}$
       iv) $x^2 + 4$
19. Example: Changing +5 to −5 is incorrect. Example: Change the sign so that it is positive.
20. 83 mm
21. Example: when you want just the speed of something and not the velocity
22. Example: signed because you want positive for up, negative for down, and zero for the top
23. a) 176 cm
    b) 4; 5; 2; 1; 4; 8; 1; 1; 2; 28 is the sum
    c) 3.11
    d) It means that most of the players are within 3.11 cm of the mean.
24. a) i) $x = 1$, $x = -3$
       ii) $x = 1$, $x = -5$; you can verify by trying them in the equation.
    b) It has no zeros. This method can only be used for functions that have zeros.
25. Example: Squaring a number makes it positive, while the square root returns only the positive root.

7.2 Absolute Value Functions, pages 375 to 379

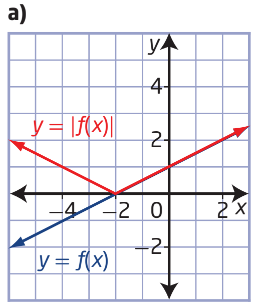
1. a)

x	$y = \lvert f(x)\rvert$
−2	3
−1	1
0	1
1	3
2	5

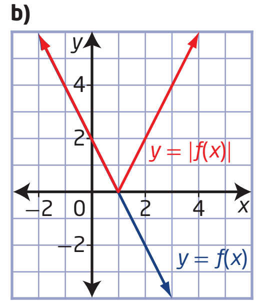
b)

x	$y = \lvert f(x)\rvert$
−2	0
−1	2
0	2
1	0
2	4

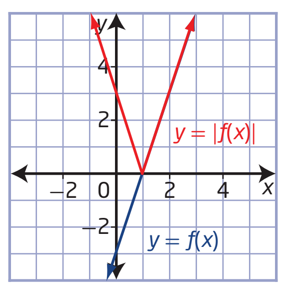
2. (−5, 8)
3. x-intercept: 3; y-intercept: 4
4. x-intercepts: −2, 7; y-intercept: $\frac{3}{2}$
5. a)



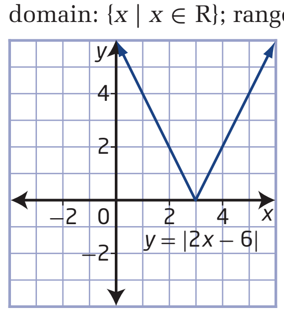
b)



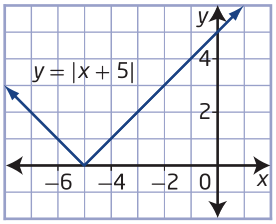
c)



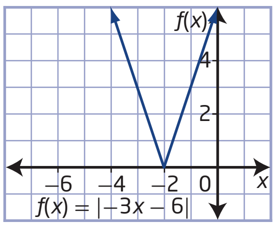
6. a) x-intercept: 3; y-intercept: 6;
   domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y \geq 0, y \in \mathrm{R}\}$



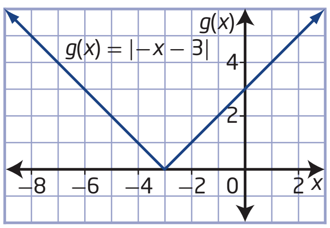
b) x-intercept: -5; y-intercept: 5;
domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y \geq 0, y \in \mathrm{R}\}$



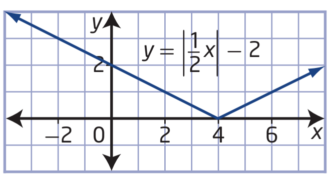
c) x-intercept: -2; y-intercept: 6;
domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y \geq 0, y \in \mathrm{R}\}$



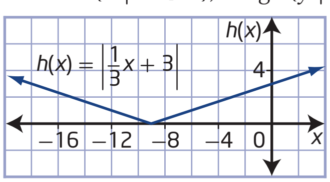
d) x-intercept: -3; y-intercept: 3;
domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y \geq 0, y \in \mathrm{R}\}$



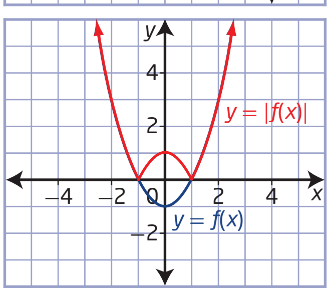
e) x-intercept: 4; y-intercept: 2;
domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y \geq 0, y \in \mathrm{R}\}$



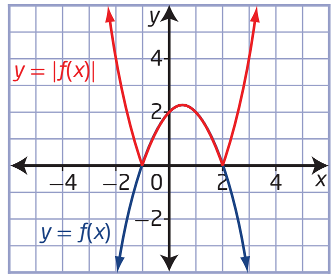
f) x-intercept: -9; y-intercept: 3;
domain $\{x \mid x \in \mathrm{R}\}$; range $\{y \mid y \geq 0, y \in \mathrm{R}\}$



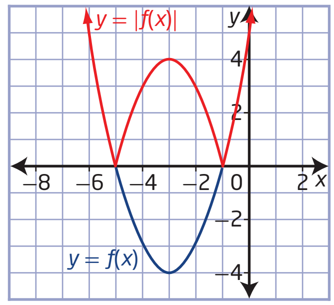
7. a)



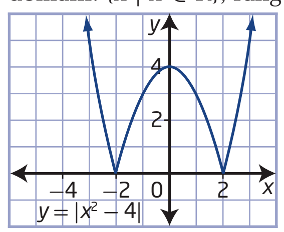
b)



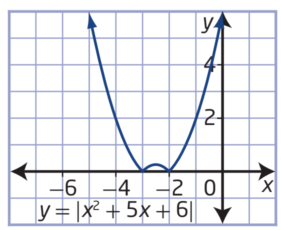
c)



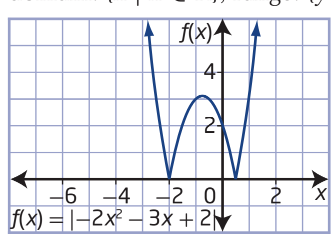
8. a) x-intercepts: -2, 2; y-intercept: 4;
domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y \geq 0, y \in \mathrm{R}\}$



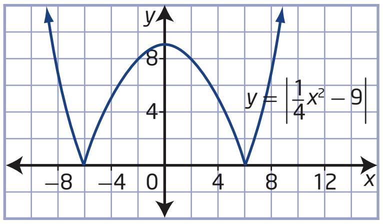
b) x-intercepts: -3, -2; y-intercept: 6;
domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y \geq 0, y \in \mathrm{R}\}$



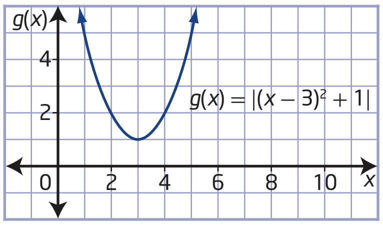
c) x-intercepts: -2, 0.5; y-intercept: 2;
domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y \geq 0, y \in \mathrm{R}\}$



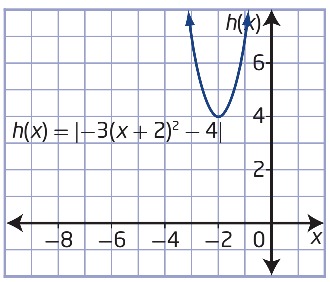
d) x-intercepts: -6, 6; y-intercept: 9; domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y \geq 0, y \in \mathrm{R}\}$



e) y-intercept: 10; domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y \geq 1, y \in \mathrm{R}\}$



f) y-intercept: 16; domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y \geq 4, y \in \mathrm{R}\}$



9. a) $y = 2x - 2$ if $x \geq 1$
$y = 2 - 2x$ if $x < 1$

b) $y = 3x + 6$ if $x \geq -2$
$y = -3x - 6$ if $x < -2$

c) $y = \frac{1}{2}x - 1$ if $x \geq 2$
$y = 1 - \frac{1}{2}x$ if $x < 2$

10. a) $y = 2x^2 - 2$ if $x \leq -1$ or $x \geq 1$
$y = -2x^2 + 2$ if $-1 < x < 1$

b) $y = (x - 1.5)^2 - 0.25$ if $x \leq 1$ or $x \geq 2$
$y = -(x - 1.5)^2 + 0.25$ if $1 < x < 2$

c) $y = 3(x - 2)^2 - 3$ if $x \leq 1$ or $x \geq 3$
$y = -3(x - 2)^2 + 3$ if $1 < x < 3$

11. a) $y = x - 4$ if $x \geq 4$
$y = 4 - x$, if $x < 4$

b) $y = 3x + 5$ if $x \geq -\frac{5}{3}$
$y = -3x - 5$ if $x < -\frac{5}{3}$

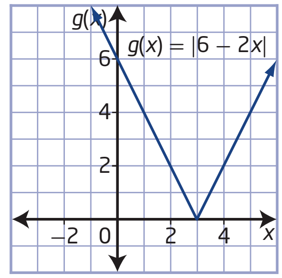
c) $y = -x^2 + 1$ if $-1 \leq x \leq 1$
$y = x^2 - 1$ if $x < -1$ or $x > 1$

d) $y = x^2 - x - 6$ if $x \leq -2$ or $x \geq 3$
$y = -x^2 + x + 6$ if $-2 < x < 3$

12. a)

x	$g(x)$
−1	8
0	6
2	2
3	0
5	4

b)

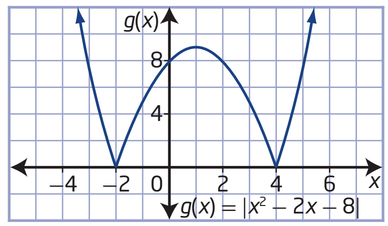


c) domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y \geq 0, y \in \mathrm{R}\}$

d) $y = 6 - 2x$ if $x \leq 3$
$y = 2x - 6$ if $x > 3$

13. a) y-intercept: 8; x-intercepts: -2, 4

b)

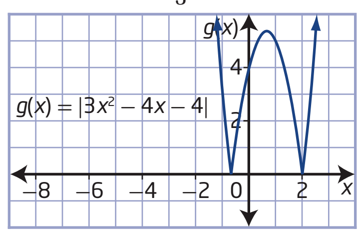


c) domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y \geq 0, y \in \mathrm{R}\}$

d) $y = x^2 - 2x - 8$ if $x \leq -2$ or $x \geq 4$;
$y = -x^2 + 2x + 8$ if $-2 < x < 4$

14. a) x-intercepts: $-\frac{2}{3}$, 2; y-intercept: 4

b)

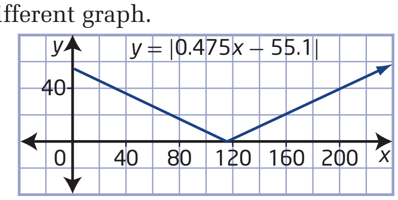


c) domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y \geq 0, y \in \mathrm{R}\}$

d) $y = 3x^2 - 4x - 4$ if $x \leq -\frac{2}{3}$ or $x \geq 2$;
$y = -3x^2 + 4x + 4$ if $-\frac{2}{3} < x < 2$

15. Michael is right. Since the vertex of the original function is below the x-axis, the absolute value function will have a different range and a different graph.

16. a)

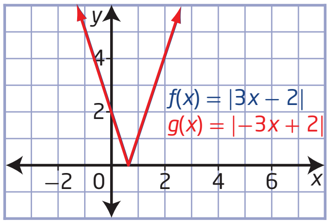


b) (116, 0)

c) (236, 57) is where the puck will be at the far side of the table, which is right in the middle of the goal.

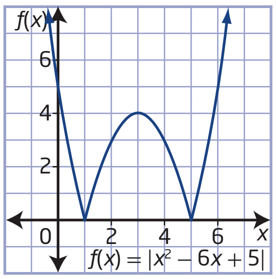
17. The distance travelled is 13 m.

18. a) The two graphs are identical. They are identical because one is the negative of the other but since they are in absolute value brackets there is no change.



b) $f(x) = |-4x - 3|$

19. $f(x) = |-x^2 + 6x - 5|$



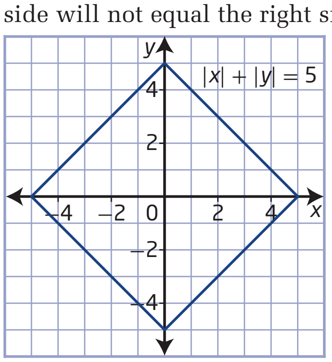
20. $a = -4$, $b = 6$ or $a = 4$, $b = -6$

21. $b = 4$; $c = -12$

22. Example: The square of something is always positive, so taking the absolute value does nothing.

23. Example: No, it is not true for all $x, y \in R$. For instance, if x and y are of different sign the left side will not equal the right side.

24.



25.

Case	$\lvert x\rvert\lvert y\rvert$	$\lvert xy\rvert$
$x \geq 0, y \geq 0$	xy	xy
$x \geq 0, y < 0$	$x(-y)$	$-xy$
$x < 0, y \geq 0$	$(-x)y$	$-xy$
$x < 0, y < 0$	$(-x)(-y)$	xy

26. Example: They have the same shape but different positions.

27. Example: Graph the functions, taking care to allow them only in their specified domain.

28. If the discriminant is less than or equal to 0 and $a > 0$, then the graphs will be equivalent.

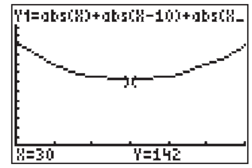
29. Examples:

Step 1 Yes.

Step 2 Absolute value is needed because the facility could be to the east or west of each town.

total $= |x| + |x - 10| + |x - 17| + |x - 30| + |x - 42| + |x - 55| + |x - 72|$

Step 3 x: [0, 60, 10]
y: [−30, 300, 20]



Step 4 The point (30, 142) on the graph shows that there is a point that minimizes the distance to each city. The point represents a place 30 km east of Allenby and results in a total distance from all towns of 142 km.

30. a) $y = |(x - 3)^2 + 7|$ **b)** $y = \left|\frac{4}{5}(x + 3)^2\right|$
c) $y = |-x^2 - 6|$ **d)** $y = |5(x + 3)^2 + 3|$

7.3 Absolute Value Equations, pages 389 to 391

1. a) $x = -7, x = 7$ **b)** $x = -4, x = 4$
c) $x = 0$ **d)** no solution

2. a) $x = -6, x = 14$ **b)** $x = -5, x = -1$
c) $x = -14, x = -2$ **d)** no solution

3. a) $|x| = 2$ **b)** $|x - 2| = 6$
c) $|x - 4| = 5$

4. a) $x = -19, x = 5$ **b)** $x = \frac{2}{3}, x = 2$
c) no solution **d)** $x = 3.5, x = 10.5$

5. a) no solution **b)** $x = -9, x = 1$
c) $m = -\frac{1}{3}, m = 3$ **d)** no solution
e) $a = -\frac{11}{3}, a = -3$

6. a) $x = -3, x = \sqrt{3}$ **b)** $x = 2, x = 3$
c) $x \leq -3$ or $x \geq 3$
d) $x = \frac{1 + \sqrt{5}}{2}, x = \frac{\sqrt{5} - 1}{2}$
e) $x = -4, x = -2, x = 4, x = 6$

7. a) $|d - 18| = 0.5$
b) 17.5 mm and 18.5 mm are allowed

8. a) $|c - 299\,792\,456.2| = 1.1$
b) 299 792 455.1 m/s or 299 792 457.3 m/s

9. a) $|V - 50\,000| = 2000$ **b)** 48 000 L, 52 000 L

10. a) 2.2, 11.8 **b)** $|x - 7| = 4.8$

11. a) 66.5 g **b)** 251 mL and 265 mL

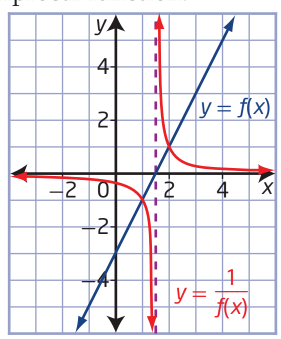
12. a) perigee: 356 400 km; apogee: 406 700 km
b) Example: The moon is usually around 381 550 km away plus or minus 25 150 km.

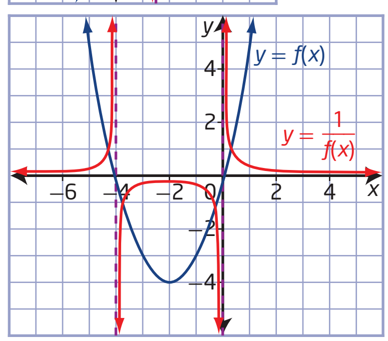
13. a) greater than or equal to zero
b) less than or equal to zero

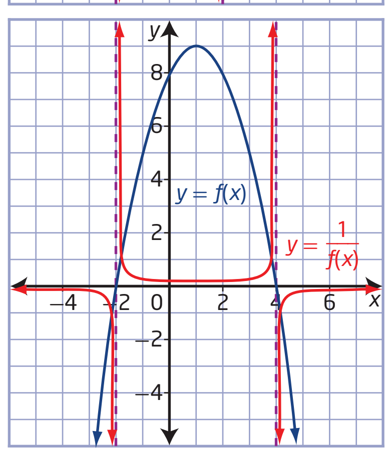
14. a) $x = \frac{b + c}{a}$ if $x \geq 0$, $x = \frac{-b - c}{a}$ if $x < 0$; $b + c \geq 0$, $a \neq 0$
b) $x = b + c$ if $x \geq b$, $x = b - c$ if $x < b$; $c \geq 0$

15. Andrea is correct. Erin did not choose the two cases correctly.
16. $|t - 11.5| = 2.5$; $t = 9$ °C, $t = 14$ °C
17. a) $|x - 81| = 16.2$; 64.8 mg, 97.2 mg
    b) Example: They might lean toward 97.2 mg because it would provide more relief because there is more of the active ingredient.
18. $|t - 10| = 2$
19. a) sometimes true; $x \neq -1$
    b) sometimes true; if $x = -a$, then the solution is 0. For all other values of x, the solution is greater than 0.
    c) always true
20. Examples:
    a) $|x - 3| = 5$ b) $|x| = -2$
    c) $|x| = 0$ d) $|x| = 5$
21. Yes; the positive case is $ax + b = 0$, which always has a solution.
22. a) $|x - 3| = 4$ b) $|x^2 - 4| = 5$
23. Example: The first equation has no solution because an absolute value expression cannot equal a negative number. The second equation has two solutions because the absolute value expression equates to a positive number, so two cases are possible.
24. Example: When solving each case, the solutions generated are for the domain $\{x \mid x \in R\}$. However, since each case is only valid for a specific domain, solutions outside of that domain are extraneous.

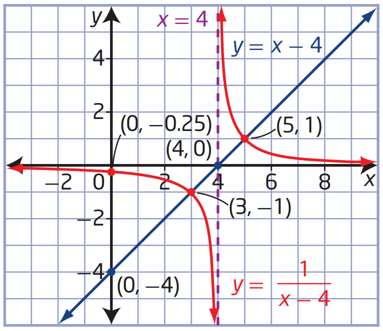
7.4 Reciprocal Functions, pages 403 to 409

1. a) $y = \frac{1}{2 - x}$ b) $y = \frac{1}{3x - 5}$
   c) $y = \frac{1}{x^2 - 9}$ d) $y = \frac{1}{x^2 - 7x + 10}$
2. a) i) $x = -5$ ii) $y = \frac{1}{x + 5}$ iii) $x \neq -5$
   iv) The zeros of the original function are the non-permissible values of the reciprocal function.
   v) $x = -5$
   b) i) $x = -\frac{1}{2}$ ii) $y = \frac{1}{2x + 1}$ iii) $x \neq -\frac{1}{2}$
   iv) The zeros of the original function are the non-permissible values of the reciprocal function.
   v) $x = -\frac{1}{2}$
   c) i) $x = -4, x = 4$ ii) $y = \frac{1}{x^2 - 16}$
   iii) $x \neq -4, x \neq 4$
   iv) The zeros of the original function are the non-permissible values of the reciprocal function.
   v) $x = -4, x = 4$
   d) i) $x = 3, x = -4$ ii) $y = \frac{1}{x^2 + x - 12}$
   iii) $x \neq 3, x \neq -4$
   iv) The zeros of the original function are the non-permissible values of the reciprocal function.
   v) $x = 3, x = -4$
3. a) $x = 2$ b) $x = -\frac{7}{3}$
   c) $x = 2, x = -4$ d) $x = 4, x = 5$
4. When $x = 3$, there is a division by zero, which is undefined.
5. a) no x-intercepts, y-intercept: $\frac{1}{5}$
   b) no x-intercepts, y-intercept: $-\frac{1}{4}$
   c) no x-intercepts, y-intercept: $-\frac{1}{9}$
   d) no x-intercepts, y-intercept: $\frac{1}{12}$
6. Example: Locate zeros and invariant points. Use these points to help sketch the graph of the reciprocal function.
   a)


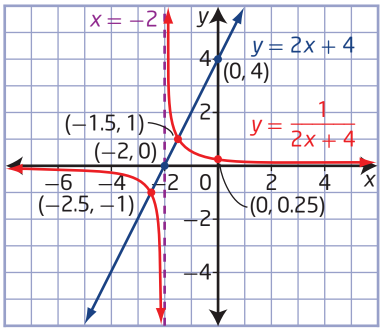
   b)


   c)


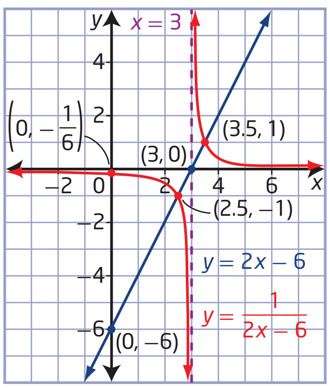
7. a)



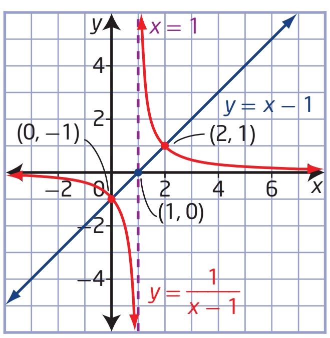
b)



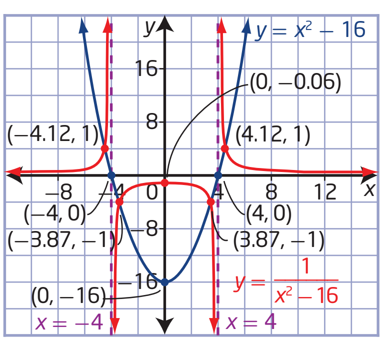
c)



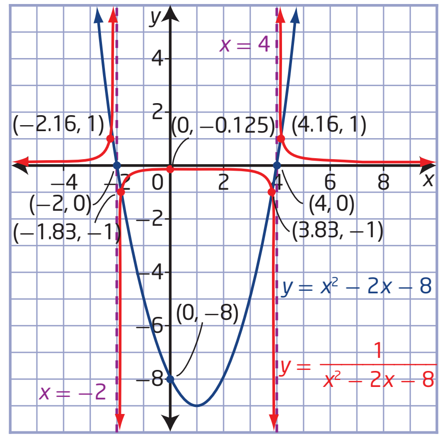
d)



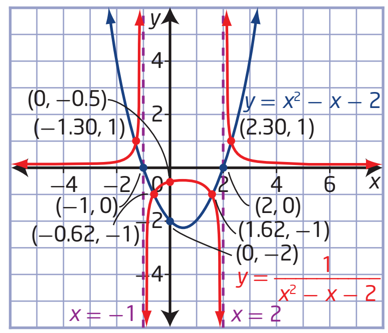
8. a)



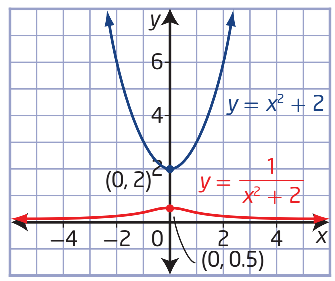
b)



c)

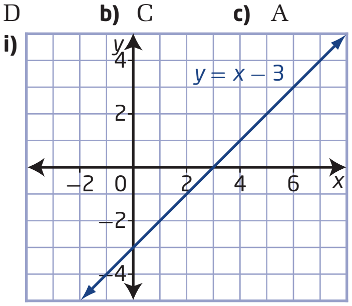


d)



9. a) D **b)** C **c)** A **d)** B

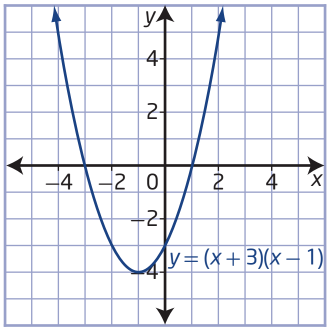
10. a) i)



ii) Example: Use the vertical asymptote to find the zero of the function. Then, use the invariant point and the x-intercept to graph the function.

iii) $y = x - 3$

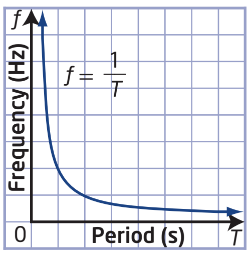
b) i)



ii) Example: Use the vertical asymptotes to find the zeros of the function. Then, use the given point to determine the vertex and then graph the function.

iii) $y = (x + 3)(x - 1)$

11. a)

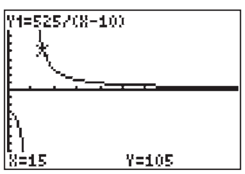


b) $y = T$

c) 0.4 Hz

d) 0.625 s

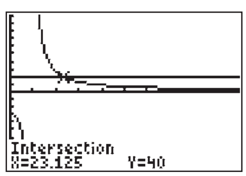
12. a)



b) $\{d \mid d > 10, d \in \mathrm{R}\}$

c) 17.5 min

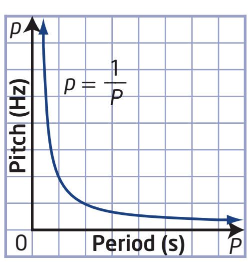
d) 23.125 m; it means that the diver has a maximum of 40 min at a depth of 23.125 m.



e) Yes; at large depths it is almost impossible to not stop for decompression.

13. a) $p = \frac{1}{P}$

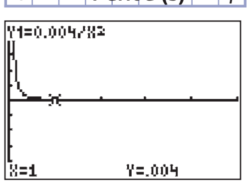
b)



c) 20.8 Hz

14. a) $I = 0.004\frac{1}{d^2}$

b)

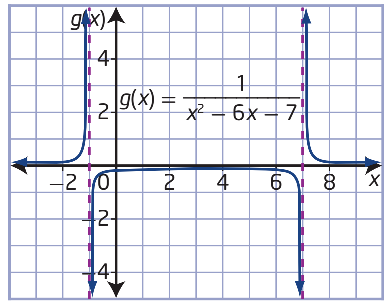


c) 0.000 16 W/m²

15. a) Example: Complete the square to change it to vertex form.

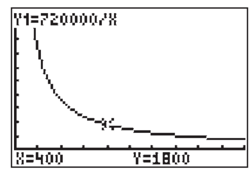
b) Example: The vertex helps with the location of the maximum for the U-shaped section of the graph of $g(x)$.

c)



16. a) $k = 720\ 000$

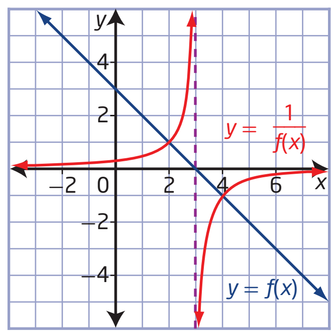
b)



c) 1800 days

d) 1440 workers

17.



18. a) False; only if the function has a zero is this true.

b) False; only if the function has a zero is this true.

c) False; sometimes there is an undefined value.

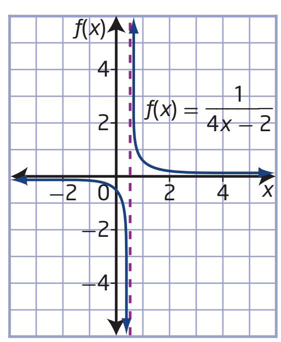
19. a) Both students are correct. The non-permissible values are the roots of the corresponding equation.

b) Yes

20. a) $v = 60$ mm

b) $f = 205.68$ mm

21. Step 1



Step 2

a)

x	$f(x)$
0	−0.5
0.4	−2.5
0.45	−5
0.47	−8.33
0.49	−25
0.495	−50
0.499	−250

x	$f(x)$
1	0.5
0.6	2.5
0.55	5
0.53	8.33
0.51	25
0.505	50
0.501	250

b) The function approaches infinity or negative infinity. The function will always approach infinity or negative infinity.

Step 3

a)

x	$f(x)$
−10	$-\frac{1}{42}$
−100	$-\frac{1}{402}$
−1000	$-\frac{1}{4002}$
−10 000	$-\frac{1}{40\,002}$
−100 000	$-\frac{1}{400\,002}$

x	$f(x)$
10	$\frac{1}{38}$
100	$\frac{1}{398}$
1000	$\frac{1}{3998}$
10 000	$\frac{1}{39\,998}$
100 000	$\frac{1}{399\,998}$

b) The function approaches zero.

22.

$y = f(x)$	$y = \frac{1}{f(x)}$
The absolute value of the function gets very large.	The absolute value of the function gets very small.
Function values are positive.	Reciprocal values are positive.
Function values are negative.	Reciprocal values are negative.
The zeros of the function are the x-intercepts of the graph.	The zeros of the function are the vertical asymptotes of the graph.
The value of the function is 1.	The value of the reciprocal function is 1.
The absolute value of the function approaches zero.	The absolute value of the reciprocal approaches infinity or negative infinity.
The value of the function is −1.	The value of the reciprocal function is −1.

Chapter 7 Review, pages 410 to 412

1. a) 5 **b)** 2.75 **c)** 6.7

2. $-4, -2.7, \left|1\frac{1}{2}\right|, |-1.6|, \sqrt{9}, |-3.5|, \left|-\frac{9}{2}\right|$

3. a) 9 **b)** 2 **c)** 18.75 **d)** 20

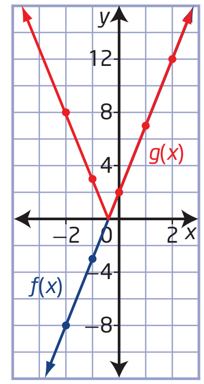
4. 43.8 km

5. a) \$2.12 **b)** \$4.38

6. a)

x	$f(x)$	$g(x)$
−2	−8	8
−1	−3	3
0	2	2
1	7	7
2	12	12

b)



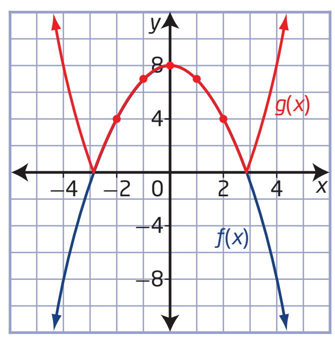
c) $f(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$; $g(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

d) Example: They are the same graph except the absolute value function never goes below zero; instead it reflects back over the x-axis.

7. a)

x	$f(x)$	$g(x)$
−2	4	4
−1	7	7
0	8	8
1	7	7
2	4	4

b)



c) $f(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \leq 8, y \in \mathrm{R}\}$; $g(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

d) Example: They are the same graph except the absolute value function never goes below zero; instead it reflects back over the x-axis.

8. a) $y = 2x - 4$ if $x \geq 2$
$y = 4 - 2x$ if $x < 2$

b) $y = x^2 - 1$ if $x \leq -1$ or $x \geq 1$
$y = 1 - x^2$ if $-1 < x < 1$

9. a) The functions have different graphs because the initial graph goes below the x-axis. The absolute value brackets reflect anything below the x-axis above the x-axis.
   b) The functions have the same graphs because the initial function is always positive.

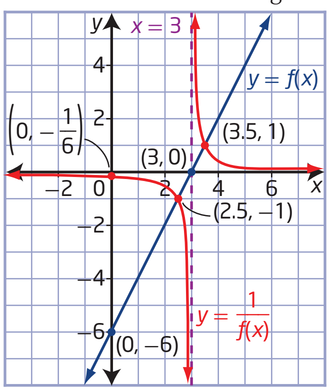
10. $a = 15$, $b = 10$

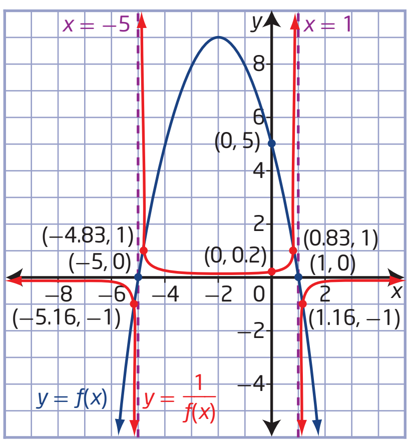
11. a) $x = -3.5$, $x = 5.5$
   b) no solution
   c) $x = -3$, $x = 3$, $x \approx -1.7$, $x \approx 1.7$
   d) $m = -1$, $m = 5$

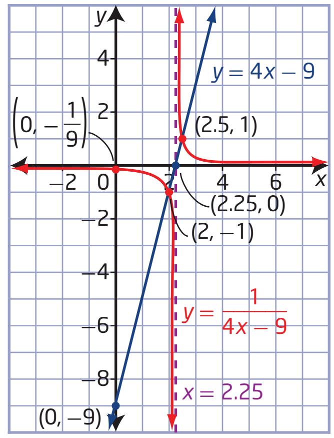
12. a) $q = -11$, $q = -7$ b) $x = \frac{1}{4}$, $x = \frac{2}{3}$
   c) $x = 0$, $x = 5$, $x = 7$
   d) $x = \frac{3}{2}$, $x = \frac{-1 + \sqrt{21}}{4}$

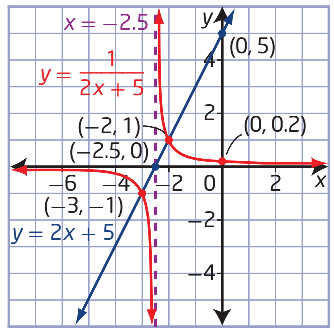
13. a) first low tide 2.41 m; first high tide 5.74 m
   b) The total change is 8.5 m.

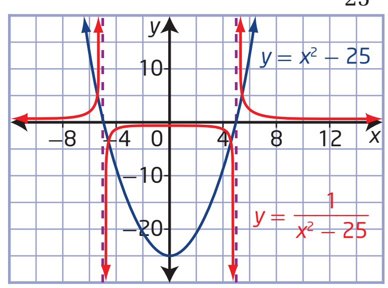
14. The two masses are 24.78 kg and 47.084 kg.

15. a)


b)


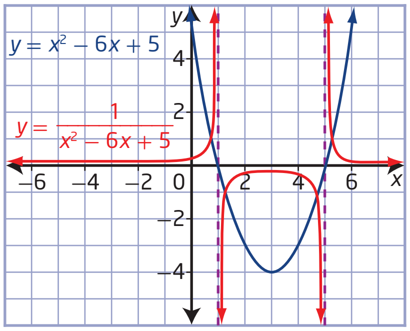
16. a)


b)


17. a) i) $y = \frac{1}{x^2 - 25}$
   ii) The non-permissable values are $x = -5$ and $x = 5$. The equations of the vertical asymptotes are $x = -5$ and $x = 5$.
   iii) no x-intercepts; y-intercept: $-\frac{1}{25}$
   iv)


b) i) $y = \frac{1}{x^2 - 6x + 5}$
   ii) The non-permissable values are $x = 5$ and $x = 1$. The equations of the vertical asymptotes are $x = 5$ and $x = 1$.
   iii) no x-intercept; y-intercept $\frac{1}{5}$

iv)

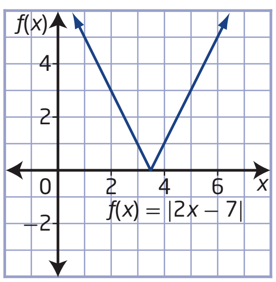


18. a) 240 N **b)** 1.33 m

c) If the distance is doubled the force is halved. If the distance is tripled only a third of the force is needed.

Chapter 7 Practice Test, pages 413 to 414

1. B
2. C
3. D
4. A
5. B
6. a)



b) x-intercept: $\frac{7}{2}$; y-intercept: 7

c) domain: $\{x \mid x \in R\}$; range: $\{y \mid y \geq 0, y \in R\}$

d) $y = 2x - 7$ if $x \geq \frac{7}{2}$

$y = 7 - 2x$ if $x < \frac{7}{2}$

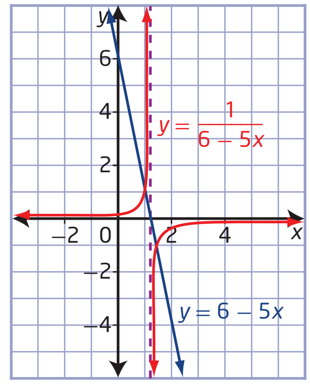
7. $x = 1, x = \frac{2}{3}$

8. $w = 4, w = \frac{2}{3}$

9. Example: In Case 1, the mistake is that after taking the absolute value brackets off, the inside term was incorrectly copied down. It should have been $x - 4$. Then, there are no solutions from Case 1. In Case 2, the mistake is that after taking the absolute value brackets off, the inside term was incorrectly multiplied by negative one. It should have been $-x + 4$. Then, the solutions are $x = \frac{-5 + \sqrt{41}}{2}$ and $x = \frac{-5 - \sqrt{41}}{2}$.

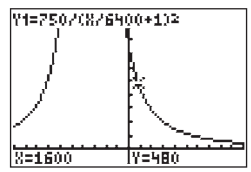
10. a) $y = \frac{1}{6 - 5x}$ **b)** $x = \frac{6}{5}$

c) Example: Use the asymptote already found and the invariant points to sketch the graph.



11. a) $y = |2.5x|$ **b)** 43.6° **c)** 39.3°

12. a)



b) i) 748.13 N **ii)** 435.37 N

c) more than 25 600 km will result in a weight less than 30 N.

Cumulative Review, Chapters 5–7, pages 416 to 417

1. $\sqrt{18x^3y^6}$

2. $4abc^2\sqrt{3ac}$

3. $\sqrt[3]{8}, 2\sqrt{3}, \sqrt{18}, \sqrt{36}, 2\sqrt{9}, 3\sqrt{6}$

4. a) $9\sqrt{2a}, a \geq 0$

b) $11x\sqrt{5}, x \geq 0$

5. a) $-16\sqrt[3]{3}$ **b)** $3\sqrt{2}$

c) $12a - 12\sqrt{a} + 2\sqrt{3a} - 2\sqrt{3}, a \geq 0$

6. a) $\sqrt{3}$ **b)** $4 - 2\sqrt{3}$

c) $-3 - 3\sqrt{2}$

7. $x = 3$

8. a) 430 ft

b) Example: The velocity would decrease with an increasing radius because of the expression $h - 2r$.

9. a) $\frac{a}{4b^3}, a \neq 0, b \neq 0$ **b)** $\frac{-1}{x - 4}, x \neq 4$

c) $\frac{(x - 3)^2(x + 5)}{(x + 2)(x + 1)(x - 1)}, x \neq 1, -1, -2, 3$

d) $\frac{1}{6}, x \neq 0, 2$

e) $1, x \neq -3, -2, 2, 3$

10. a) $\dfrac{a^2 + 11a - 72}{(a + 2)(a - 7)}$, $a \neq -2, 7$

b) $\dfrac{3x^3 + x^2 - 11x + 12}{(x + 4)(x - 2)(x + 2)}$, $x \neq -4, -2, 2$

c) $\dfrac{2x^2 - x - 15}{(x - 5)(x + 5)(x + 1)}$, $x \neq -5, -1, 5$

11. Example: No; they are not equivalent because the expression should have the restriction of $x \neq -5$.

12. $x = 12$

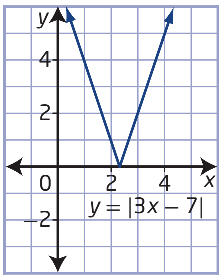
13. $\frac{1}{4}$

14. $|4 - 6|, |-5|, |8.4|, |2(-4) - 5|$

15. a) $y = 3x - 6$ if $x \geq 2$
$y = 6 - 3x$ if $x < 2$

b) $y = \frac{1}{3}(x - 2)^2 - 3$ if $x \leq -1$ or $x \geq 5$
$y = -\frac{1}{3}(x - 2)^2 + 3$ if $-1 < x < 5$

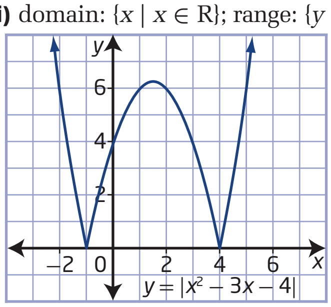
16. a) i)



ii) x-intercept: $\frac{7}{3}$; y-intercept: 7

iii) domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y \geq 0, y \in \mathrm{R}\}$

b) i)



ii) x-intercepts: $-1, 4$; y-intercept: 4

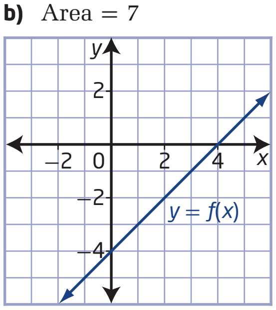
iii) domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y \geq 0, y \in \mathrm{R}\}$

17. a) $x = 5, x = -4$ b) $x = 3, x = -3$

18. a) Example: Absolute value must be used because area is always positive.

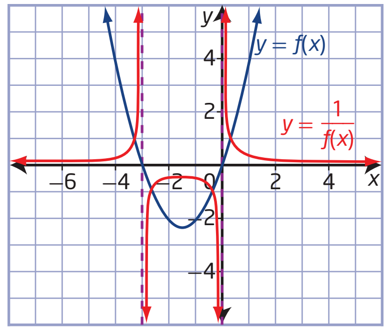
b) Area = 7

19.

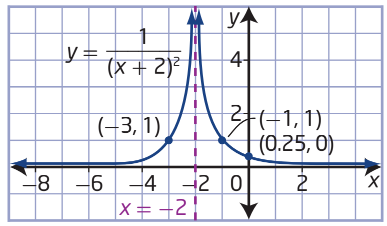


$f(x) = x - 4$

20.



21.



22. a) Example: The shape, range, and y-intercept will be different for $y = |f(x)|$.

b) Example: The graph of the reciprocal function has a horizontal asymptote at $y = 0$ and a vertical asymptote at $x = \frac{1}{3}$.

Unit 3 Test, pages 418 to 419

1. C
2. D
3. B
4. C
5. D
6. B
7. D
8. B
9. 3

10. $\dfrac{\sqrt{10}}{6}$

11. 28

12. -2

13. $-2, 2$

14. $5, 3\sqrt{7}, 6\sqrt{2}, 4\sqrt{5}$

15. a) Example: Square both sides.

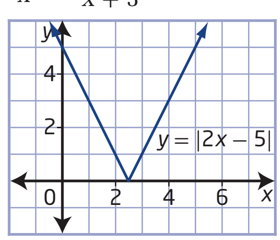
b) $x \geq 2.5$ c) There are no solutions.

16. $\dfrac{4(2x + 5)}{(x - 4)}$, $x \neq -2.5, -2, 1, 0.5, 4$

17. Example:

a) $\dfrac{2x}{x} = \dfrac{x + 10}{x + 3}$ b) $x \neq -3, 0$ c) $x = 4$

18. a)



b) y-intercept: 5; x-intercept: $\frac{5}{2}$

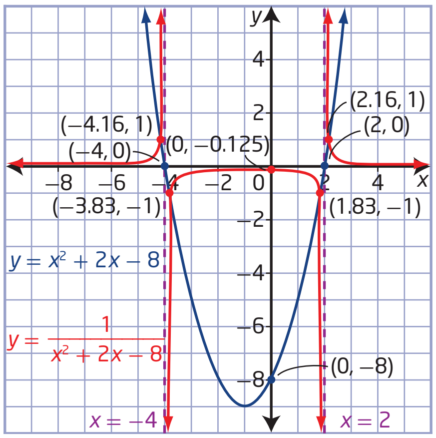
c) domain: $\{x \mid x \in R\}$; range: $\{y \mid y \geq 0, y \in R\}$

d) $y = 2x - 5$ if $x \geq \frac{5}{2}$

$y = 5 - 2x$ if $x < \frac{5}{2}$

19. $x = \frac{3 \pm \sqrt{17}}{2}$, 1, 2

20.



21. a) $t = \frac{72}{s}$ **b)** 4.97 ft/s **c)** 11.43 s

Chapter 8 Systems of Equations

8.1 Solving Systems of Equations Graphically, pages 435 to 439

1. a) System A models the situation: to go off a ramp at different heights means two positive vertical intercepts, and in this system the launch angles are different, causing the bike with the lower trajectory to land sooner. System B is not correct because it shows both jumps starting from the same height. System C has one rider start from zero, which would mean no ramp. In System D, a steeper trajectory would mean being in the air longer but the rider is going at the same speed.

b) The rider was at the same height and at the same time after leaving the jump regardless of which ramp was chosen.

2. For $(0, -5)$: In $y = -x^2 + 4x - 5$:

Left Side	Right Side
$y = -5$	$-x^2 + 4x - 5$
	$= -(0)^2 + 4(0) - 5$
	$= -5$

Left Side = Right Side

In $y = x - 5$:

Left Side	Right Side
$y = -5$	$x - 5$
	$= 0 - 5$
	$= -5$

Left Side = Right Side

For $(3, -2)$: In $y = -x^2 + 4x - 5$:

Left Side	Right Side
$y = -2$	$-x^2 + 4x - 5$
	$= -(3)^2 + 4(3) - 5$
	$= -2$

Left Side = Right Side

In $y = x - 5$:

Left Side	Right Side
$y = -2$	$x - 5$
	$= 3 - 5$
	$= -2$

Left Side = Right Side

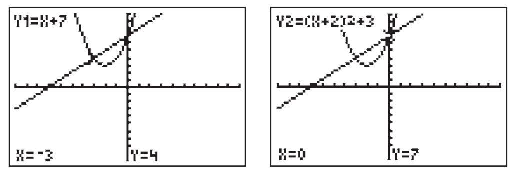
So, both solutions are verified.

3. a) linear-quadratic; $(-4, 1)$ and $(-1, -2)$

b) quadratic-quadratic; no solution

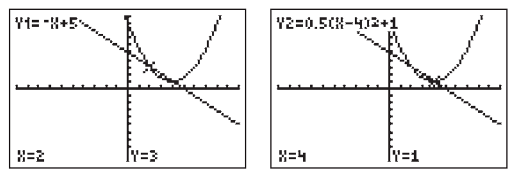
c) linear-quadratic; $(1, -4)$

4. a)



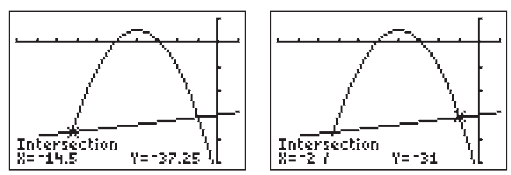
$(-3, 4)$ and $(0, 7)$

b)



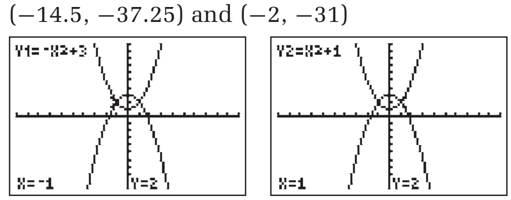
$(2, 3)$ and $(4, 1)$

c)



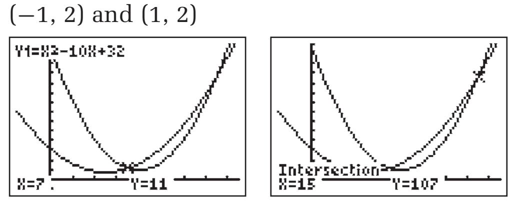
$(-14.5, -37.25)$ and $(-2, -31)$

d)



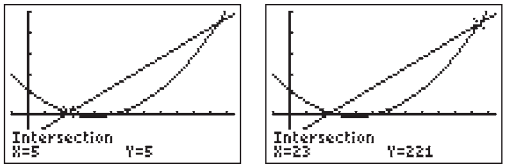
$(-1, 2)$ and $(1, 2)$

e)



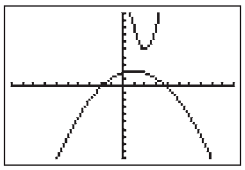
$(7, 11)$ and $(15, 107)$

5. a)



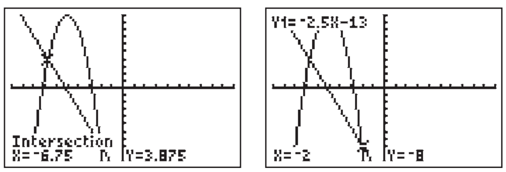
(5, 5) and (23, 221) or $d = 5$, $h = 5$ and $d = 23$, $h = 221$

b)



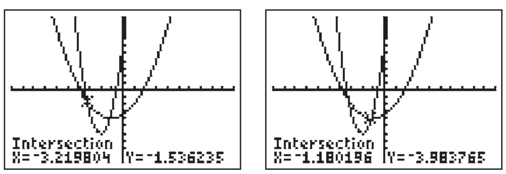
no solution

c)



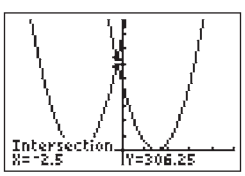
$(-6.75, 3.875)$ and $(-2, -8)$ or $v = -6.75$, $t = 3.875$ and $v = -2$, $t = -8$

d)



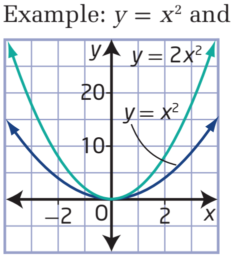
$(-3.22, -1.54)$ and $(-1.18, -3.98)$ or $n = -3.22$, $m = -1.54$ and $n = -1.18$, $m = -3.98$

e)



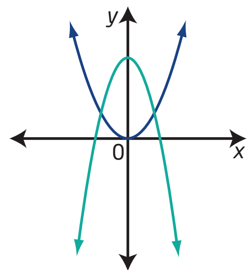
$(-2.5, 306.25)$ or $h = -2.5$, $t = 306.25$

6. The two parabolas have the same vertex, but different values of a.
Example: $y = x^2$ and $y = 2x^2$.

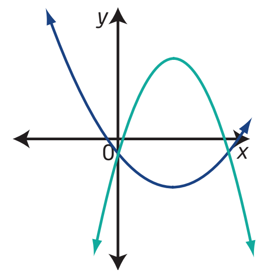


7. Examples:

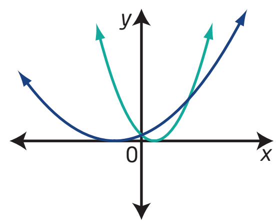
a)



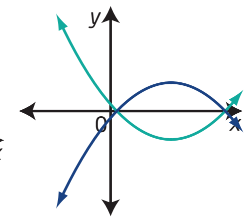
b)



c)



d)



8. Examples:

a) $y = x - 3$ **b)** $y = -2$ **c)** $y = x - 1$

9. a) (100, 3800) and (1000, 8000)

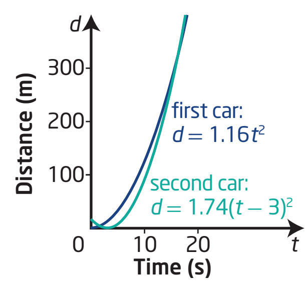
b) When he makes and sells either 100 or 1000 shirts, Jonas makes no profit as costs equal revenue.

c) Example: (550, 15 500). This quantity (550 shirts) has the greatest difference between cost and revenue.

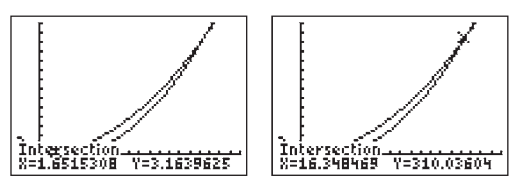
10. (0, 3.9) and (35.0, 3.725)

11. a) $d = 1.16t^2$ and $d = 1.74(t - 3)^2$

b) A suitable domain is $0 \leq t \leq 23$.

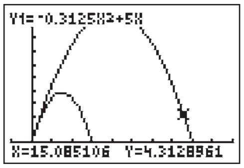


c)

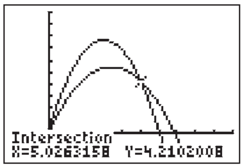


(1.65, 3.16) and (16.35, 310.04) While (1.65, 3.16) is a graphical solution to the system, it is not a solution to the problem since the second car starts 3 s after the first car.

d) At 16.35 s after the first car starts, both cars have travelled the same distance.

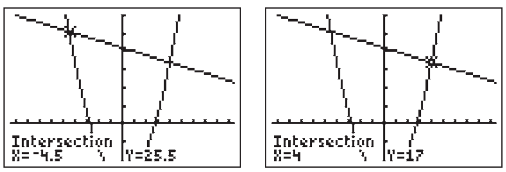
12. a) 

Both start at (0, 0), at the fountain, and they have one other point in common, approximately (0.2, 1.0). The tallest stream reaches higher and farther than the smaller stream.

b) 

They both start at (0, 0), but the second stream passes through the other fountain's spray 5.03 m from the fountain, at a height of 4.21 m.

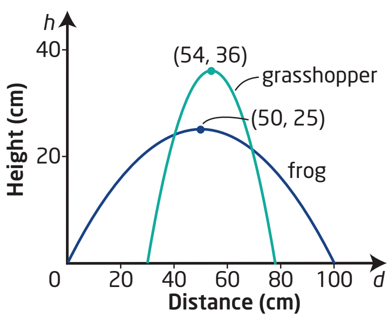
13. a) Let x represent the smaller integer and y the larger integer. $x + y = 21$, $2x^2 - 15 = y$.

b) 

One point of intersection does not give integers. The two integers are 4 and 17.

14. a) The blue line and the parabola intersect at (2, 2). The green line and the parabola intersect at (−4.54, −2.16).

b) Example: There is one possible location to leave the jump and one location for the landing.

15. a) 

b) Frog: $y = -0.01(x - 50)^2 + 25$
Grasshopper: $y = -0.0625(x - 54)^2 + 36$

c) (40.16, 24.03) and (69.36, 21.25)

d) These are the locations where the frog and grasshopper are at the same distance and height relative to the frog's starting point. If the frog does not catch the grasshopper at the first point, there is another opportunity. However, we do not know anything about time, i.e., the speed of either one, so the grasshopper may be gone.

16. a) (0, 0) and approximately (1.26, 1.59) Since 0 is a non-permissible value for x and y, the point (0, 0) is not a solution to this system.

b) 1.26 cm

c) $V = lwh$
$V = 1.26 \times 1.26 \times 1.26$
$V = 2.000\ 376$
So, the volume is very close to 2 cm^3.

d) If x represents the length of one side, then $V = x^3$. For a volume of 2 cm^3, $2 = x^3$. Then, $x = \sqrt[3]{2}$ or approximately 1.26. Menaechmus did not have a calculator to find roots.

17. Examples:

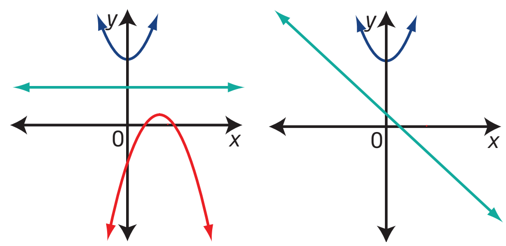
a) $y = x^2 + 1$ and $y = x + 3$

b) $y = x^2 + 1$ and $y = -(x - 1)^2 + 6$

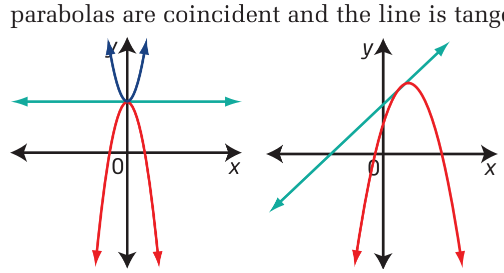
c) $y = x^2 + 1$ and $y = (x + 1)^2 - 2x$

18. Examples:

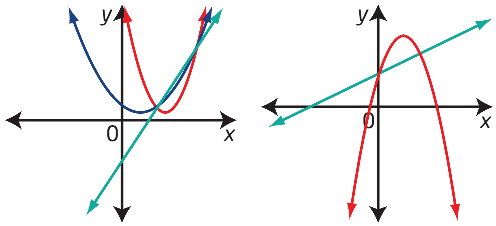
No solution: Two parabolas do not intersect and the line is between them, intersecting neither, or the parabolas are coincident and the line does not intersect them.



One solution: Two parabolas intersect once, with a line tangent to both curves, or the parabolas are coincident and the line is tangent.



Two solutions: Two parabolas intersect twice, with a line passing through both points of intersection, or the parabolas are coincident and the line passes through two points on them.



19. Example: Similarities: A different number of solutions are possible. It can be solved graphically or algebraically. Differences: Some systems involving quadratic equations cannot be solved by elimination. The systems in this section involve equations that are more difficult to solve.

20. a) Two solutions. The y-intercept of the line is above the vertex, and the parabola opens upward.

b) No solution. The parabola's vertex is at (0, 3) and it opens upward, while the line has y-intercept -5 and negative slope.

c) Two solutions. One vertex is directly above the other. The upper parabola has a smaller vertical stretch factor.

d) One solution. They share the same vertex. One opens upward, the other downward.

e) No solution. The first parabola has its vertex at (3, 1) and opens upward. The second parabola has it vertex at (3, -1) and opens downward.

f) An infinite number of solutions. When the first equation is expanded, it is exactly the same as the second equation.

8.2 Solving Systems of Equations Algebraically, pages 451 to 456

1. In $k + p = 12$:
Left Side
$k + p$
$= 5 + 7$
$= 12$
= Right Side

In $4k^2 - 2p = 86$:
Left Side
$4k^2 - 2p$
$= 4(5)^2 - 2(7)$
$= 86$
= Right Side

So, (5, 7) is a solution.

2. In $18w^2 - 16z^2 = -7$:

$$\begin{aligned}\text{Left Side} &= 18\left(\frac{1}{3}\right)^2 - 16\left(\frac{3}{4}\right)^2\\ &= 18\left(\frac{1}{9}\right) - 16\left(\frac{9}{16}\right)\\ &= 2 - 9\\ &= -7\\ &= \text{Right Side}\end{aligned}$$

In $144w^2 + 48z^2 = 43$:

$$\begin{aligned}\text{Left Side} &= 144\left(\frac{1}{3}\right)^2 + 48\left(\frac{3}{4}\right)^2\\ &= 16 + 27\\ &= 43\\ &= \text{Right Side}\end{aligned}$$

So, $\left(\frac{1}{3}, \frac{3}{4}\right)$ is a solution.

3. a) (-6, 38) and (2, 6) **b)** (0.5, 4.5)

c) (-2, 10) and (2, 30)

d) (-2.24, -1.94) and (2.24, 15.94)

e) no solution

4. a) $\left(-\frac{1}{2}, 4\right)$ and (3, 25)

b) (-0.5, 14.75) and (8, 19)

c) (-1.52, -2.33) and (1.52, 3.73)

d) (1.41, -4) and (-1.41, -4)

e) There are an infinite number of solutions.

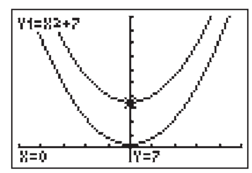
5. a) (2.71, -1.37) and (0.25, 0.78)

b) (-2.41, 10.73) and (2.5, 10)

c) (0.5, 6.25) and (0.75, 9.3125)

6. a) They are both correct.

b) Graph $n = m^2 + 7$ and $n = m^2 + 0.5$ to see that there is no point of intersection.



7. a) Yes. Multiplying by (-1) and then adding is equivalent to subtraction.

b) Yes.

c) Example: Adding is easier for most people. Subtracting with negative signs can be error prone.

8. $m = 6$, $n = 40$

9. a) $7x + y + 13$ **b)** $5x^2 - x$

c) $60 = 7x + y + 13$ and $10y = 5x^2 - x$. Since the perimeter and the area are both based on the same dimensions, x and y must represent the same values. You can solve the system to find the actual dimensions.

d) (5, 12); the base is 24 m, the height is 10 m and the hypotenuse is 26 m.

e) A neat verification uses the Pythagorean Theorem: $24^2 + 10^2 = 676$ and $26^2 = 676$.
Alternatively, in the context:
Perimeter $= 24 + 10 + 26 = 60$
Area $= \frac{1}{2}(24)(10) = 120$

10. a) $x - y = -30$ and $y + 3 + x^2 = 189$

b) (12, 42) or (-13, 17)

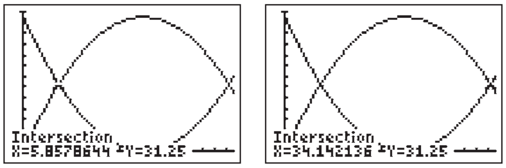
c) For 12 and 42:
$12 - 42 = -30$ and
$42 + 3 + 12^2 = 189$
For -13 and 17: $-13 - 17 = -30$ and
$17 + 3 + (-13)^2 = 189$
So, both solutions check.

11. a) $C = 2\pi r$, $C = 3\pi r^2$

b) $r = \frac{2}{3}$, $C = \frac{4\pi}{3}$, $A = \frac{4\pi}{9}$

12. a) 5.86 m and 34.14 m **b)** 31.25 J

c)



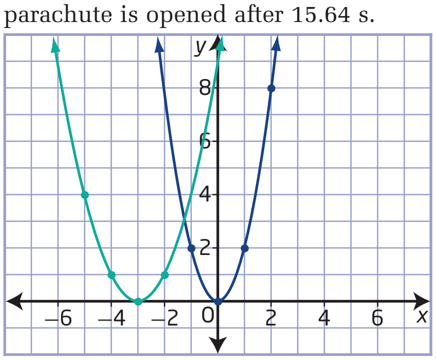
d) Find the sum of the values of E_k and E_p at several choices for d. Observe that the sum is constant, 62.5. This can be deduced from the graph because each is a reflection of the other in the horizontal line $y = 31.25$.

13. a) approximately 15.64 s

b) approximately 815.73 m

c)
$$\begin{aligned} h(t) &= -4.9t^2 + 2015 \\ &= -4.9(15.64)^2 + 2015 \\ &\approx 816.4 \end{aligned}$$
$$\begin{aligned} h(t) &= -10.5t + 980 \\ &= -10.5(15.64) + 980 \\ &\approx 815.78 \end{aligned}$$
The solution checks. Allowing for rounding errors, the height is about 816 m when the parachute is opened after 15.64 s.

14. a)



b) $(-1.3, 3)$ **c)** $y = 2x^2$ and $y = (x + 3)^2$

d) $(-1.24, 3.09)$ and $(7.24, 104.91)$ Example: The estimate was close for one point, but did not get the other.

15. a) For the first fragment, substitute $v_0 = 60$, $\theta = 45°$, and $h_0 = 2500$:
$$h(x) = -\frac{4.9}{(v_0 \cos \theta)^2}x^2 + (\tan \theta)x + h_0$$
$$h(x) = -\frac{4.9}{(60 \cos 45°)^2}x^2 + (\tan 45°)x + 2500$$
$$h(x) \approx -0.003x^2 + x + 2500$$
For the second fragment, substitute $v_0 = 60$, $\theta = 60°$, and $h_0 = 2500$:
$$h(x) = -\frac{4.9}{(v_0 \cos \theta)^2}x^2 + (\tan \theta)x + h_0$$
$$h(x) = -\frac{4.9}{(60 \cos 60°)^2}x^2 + (\tan 60°)x + 2500$$
$$h(x) \approx -0.005x^2 + 1.732x + 2500$$

b) $(0, 2500)$ and $(366, 2464.13)$

c) Example: This is where the fragments are at the same height and the same distance from the summit.

16. a) The solution for the system of equations will tell the horizontal distance from and the height above the base of the mountain, where the charge lands.

b) 150.21 m

17. a) 103 items **b)** \$377 125.92

18. a) $(-3.11, 0.79)$, $(3.11, 0.79)$ and $(0, 16)$

b) Example: 50 m^2

19. a) $(2, 6)$ **b)** $y = -\frac{1}{4}x + \frac{13}{2}$ **c)** 2.19 units

20. $(2, 1.5)$ and $(-1, 3)$

21. $y = 0.5(x + 1)^2 - 4.5$ and $y = -x - 4$ or $y = 0.5(x + 1)^2 - 4.5$ and $y = 2x - 4$

22. Example: Graphing is relatively quick using a graphing calculator, but may be time-consuming and inaccurate using pencil and grid paper. Sometimes, rearranging the equation to enter into the calculator is a bit tricky. The algebraic methods will always give an exact answer and do not rely on having technology available. Some systems of equations may be faster to solve algebraically, especially if one variable is easily eliminated.

23. $(-3.39, -0.70)$ and $(-1.28, 4.92)$

24. Example: Express the quadratic in vertex form, $y = (x - 2)^2 - 2$. This parabola has its minimum at $(2, -2)$ and its y-intercept at 2. The linear function has its y-intercept at -2 and has a negative slope so it is never close to the parabola. Algebraically,
$$\begin{aligned} -\frac{1}{2}x - 2 &= x^2 - 4x + 2 \\ -x - 4 &= 2x^2 - 8x + 4 \\ 0 &= 2x^2 - 7x + 8 \end{aligned}$$
This quadratic equation has no real roots. Therefore, the graphs do not intersect.

25. Step 1: Example: In a standard viewing window, it looks like there are two solutions when $b > 0$, one solution when $b = 0$, and no solution when $b < 0$

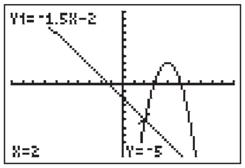
Step 2: There are two solutions when $b > -\frac{1}{4}$, one solution when $b = -\frac{1}{4}$, and no solution when $b < -\frac{1}{4}$.

Steps 3 and 4: two solutions when $|m| > 2$, one solution when $m = \pm 2$, and no solution when $|m| < 2$

Step 5: For $m = 1$: two solutions when $b > 0$, one solution when $b = 0$, and no solution when $b < 0$; for $m = -1$: two solutions when $b < 0$, one solution when $b = 0$, and no solution when $b > 0$; two solutions when $|m| > 2b$, one solution when $m = \pm 2b$, and no solution when $|m| < 2b$

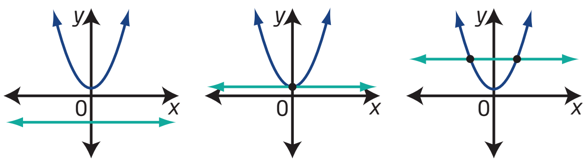
Chapter 8 Review, pages 457 to 458

1. a) (2, −5)

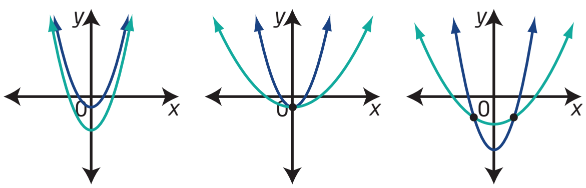
b) 

c) (6.75, −12.125)

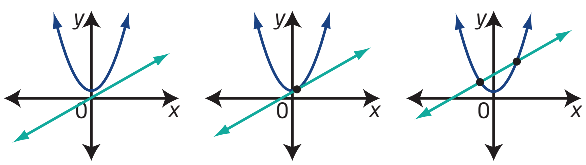
2. a) no solution, one solution, two solutions

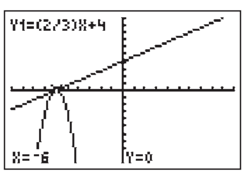


b) no solution, one solution, two solutions

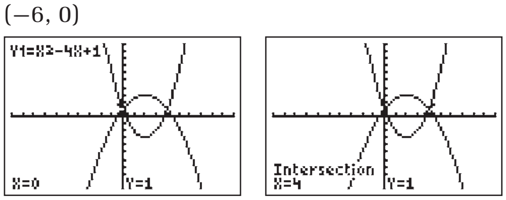


c) no solution, one solution, two solutions



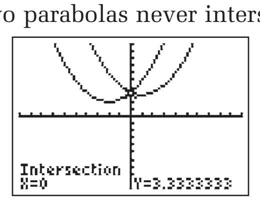
3. a) 

(−6, 0)

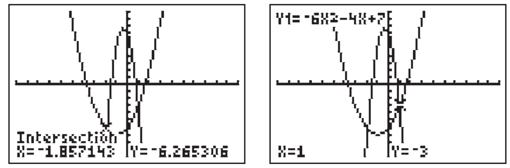
b) 

(0, 1) and (4, 1)

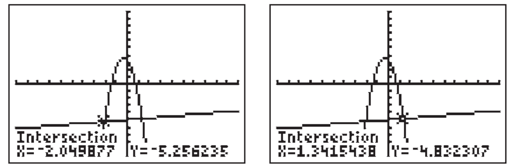
4. Example: Adam is not correct. For all values of x, $x^2 + 3$ is always 2 greater than $x^2 + 1$ and the two parabolas never intersect.

5. a) 

(0, 3.33) or $x = 0$, $p = 3\frac{1}{3}$

b) 

(−1.86, −6.27) and (1, −3)

c) 

(−2.05, −5.26) and (1.34, −4.83) or $d = -2.05$, $t = -5.26$ and $d = -1.34$, $t = -4.83$

6. a) road arch: $y = -\frac{3}{32}(x - 8)^2 + 6$

river arch: $y = -\frac{1}{18}(x - 24)^2 + 8$

b) (14.08, 2.53)

c) Example: the location and height of the support footing

7. a) Example: The first equation models the horizontal distance travelled and the height of the ball; it would follow a parabolic path that opens downward. The linear equation models the profile of the hill with a constant slope.

b) $d = 0$, $h = 0$ and $d = 14.44$, $h = 7.22$

c) Example: The point (0, 0) represents the starting point, where the ball was kicked. The point (14.44, 7.22) is where the ball would land on the hill. The coordinates give the horizontal distance and vertical distance from the point that the ball was kicked.

8. a) Example: (2, −3) and (6, 5)

b) (2, −3) and (6, 5)

9. The solution $\left(\frac{1}{2}, 1\right)$ is correct.

10. a) $\left(\frac{1}{2}, \frac{5}{2}\right)$ and $\left(-\frac{5}{3}, -4\right)$; substitution, because the first equation is already solved for p

b) (3.16, −13) and (−3.16, −13); elimination, because it is easy to make opposite coefficients for the y-terms

c) $\left(-\frac{2}{3}, -\frac{53}{9}\right)$ and (2, −1); elimination after clearing the fractions

d) (0.88, 0.45) and (−1.88, 3.22); substitution after isolating y in the second equation

11. a) 0 m and 100 m **b)** 0 m and 10 m

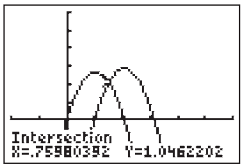
12. a) the time when both cultures have the same rate of increase of surface area

b) (0, 0) and (6.67, 0.02)

c) The point (0, 0) represents the starting point. In 6 h 40 min, the two cultures have the same rate of increase of surface area.

Chapter 8 Practice Test, pages 459 to 460

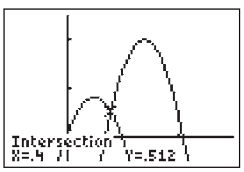
1. C
2. C
3. B
4. D
5. D
6. $n = -4$
7. a) $\left(\frac{3}{4}, -\frac{35}{16}\right)$ and $(-2, -7)$ **b)** $(1, -4)$
8. a)



(0.76, 1.05)

b) Example: At this time, 0.76 s after Sophie starts her jump, both dancers are at the same height above the ground.

9. a) perimeter: $8y = 4x + 28$
area: $6y + 3 = x^2 + 14x + 48$
b) The perimeter is 16 m. The area is 15 m^2.
10. a) $y = \frac{1}{3}x^2$ and $y = \frac{1}{2}(x - 1)^2$
b) (5.45, 9.90) and (0.55, 0.10)
11. a)



b) Example: At this point, a horizontal distance of 0.4 cm and a vertical distance of 0.512 cm from the start of the jump, the second part of the jump begins.

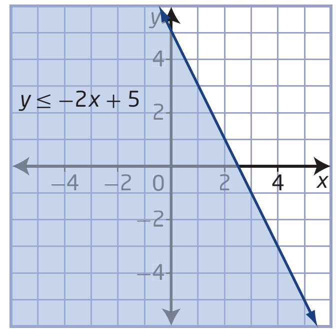
12. A(−3.52, 0), B(7.52, 0), C(6.03, 14.29)
area = 78.88 square units

Chapter 9 Linear and Quadratic Inequalities

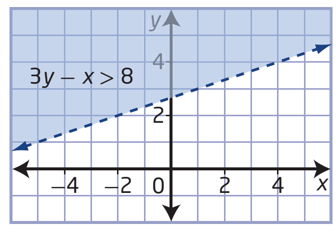
9.1 Linear Inequalities in Two Variables, pages 472 to 475

1. a) (6, 7), (12, 9)
b) (−6, −12), (4, −1), (8, −2)
c) (12, −4), (5, 1) **d)** (3, 1), (6, −4)
2. a) (1, 0), (−2, 1) **b)** (−5, 8), (4, 1)
c) (5, 1) **d)** (3, −1)
3. a) $y \le x + 3$; slope of 1; y-intercept of 3; the boundary is a solid line.
b) $y > 3x + 5$; slope of 3; y-intercept of 5; the boundary is a dashed line.
c) $y > -4x + 7$; slope of −4; y-intercept of 7; the boundary is a dashed line.
d) $y \ge 2x - 10$; slope of 2; y-intercept of −10; the boundary is a solid line.
e) $y \ge -\frac{4}{5}x + 4$; slope of $-\frac{4}{5}$; y-intercept of 4; the boundary is a solid line.
f) $y > \frac{1}{2}x - 5$; slope of $\frac{1}{2}$; y-intercept of −5; the boundary is a dashed line.

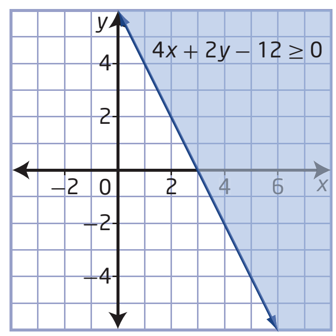
4. a)



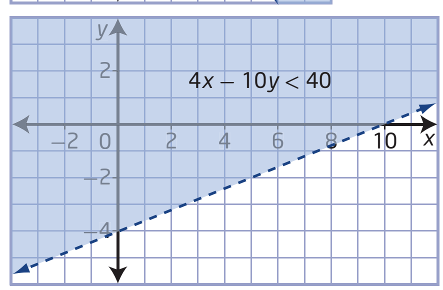
b)



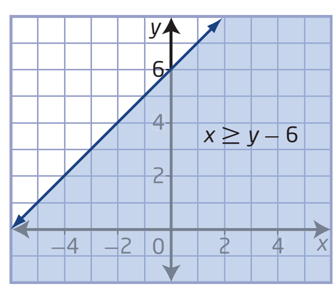
c)



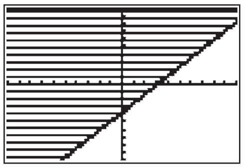
d)



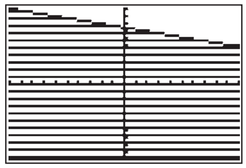
e)



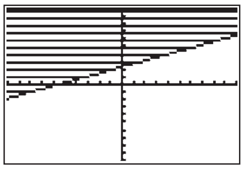
5. a) $y \geq \frac{6}{5}x - \frac{18}{5}$



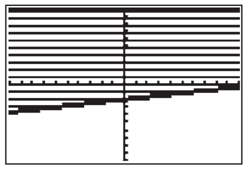
b) $y < -\frac{1}{4}x + \frac{15}{2}$



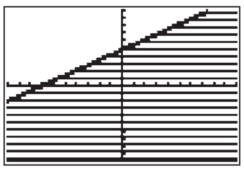
c) $y > \frac{5}{12}x + \frac{7}{3}$



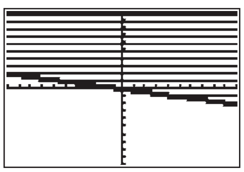
d) $y \geq \frac{1}{6}x - \frac{11}{6}$



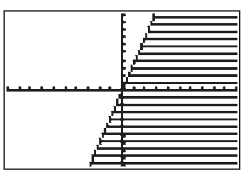
e) $y \leq \frac{36}{53}x + \frac{260}{53}$



6. $y \geq -\frac{1}{5}x$

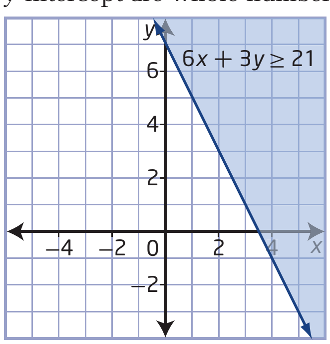


7. $y < \frac{7}{2}x$

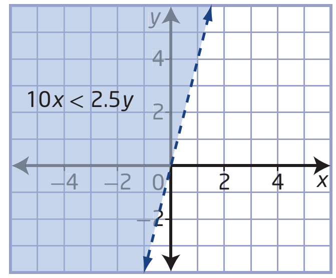


8. Examples:

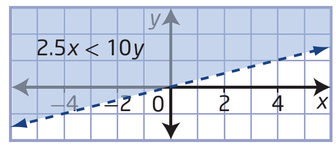
a) Graph by hand because the slope and the y-intercept are whole numbers.



b) Graph by hand because the slope and the y-intercept are whole numbers.

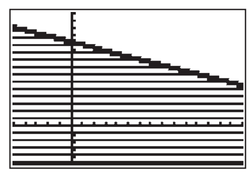


c) Graph by hand because the slope is a simple fraction and the y-intercept is 0.

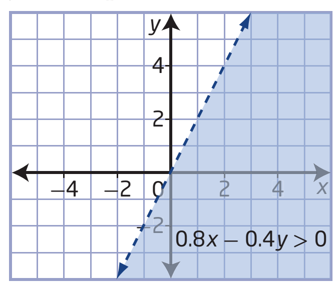


d) Graph using technology because the slope and the y-intercept are complicated fractions.

$y \leq -\frac{489}{1279}x + \frac{14\,500}{1279}$



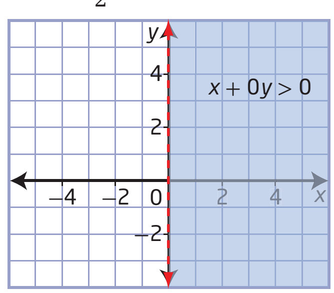
e) Graph by hand because the slope and the y-intercept are whole numbers.



9. a) $y < \frac{1}{4}x + 2$ **b)** $y < -\frac{1}{4}x$

c) $y > \frac{3}{2}x - 4$ **d)** $y \leq -\frac{3}{4}x + 5$

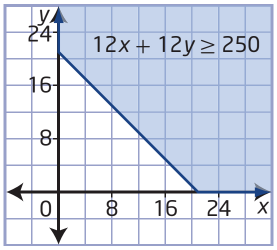
10.



The graph of this solution is everything to the right of the y-axis.

11. a) $12x + 12y \geq 250$, where x represents the number of moccasins sold, $x \geq 0$, and y represents the hours worked, $y \geq 0$.

b)

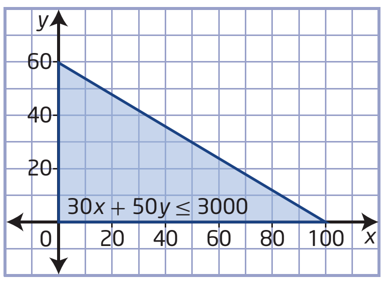


c) Example: (4, 20), (8, 16), (12, 12)

d) Example: If she loses her job, then she will still have a source of income.

12. a) $30x + 50y \leq 3000$, $x \geq 0$, $y \geq 0$, where x represents the hours of work and y represents the hours of marketing assistance.

b)



13. $0.3x + 0.05y \leq 100$, $x \geq 0$, $y \geq 0$, where x represents the number of minutes used and y represents the megabytes of data used; she should stay without a plan if her usage stays in the region described by the inequality.

14. $60x + 45y \leq 50$, $x \geq 0$, $y \geq 0$, where x represents the area of glass and y represents the mass of nanomaterial.

15. $125x + 55y \leq 7000$, $x \geq 0$, $y \geq 0$, where x represents the hours of ice rental and y represents the hours of gym rental.

16. Example:

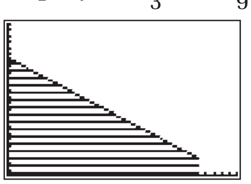
a) $y = x^2$ **b)** $y \geq x^2$; $y < x^2$

c) This does satisfy the definition of a solution region. The boundary is a curve not a line.

17. $y \geq \frac{3}{4}x + 384$, $0 \leq x \leq 512$; $y \leq -\frac{3}{4}x + 384$, $0 \leq x \leq 512$; $y \geq -\frac{3}{4}x + 1152$, $512 \leq x \leq 1024$; $y \leq \frac{3}{4}x - 384$, $512 \leq x \leq 1024$

18. Step 1 $60x + 90y \leq 35\,000$

Step 2 $y \leq -\frac{2}{3}x + \frac{3500}{9}$, $0 \leq x \leq 500$, $y \geq 0$



(0, 0), approximately (0, 388.9) and (500, 55.6), and (500, 0); y-intercept: the maximum number of megawatt hours of wind power that can be produced; x-intercept: the maximum number of megawatt hours of hydroelectric power that can be produced

Step 3 Example: It would be very time-consuming to attempt to find the revenue for all possible combinations of power generation. You cannot be certain that the spreadsheet gives the maximum revenue.

Step 4 The maximum revenue is \$53 338, with 500 MWh of hydroelectric power and approximately 55.6 MWh of wind power.

19. Example:

	Example 1	Example 2	Example 3	Example 4
Linear Inequality	$y \geq x$	$y \leq x$	$y > x$	$y < x$
Inequality Sign	$\geq$	$\leq$	$>$	$<$
Boundary Solid/Dashed	Solid	Solid	Dashed	Dashed
Shaded Region	Above	Below	Above	Below

20. Example: Any scenario with a solution that has the form $5y + 3x \leq 150$, $x \geq 0$, $y \geq 0$ is correct.

21. a) 48 units²

b) The y-intercept is the height of the triangle. The larger it gets, the larger the area gets.

c) The slope of the inequality dictates where the x-intercept will be, which is the base of the triangle. Steeper slope gives a closer x-intercept, which gives a smaller area.

d) If you consider the magnitude, then nothing changes.

9.2 Quadratic Inequalities in One Variable, pages 484 to 487

1. a) $\{x \mid 1 \leq x \leq 3, x \in \mathrm{R}\}$

b) $\{x \mid x \leq 1 \text{ or } x \geq 3, x \in \mathrm{R}\}$

c) $\{x \mid x < 1 \text{ or } x > 3, x \in \mathrm{R}\}$

d) $\{x \mid 1 < x < 3, x \in \mathrm{R}\}$

2. a) $\{x \mid x \in \mathrm{R}\}$ **b)** $\{x \mid x = 2, x \in \mathrm{R}\}$

c) no solution **d)** $\{x \mid x \neq 2, x \in \mathrm{R}\}$

3. a) not a solution **b)** solution

c) solution **d)** not a solution

4. a) $\{x \mid x \leq -10 \text{ or } x \geq 4, x \in \mathrm{R}\}$

b) $\{x \mid x < -12 \text{ or } x > -2, x \in \mathrm{R}\}$

c) $\left\{x \mid x < -\frac{5}{3} \text{ or } x > \frac{7}{2}, x \in \mathrm{R}\right\}$

d) $\left\{x \mid -2 - \frac{\sqrt{6}}{2} \leq x \leq 2 + \frac{\sqrt{6}}{2}, x \in \mathrm{R}\right\}$

5. a) $\{x \mid -6 \le x \le 3, x \in R\}$
b) $\{x \mid x \le -3 \text{ or } x \ge -1, x \in R\}$
c) $\left\{x \mid \frac{3}{4} < x < 6, x \in R\right\}$
d) $\{x \mid -8 \le x \le 2, x \in R\}$

6. a) $\{x \mid -3 < x < 5, x \in R\}$
b) $\{x \mid x < -12 \text{ or } x > -1, x \in R\}$
c) $\{x \mid x \le 1 - \sqrt{6} \text{ or } x \ge 1 + \sqrt{6}, x \in R\}$
d) $\left\{x \mid x \le -8 \text{ or } x \ge \frac{1}{2}, x \in R\right\}$

7. a) $\{x \mid -8 \le x \le -6, x \in R\}$
b) $\{x \mid x \le -4 \text{ or } x \ge 7, x \in R\}$
c) There is no solution.
d) $\{x \mid x < -\frac{7}{2} \text{ or } x > \frac{9}{2}, x \in R\}$

8. a) $\{x \mid 2 < x < 8, x \in R\}$
Example: Use graphing because it is a simple graph to draw.
b) $\left\{x \mid x \le -\frac{3}{4} \text{ or } x \ge \frac{5}{3}, x \in R\right\}$
Example: Use sign analysis because it is easy to factor.
c) $\{x \mid 1 - \sqrt{13} \le x \le 1 + \sqrt{13}, x \in R\}$
Example: Use test points and the zeros.
d) $\{x \mid x \ne 3, x \in R\}$
Example: Use case analysis because it is easy to factor and solve for the inequalities.

9. a) $\left\{x \mid \frac{13 - \sqrt{145}}{2} \le x \le \frac{13 + \sqrt{145}}{2}, x \in R\right\}$
b) $\{x \mid x < -12 \text{ or } x > 2, x \in R\}$
c) $\left\{x \mid x < \frac{5}{2} \text{ or } x > 4, x \in R\right\}$
d) $\left\{x \mid x \le -\frac{8}{3} \text{ or } x \ge \frac{7}{2}, x \in R\right\}$

10. a) Ice equal to or thicker than $\frac{5\sqrt{30}}{3}$ cm, or about 9.13 cm, will support the weight of a vehicle.
b) $9h^2 \ge 1500$
c) Ice equal to or thicker than $\frac{10\sqrt{15}}{3}$ cm, or about 12.91 cm, will support the weight of a vehicle.
d) Example: The relationship between ice strength and thickness is not linear.

11. a) $\pi x^2 \le 630\,000$, where x represents the radius, in metres.
b) $0 \le x \le \sqrt{\frac{630\,000}{\pi}}$ **c)** $0 \text{ m} \le x \le 447.81 \text{ m}$

12. a) 2 years or more
b) One of the solutions is negative, which does not make sense in this problem. Time cannot be negative.
c) $-t^2 + 14 \le 5$; $t \ge 3$; 3 years or more

13. $\frac{x^2}{2} + x \ge 4$; the shorter leg should be greater than or equal to 2 cm.

14. a) $a > 0$; $b^2 - 4ac \le 0$ **b)** $a < 0$; $b^2 - 4ac = 0$
c) $a \ne 0$; $b^2 - 4ac > 0$

15. Examples:
a) $x^2 - 5x - 14 \le 0$ **b)** $x^2 - 11x + 10 > 0$
c) $3x^2 - 23x + 30 \le 0$ **d)** $20x^2 + 19x + 3 > 0$
e) $x^2 + 6x + 2 \ge 0$ **f)** $x^2 + 1 > 0$
g) $x^2 + 1 < 0$

16. $\{x \mid x \le -\sqrt{6} \text{ or } -\sqrt{2} \le x \le \sqrt{2} \text{ or } x \ge \sqrt{6}, x \in R\}$

17. a) It is the solution because it is the set of values for which the parabola lies above the line.
b) $-x^2 + 13x - 12 \ge 0$
c) $\{x \mid 1 \le x \le 12, x \in R\}$
d) They are the same solutions. The inequality was just rearranged in part c).

18. They all require this step because you need the related function to work with.

19. Answers may vary.

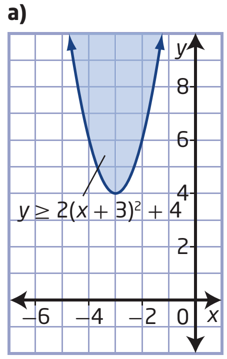
20. a) The solution is incorrect. He switched the inequality sign when he added 2 to both sides in the first step.
b) $\{x \mid -3 \le x \le -2, x \in R\}$

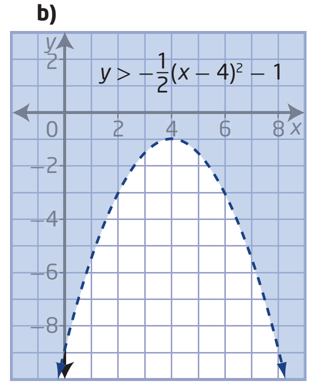
9.3 Quadratic Inequalities in Two Variables, pages 496 to 500

1. a) $(2, 6)$, $(-1, 3)$
b) $(2, -2)$, $(0, -6)$, $(-2, -15)$
c) None
d) $(-4, 2)$, $(1, 3.5)$, $(3, 2.5)$

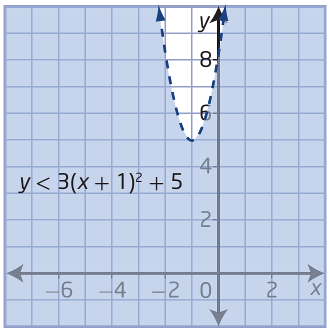
2. a) $(0, 1)$, $(1, 0)$, $(3, 6)$, $(-2, 15)$
b) $(-2, -3)$, $(0, -8)$
c) $(2, 9)$
d) $(-2, 2)$, $(-3, -2)$

3. a) $y < -x^2 - 4x + 5$ **b)** $y \le \frac{1}{2}x^2 - x + 3$
c) $y \ge -\frac{1}{4}x^2 - x + 3$ **d)** $y > 4x^2 + 5x - 6$

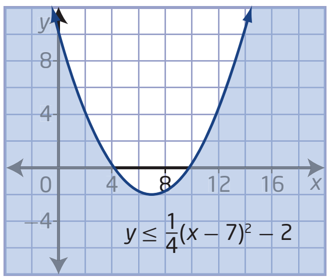
4. a)


b)


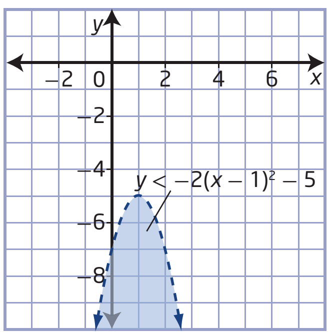
c)



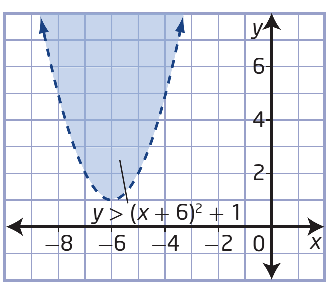
d)



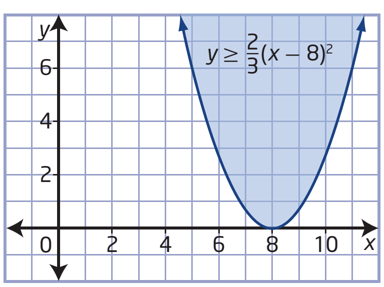
5. a)



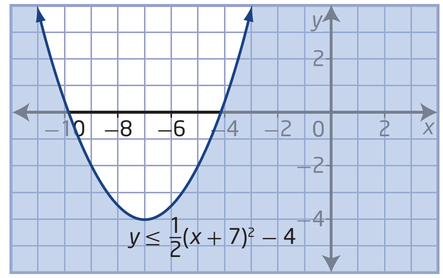
b)



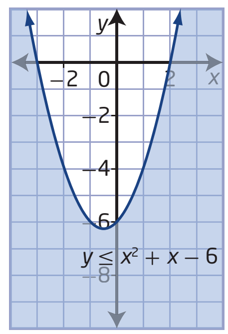
c)



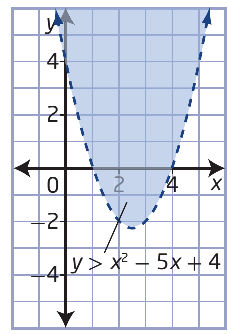
d)



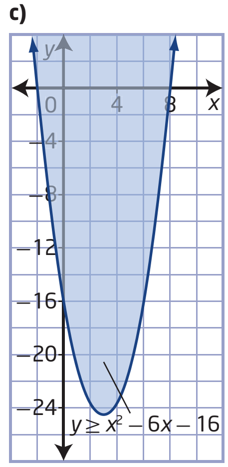
6. a)



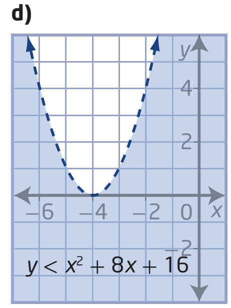
b)



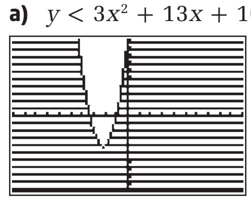
c)



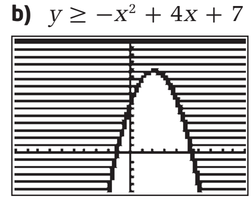
d)



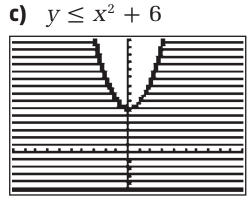
7. a) $y < 3x^2 + 13x + 10$



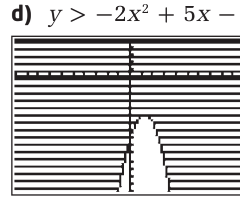
b) $y \geq -x^2 + 4x + 7$



c) $y \leq x^2 + 6$



d) $y > -2x^2 + 5x - 8$



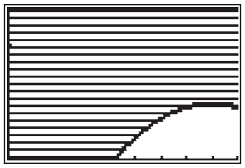
8. a) $y \geq \frac{1}{2}x^2 + 1$

b) $y > -\frac{1}{3}(x - 1)^2 + 3$

9. a) $y = -\frac{1}{625}(x - 50)^2 + 4$

b) $y < -\frac{1}{625}(x - 50)^2 + 4$, $0 \le x \le 100$

10. a) $L \ge -0.000\,125a^2 + 0.040a - 2.442$, $0 \le a \le 180$, $L \ge 0$



b) any angle greater than or equal to approximately 114.6° and less than or equal to 180°

11. a) $y < -0.03x^2 + 0.84x - 0.08$

b) $0 \le -0.03x^2 + 0.84x - 0.28$
$\{x \mid 0.337\ldots \le x \le 27.662\ldots, x \in \mathrm{R}\}$

c) The width of the river is 27.325 m.

12. a) $0 < -2.944t^2 + 191.360t - 2649.6$

b) Between 20 s and 45 s is when the jet is above 9600 m.

c) 25 s

13. a) $y = -0.04x^2 + 5$ b) $0 \le -0.04x^2 + 5$

14. a) $y \le -x^2 + 20x$ or $y \le -1(x - 10)^2 + 100$

b) $-x^2 + 20x - 50 \ge 0$; she must have between 3 and 17 ads.

15. $y \le -0.0001x^2 - 600$ and $y \ge -0.0002x^2 - 700$

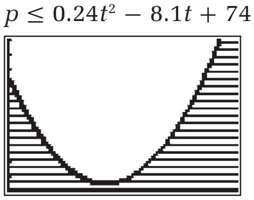
16. a) $y = (4 + 0.5x)(400 - 20x)$ or $y = -10x^2 + 120x + 1600$; x represents the number of \$0.50 increases and y represents the total revenue.

b) $0 \le -10x^2 + 120x - 200$; to raise \$1800 the price has to be between \$5 and \$9.

c) $0 \le -10x^2 + 120x$; to raise \$1600 the price has to be between \$4 and \$10.

17. a) $0 \le 0.24x^2 - 8.1x + 64$; from approximately 12.6 years to 21.1 years after the year 2000

b) $p \le 0.24t^2 - 8.1t + 74$, $t \ge 0$, $p \ge 0$



Only the portion of the graph from $t = 0$ to $t \approx 16.9$ and from $p = 0$ to $p = 100$ is reasonable. This represents the years over which the methane produced goes from a maximum percent of 100 to a minimum percent around 16.9 years.

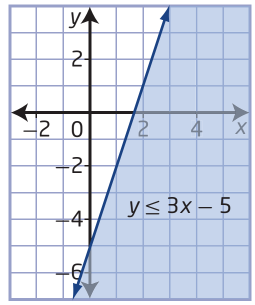
c) from approximately 12.6 years to 16.9 years after the year 2000

d) He should take only positive values of x from 0 to 16.9, because after that the model is no longer relevant.

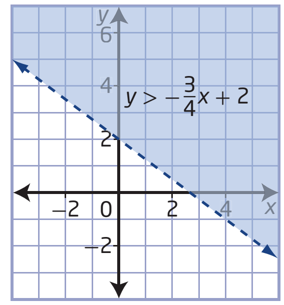
18. Answers may vary.

Chapter 9 Review, pages 501 to 503

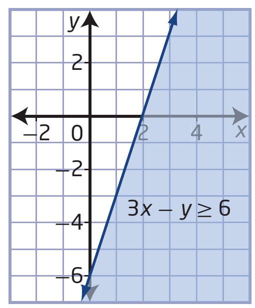
1. a)



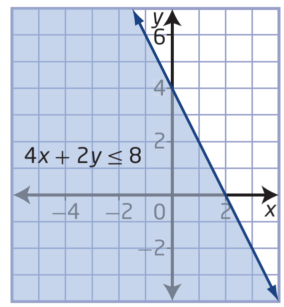
b)



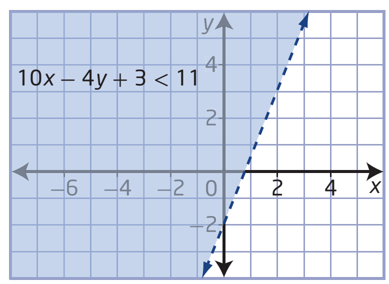
c)



d)



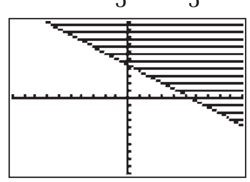
e)

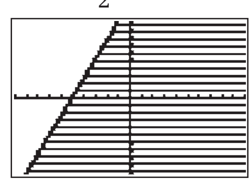


2. a) $y \ge 2x + 3$ b) $y > 0.25x - 1$

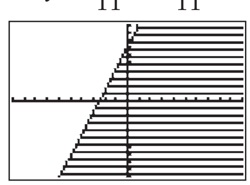
c) $y < -3x + 2$ d) $y \le -0.75x + 2$

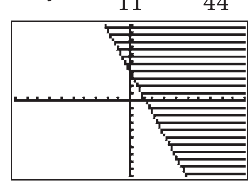
3. a) $y > -\frac{4}{5}x + \frac{22}{5}$ b) $y \le \frac{5}{2}x + 13$



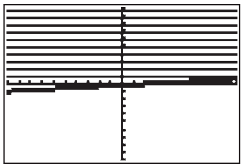


c) $y < \frac{32}{11}x + \frac{80}{11}$ d) $y > -\frac{31}{11}x + \frac{165}{44}$



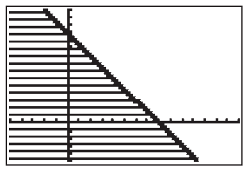


e) $y \geq \frac{1}{12}x$



4. a) $15x + 10y \leq 120$, where x represents the number of movies and y represents the number of meals.

b) $y \leq -1.5x + 12$

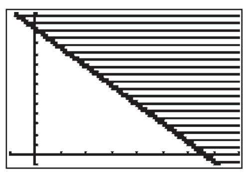


c) The region below the line in quadrant I ($x \geq 0$, $y \geq 0$) shows which combinations will work for her budget. The values of x and y must be whole numbers.

5. a) \$30 for a laptop and \$16 for a DVD player

b) $30x + 16y \geq 1000$, where x represents the number of laptops sold and y represents the number DVD player sold.

c) $y \geq -1.875x + 62.5$



The region above the line in quadrant I shows which combinations will give the desired commission. The values of x and y must be whole numbers.

6. a) $\{x \mid x < -7 \text{ or } x > 9, x \in \mathrm{R}\}$

b) $\{x \mid x \leq -2.5 \text{ or } x \geq 6, x \in \mathrm{R}\}$

c) $\{x \mid -12 < x < 4, x \in \mathrm{R}\}$

d) $\{x \mid x \leq 3 - \sqrt{5} \text{ or } x \geq 3 + \sqrt{5}, x \in \mathrm{R}\}$

7. a) $\left\{x \mid -\frac{4}{3} \leq x \leq \frac{1}{2}, x \in \mathrm{R}\right\}$

b) $\left\{x \mid \frac{5 - \sqrt{21}}{4} < x < \frac{5 + \sqrt{21}}{4}, x \in \mathrm{R}\right\}$

c) $\{x \mid -4 \leq x \leq 8, x \in \mathrm{R}\}$

d) $\left\{x \mid x \leq \frac{-6 - 2\sqrt{14}}{5} \text{ or } x \geq \frac{-6 + 2\sqrt{14}}{5}, x \in \mathrm{R}\right\}$

8. a) $\left\{x \mid \frac{6 - 2\sqrt{3}}{3} \leq x \leq \frac{6 + 2\sqrt{3}}{3}, x \in \mathrm{R}\right\}$

b) The path has to be between those two points to allow people up to 2 m in height to walk under the water.

9. The length can be anything up to and including 6 m. The width is just half the length, so it is a maximum of 3 m.

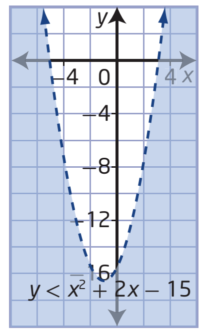
10. a) 104.84 km/h

b) $0.007v^2 + 0.22v \leq 50$

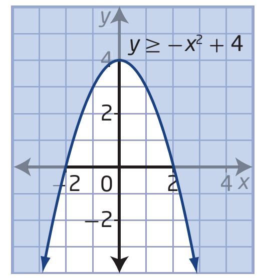
c) The solution to the inequality within the given context is $0 < v \leq 70.25$. The maximum stopping speed of 70.25 km/h is not half of the answer from part a) because the function is quadratic not linear.

11. a) $y \leq \frac{1}{2}(x + 3)^2 - 4$ **b)** $y > 2(x - 3)^2$

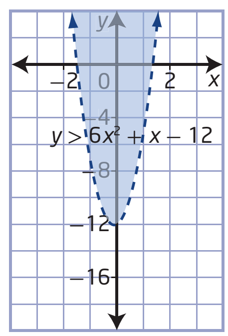
12. a)



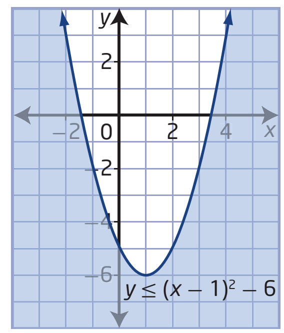
b)



c)



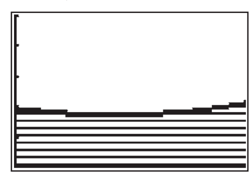
d)



13. a) $y < x^2 + 3$

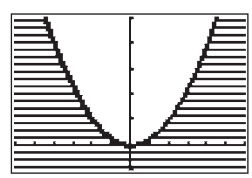
b) $y \leq -(x + 4)^2 + 2$

14. a) $y \leq 0.003t^2 - 0.052t + 1.986$, $0 \leq t \leq 20$, $y \geq 0$



b) $0.003t^2 - 0.052t - 0.014 \leq 0$; the years it was at most 2 t/ha were from 1975 to 1992.

15. a) $r \leq 0.1v^2$



You cannot have a negative value for the speed or the radius. Therefore, the domain is $\{v \mid v \geq 0, v \in \mathrm{R}\}$ and the range is $\{r \mid r \geq 0, r \in \mathrm{R}\}$.

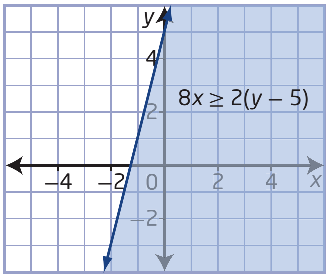
b) Any speed above 12.65 m/s will complete the loop.

16. a) $20 \leq \frac{1}{20}x^2 - 4x + 90$

b) $\{x \mid 0 \leq x \leq 25.86 \text{ or } 54.14 \leq x \leq 90, x \in \mathrm{R}\}$; the solution shows where the cable is at least 20 m high.

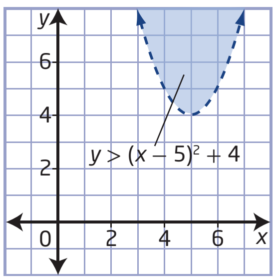
Chapter 9 Practice Test, pages 504 to 505

1. B
2. A
3. C
4. B
5. C
6.



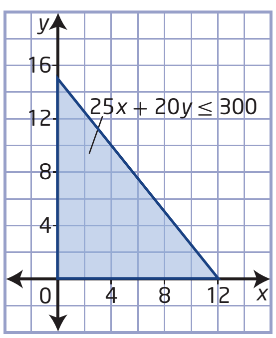
7. $\left\{x \mid -\frac{2}{3} < x < \frac{5}{4}, x \in \mathrm{R}\right\}$

8.



9. $y \geq 0.02x^2$

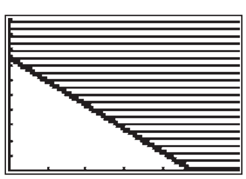
10. $25x + 20y \leq 300$, where x represents the number of hours scuba diving ($x \geq 0$) and y represents the number of hours sea kayaking ($y \geq 0$).



11. a) $50x + 80y \geq 1200$, where x represents the number of ink sketches sold ($x \geq 0$) and y represents the number of watercolours sold ($y \geq 0$).

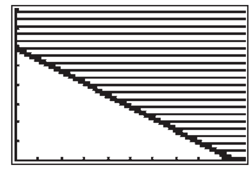
b) $y \geq -\frac{5}{8}x + 15$, $y \geq 0$, $x \geq 0$

Example: (0, 15), (2, 15), (8, 12)



c) $50x + 80y \geq 2400$, where x represents the number of ink sketches sold ($x \geq 0$) and y represents the number of watercolours sold ($y \geq 0$); the related line is parallel to the original with a greater x-intercept and y-intercept.

d) $y \geq -\frac{5}{8}x + 30$, $y \geq 0$, $x \geq 0$



12. a) Example: $f(x) = x^2 - 2x - 15$

b) Example: any quadratic function with two real zeros and whose graph opens upward

c) Example: It is easier to express them in vertex form because you can tell if the parabola opens upward and has a vertex below the x-axis, which results in two zeros.

13. a) $0.01a^2 + 0.05a + 107 < 120$

b) $\{x \mid -38.642 < x < 33.642, x \in \mathrm{R}\}$

c) The only solutions that make sense are those where x is greater than 0. A person cannot have a negative age.

Cumulative Review, Chapters 8–9, pages 508 to 509

1. a) B **b)** D **c)** A **d)** C

2. $(-2.2, 10.7)$, $(2.2, -2.7)$

3. a) $(-1, -4)$, $(2, 5)$

b) The ordered pairs represent the points where the two functions intersect.

4. a) $b > 3.75$ **b)** $b = 3.75$ **c)** $b < 3.75$

5.

<table>
<tr><th colspan="3">Solving Linear-Quadratic Systems</th></tr>
<tr><th>Substitution Method</th><th colspan="2">Elimination Method</th></tr>
<tr><td>Determine which variable to solve for.</td><td>Determine which variable to eliminate.</td><td>Multiply the linear equation as needed.</td></tr>
<tr><td>Solve the linear equation for the chosen variable.</td><td colspan="2" rowspan="2">Add a new linear equation and quadratic equation.</td></tr>
<tr><td>Substitute the expression for the variable into the quadratic equation and simplify.</td></tr>
<tr><td colspan="3">Solve New Quadratic Equation</td></tr>
<tr><td>No Solution</td><td colspan="2">Substitute the value(s) into the original linear equation to determine the corresponding value(s) of the other variable.</td></tr>
</table>

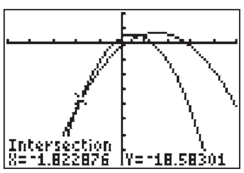
6.

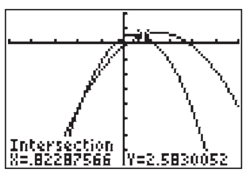
<table>
<tr><th colspan="3">Solving Quadratic-Quadratic Systems</th></tr>
<tr><th>Substitution Method</th><th colspan="2">Elimination Method</th></tr>
<tr><td>Solve one quadratic equation for the y-term.</td><td>Eliminate the y-term.</td><td>Multiply equations as needed.</td></tr>
<tr><td>Substitute the expression for the y-term into the other quadratic equation and simplify.</td><td colspan="2">Add new equations.</td></tr>
<tr><td colspan="3">Solve New Quadratic Equation</td></tr>
<tr><td>No solution.</td><td colspan="2">Substitute the value(s) into an original equation to determine the corresponding value(s) of x.</td></tr>
</table>

7. The two stocks will be the same price at \$34 and \$46.

8. Example: The number of solutions can be determined by the location of the vertex and the direction in which the parabola opens. The vertex of the first parabola is above the x-axis and it opens upward. The vertex of the second parabola is below the x-axis and it opens downward. The system will have no solution.

9. $(-1.8, -18.6)$, $(0.8, 2.6)$





10. a) $(1, 6)$ **b)** $(0, -5)$, $(1, 24)$

11. a) D **b)** A **c)** B **d)** C

12. a) $y > x^2 + 1$ **b)** $y \geq -(x + 3)^2 + 2$

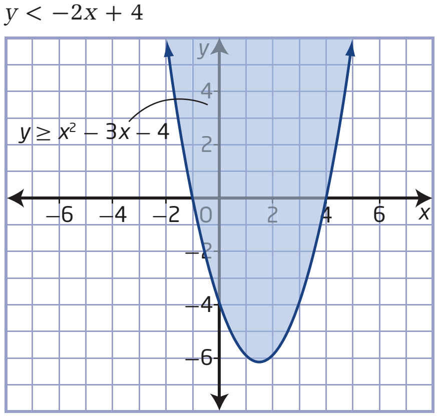
13. a) Results in a true statement. Shade the region that contains the point $(0, 0)$.

b) Results in a false statement. Shade the region that does not contain the point $(2, -5)$.

c) The point $(-1, 1)$ cannot be used as a test point. The point is on the boundary line.

14. $y < -2x + 4$

15.



16. $\{x \mid x \leq -7 \text{ or } x \geq 2.5, x \in \mathrm{R}\}$

17. The widths must be between 200 m and 300 m.

Unit 4 Test, pages 511 to 513

1. C
2. C
3. B
4. B
5. D
6. D
7. A
8. B
9. A
10. 5
11. 3
12. 1 s

13. a) $(0, 0)$, $(2, 4)$

b) The points are where the golfer is standing and where the hole is.

14. $g(x) = -(x - 6)^2 + 13$

15. $(-9, 256)$, $(-1, 0)$

16. a) Example: In the second step, she should have subtracted 2 from both sides of the inequality. It should be $3x^2 - 5x - 12 > 0$.

b) $\left\{x \mid x < -\frac{4}{3} \text{ or } x > 3, x \in \mathrm{R}\right\}$

17. The ball is above 3 m for 1.43 s.

Glossary

A

absolute value For a real number a, the absolute value is written as $|a|$ and is a positive number.

$$|a| = \begin{cases} a, \text{ if } a \geq 0 \\ -a, \text{ if } a < 0 \end{cases}$$

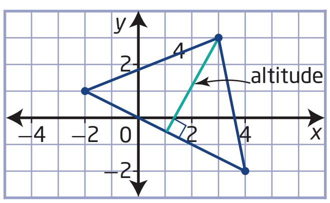
absolute value equation An equation that includes the absolute value of an expression involving a variable.

absolute value function A function that involves the absolute value of a variable.

acute angle An angle that is between 0° and 90°.

acute triangle A triangle in which each of the three interior angles is acute.

altitude (of a triangle) The perpendicular distance from a vertex to the opposite side of a triangle.



ambiguous case From the given information, the solution for the triangle is not clear: there might be one triangle, two triangles, or no triangle.

angle in standard position The position of an angle when its initial arm is on the positive x-axis and its vertex is at the origin of a coordinate grid.

arithmetic sequence A sequence in which the difference between consecutive terms is constant. An arithmetic sequence is represented by the formula for the general term $t_n = t_1 + (n - 1)d$, where t_1 is the first term, n is the number of terms, and d is the common difference.

The sequence 1, 4, 7, 10, … is arithmetic.

arithmetic series The terms of an arithmetic sequence expressed as a sum. This sum can be determined using the formula $S_n = \frac{n}{2}[2t_1 + (n - 1)d]$ or $S_n = \frac{n}{2}(t_1 + t_n)$, where n is the number of terms, t_1 is the first term, d is the common difference, and t_n is the nth term.

asymptote A line whose distance from a given curve approaches zero.

axis of symmetry A line through the vertex that divides the graph of a quadratic function into two congruent halves. The x-coordinate of the vertex defines the equation of the axis of symmetry.

B

binomial A polynomial with two terms.

For example, $x^2 + 3$, $m^2n + 4n$, and $2x - 5y$ are binomials.

boundary A line or curve that separates the Cartesian plane into two regions and may or may not be part of the solution region. Drawn as a solid line and included in the solution region if the inequality involves $\leq$ or $\geq$. Drawn as a dashed line and not included in the solution region if the inequality involves $<$ or $>$.

C

common difference The difference between successive terms in an arithmetic sequence, which may be positive or negative. The common difference, d, is equal to $t_n - t_{n-1}$.

For the sequence 1, 4, 7, 10, …, the common difference is 3.

common ratio The ratio of successive terms in a geometric sequence, which may be positive or negative. The common ratio, r, is equal to $\frac{t_n}{t_{n-1}}$.

For the sequence 1, 2, 4, 8, 16, …, the common ratio is 2.