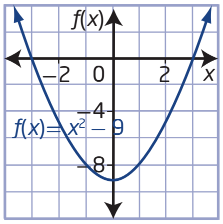
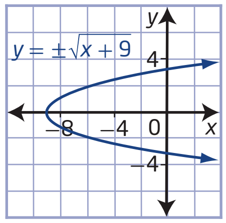
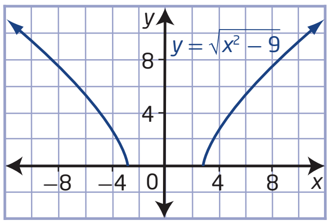
13. a)



b) $y = \pm\sqrt{x + 9}$



c) $y = \sqrt{x^2 - 9}$



d) for part a): domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \geq -9, y \in \mathrm{R}\}$; for part *b*): domain $\{x \mid x \geq -9, x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$; for part c): domain $\{x \mid x \leq -3 \text{ or } x \geq 3, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

14. Quadrant II: reflection in the *y*-axis, $y = f(-x)$; quadrant III: reflection in the *y*-axis and then the *x*-axis, $y = -f(-x)$; quadrant IV: reflection in the *x*-axis, $y = -f(x)$

15. a) Mary should have subtracted 4 from both sides in step 1. She also incorrectly squared the expression on the right side in step 2. The correct solution follows:

$2x = \sqrt{x + 1} + 4$

Step 1: $(2x - 4)^2 = (\sqrt{x + 1})^2$

Step 2: $4x^2 - 16x + 16 = x + 1$

Step 3: $4x^2 - 17x + 15 = 0$

Step 4: $(4x - 5)(x - 3) = 0$

Step 5: $4x - 5 = 0$ or $x - 3 = 0$

Step 6: $x = \frac{5}{4}$ $\quad x = 3$

Step 7: A check determines that $x = 3$ is the solution.

b) Yes, the point of intersection of the two graphs will yield the possible solution, $x = 3$.

16. $c = -3$; $P(x) = (x + 3)(x + 2)(x - 1)^2$

17. a) $\pm1, \pm2, \pm3, \pm6$

b) $P(x) = (x - 3)(x + 2)(x + 1)$

c) *x*-intercepts: −2, −1 and 3; *y*-intercept: −6

d) $-2 \leq x \leq -1$ and $x \geq 3$

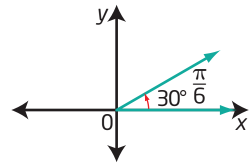
Chapter 4 Trigonometry and the Unit Circle

4.1 Angles and Angle Measure, pages 175 to 179

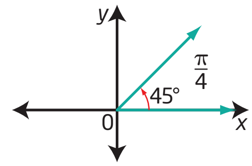
1. a) clockwise **b)** counterclockwise

c) clockwise **d)** counterclockwise

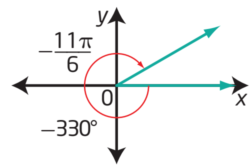
2. a) $30° = \frac{\pi}{6}$



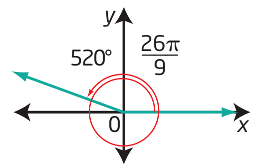
b) $45° = \frac{\pi}{4}$



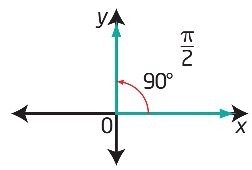
c) $-330° = -\frac{11\pi}{6}$



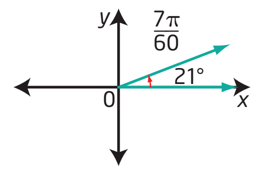
d) $520° = \frac{26\pi}{9}$



e) $90° = \frac{\pi}{2}$



f) $21° = \frac{7\pi}{60}$



3. a) $\frac{\pi}{3}$ or 1.05 **b)** $\frac{5\pi}{6}$ or 2.62

c) $-\frac{3\pi}{2}$ or −4.71 **d)** $\frac{2\pi}{5}$ or 1.26

e) $-\frac{37\pi}{450}$ or −0.26 **f)** 3π or 9.42

4. a) 30° **b)** 120°

c) −67.5° **d)** −450°

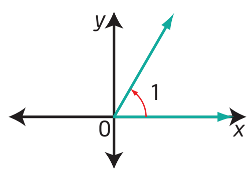
e) $\frac{180°}{\pi}$ or 57.3° **f)** $\frac{495°}{\pi}$ or 157.6°

5. a) $\frac{360°}{7}$ or 51.429° **b)** $\frac{1260°}{13}$ or 96.923°

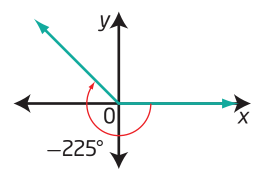
c) $\frac{120°}{\pi}$ or 38.197° **d)** $\frac{3294°}{5\pi}$ or 209.703°

e) $\frac{-1105.2°}{\pi}$ or −351.796° **f)** $\frac{-3600°}{\pi}$ or −1145.916°

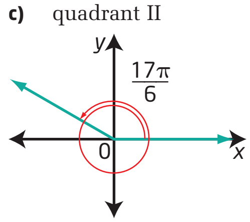
6. a) quadrant I



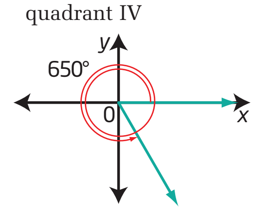
b) quadrant II



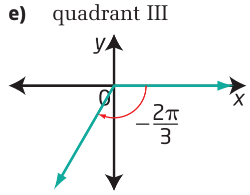
c) quadrant II



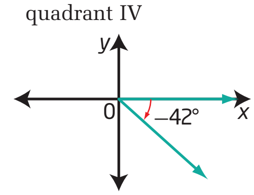
d) quadrant IV



e) quadrant III



f) quadrant IV

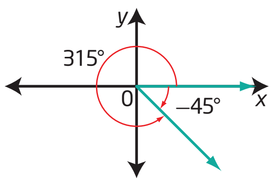


7. Examples:
   a) 432°, −288° b) $\frac{11\pi}{4}$, $-\frac{5\pi}{4}$
   c) 240°, −480° d) $\frac{7\pi}{2}$, $-\frac{\pi}{2}$
   e) 155°, −565° f) 1.5, −4.8

8. a) coterminal, $\frac{17\pi}{6} = \frac{5\pi}{6} + \frac{12\pi}{6} = \frac{5\pi}{6} + 2\pi$
   b) not coterminal c) not coterminal
   d) coterminal, $-493° = 227° - 2(360°)$

9. a) $135° \pm (360°)n,\ n \in \mathrm{N}$ b) $-\frac{\pi}{2} \pm 2\pi n,\ n \in \mathrm{N}$
   c) $-200° \pm (360°)n,\ n \in \mathrm{N}$ d) $10 \pm 2\pi n,\ n \in \mathrm{N}$

10. Example:



$-45° + 360° = 315°$, $-45° \pm (360°)n,\ n \in \mathrm{N}$

11. a) 425° b) 320°
   c) −400°, 320°, 680° d) $-\frac{5\pi}{4}$
   e) $-\frac{23\pi}{6}, \frac{\pi}{6}, \frac{13\pi}{6}$ f) $-\frac{5\pi}{3}, \frac{\pi}{3}$
   g) −3.9 h) −0.9, 5.4

12. a) 13.30 cm b) 4.80 m
   c) 15.88 cm d) 30.76 in.

13. a) 2.25 radians b) 10.98 ft
   c) 3.82 cm d) 17.10 m

14. a) $\frac{25\pi}{3}$ or 26.18 m
   b) $\frac{A_{\text{sector}}}{A_{\text{circle}}} = \frac{\text{sector angle}}{2\pi}$
   $A_{\text{sector}} = \frac{\pi r^2\left(\frac{5\pi}{3}\right)}{2\pi}$
   $A_{\text{sector}} = \frac{5\pi(5)^2}{6}$
   $A_{\text{sector}} = \frac{125\pi}{6}$
   The area watered is approximately 65.45 m^2.
   c) 16π radians or 2880°

15. a) Examples: $\frac{\pi}{12}$ radians/h, 1 revolution per day, 15°/h
   b) $\frac{100\pi}{3}$ or 104.72 radians/s
   c) 54 000°/min

16. a) 2.36 b) 135.3°

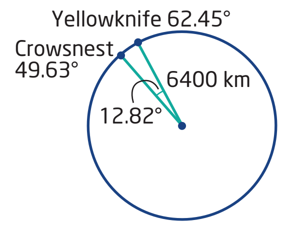
17.

	Revolutions	Degrees	Radians
a)	1 rev	360°	2π
b)	0.75 rev	270°	$\frac{3\pi}{2}$ or 4.7
c)	0.4 rev	150°	$\frac{5\pi}{6}$
d)	−0.3 rev	−97.4°	−1.7
e)	−0.1 rev	−40°	$-\frac{2\pi}{9}$ or −0.7
f)	0.7 rev	252°	$\frac{7\pi}{5}$ or 4.4
g)	−3.25 rev	−1170°	$-\frac{13\pi}{2}$ or −20.4
h)	$\frac{23}{18}$ or 1.3 rev	460°	$\frac{23\pi}{9}$ or 8.0
i)	$-\frac{3}{16}$ or −0.2 rev	−67.5°	$-\frac{3\pi}{8}$

18. Jasmine is correct. Joran's answer includes the solution when $k = 0$, which is the reference angle 78°.

19. a) 55.6 grad
   b) Use a proportion: $\frac{\text{gradians}}{\text{degrees}} = \frac{400\text{ grad}}{360°}$.
   So, measure in gradians $= \frac{10(\text{number of degrees})}{9}$.
   c) The gradian was developed to express a right angle as a metric measure. A right angle is equivalent to 100 grads.

20. a)



   b) 1432.01 km
   c) Example: Bowden (51.93° N, 114.03° W) and Airdrie (51.29° N, 114.01° W) are 71.49 km apart.

21. a) 2221.4 m/min b) 7404.7 radians/min

22. 8.5 km/h

23. 66 705.05 mph

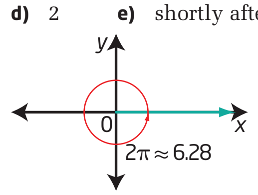
24. a) $69.375° = 69° + 0.375(60')$
   $= 69°\ 22.5'$
   $= 69°\ 22'\ 30''$
   b) i) 40° 52′ 30″ ii) 100° 7′ 33.6″
   iii) 14° 33′ 54″ iv) 80° 23′ 6″

25. a) $69°\ 22'\ 30'' = 69°22.5'$
   $= 69° + \left(\frac{22.5}{60}\right)^{\circ}$
   $= 69.375°$
   b) i) 45.508° ii) 72.263°
   iii) 105.671° iv) 28.167°

26. $A_{\text{segment}} = \frac{1}{2}r^2(\theta - \sin\theta)$

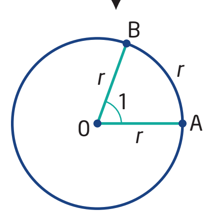
27. a) 120° b) 65° c) Examples: 3:00 and 9:00
   d) 2 e) shortly after 4:05

C1



π is 180° and 2π is 360°. 2(3.14) = 6.282 which is more than 6. Therefore, 6 radians must be less than 360°.

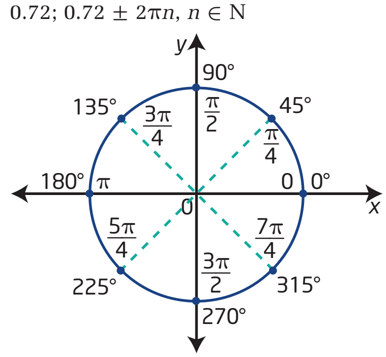
C2



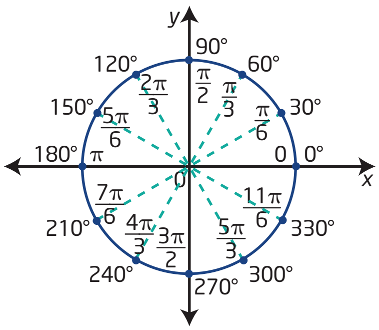
1° is a very small angle, it is $\frac{1}{360}$ of one rotation. One radian is much larger than 1°; 1 radian is the angle whose arc is the same as the radius, it is nearly $\frac{1}{6}$ of one rotation.

C3 a) 40°; $140° \pm (360°)n,\ n \in \mathrm{N}$
   b) 0.72; $0.72 \pm 2\pi n,\ n \in \mathrm{N}$

C4 a)



b)



C5 a) $x = 3$ **b)** $y = x - 3$

4.2 The Unit Circle, pages 186 to 190

1. a) $x^2 + y^2 = 16$ **b)** $x^2 + y^2 = 9$
c) $x^2 + y^2 = 144$ **d)** $x^2 + y^2 = 6.76$

2. a) No; $\left(-\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2 = \frac{5}{8} \neq 1$

b) No; $\left(\frac{\sqrt{5}}{8}\right)^2 + \left(\frac{7}{8}\right)^2 = \frac{27}{32} \neq 1$

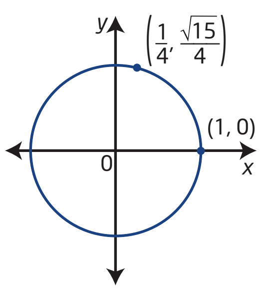
c) Yes; $\left(-\frac{5}{13}\right)^2 + \left(\frac{12}{13}\right)^2 = 1$

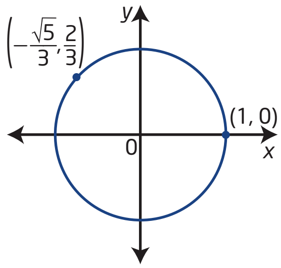
d) Yes; $\left(\frac{4}{5}\right)^2 + \left(-\frac{3}{5}\right)^2 = 1$

e) Yes; $\left(-\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{-1}{2}\right)^2 = 1$

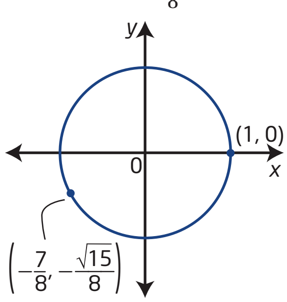
f) Yes; $\left(\frac{\sqrt{7}}{4}\right)^2 + \left(\frac{3}{4}\right)^2 = 1$

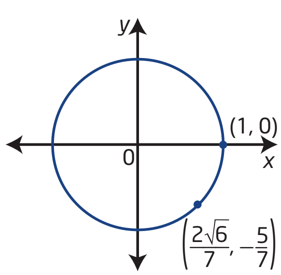
3. a) $y = \frac{\sqrt{15}}{4}$ **b)** $x = -\frac{\sqrt{5}}{3}$



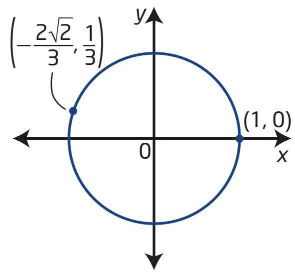


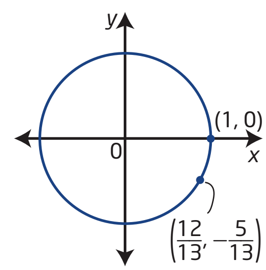
c) $y = -\frac{\sqrt{15}}{8}$ **d)** $x = \frac{2\sqrt{6}}{7}$





e) $x = -\frac{2\sqrt{2}}{3}$ **f)** $y = -\frac{5}{13}$



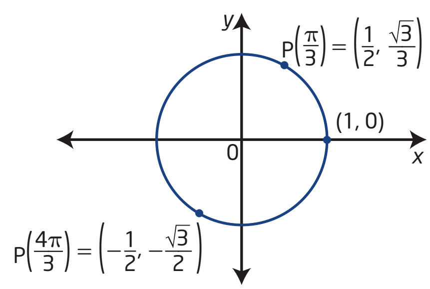


4. a) $(-1, 0)$ **b)** $(0, -1)$
c) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ **d)** $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
e) $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ **f)** $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
g) $(1, 0)$ **h)** $(0, 1)$
i) $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ **j)** $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

5. a) $\frac{3\pi}{2}$ **b)** 0 **c)** $\frac{\pi}{4}$ **d)** $\frac{3\pi}{4}$
e) $\frac{\pi}{3}$ **f)** $\frac{5\pi}{3}$ **g)** $\frac{5\pi}{6}$ **h)** $\frac{7\pi}{6}$
i) $\frac{5\pi}{4}$ **j)** π

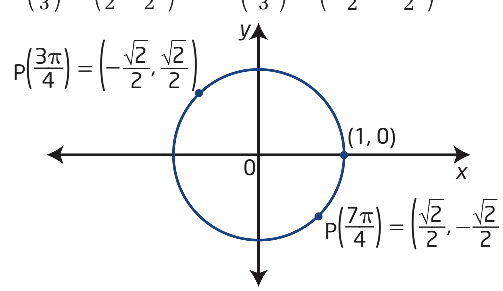
6. $\frac{5\pi}{6}$ and $-\frac{7\pi}{6}$

7. a)



If $\theta = \frac{\pi}{3}$ then $\theta + \pi = \frac{\pi}{3} + \pi$ or $\frac{4\pi}{3}$ since $P\left(\frac{\pi}{3}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and $P\left(\frac{4\pi}{3}\right) = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

b)



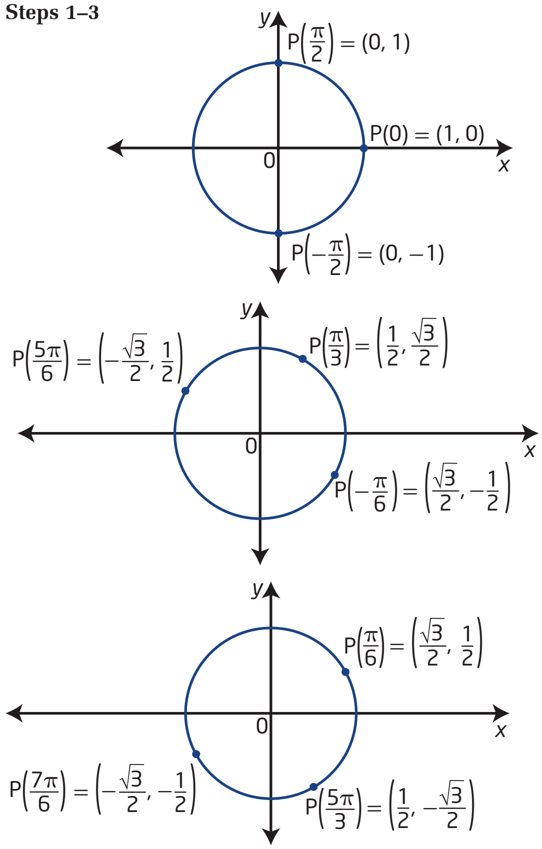
If $\theta = \frac{3\pi}{4}$ then $\theta + \pi = \frac{3\pi}{4} + \pi$ or $\frac{7\pi}{4}$ since $P\left(\frac{3\pi}{4}\right) = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ and $P\left(\frac{7\pi}{4}\right) = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

8.

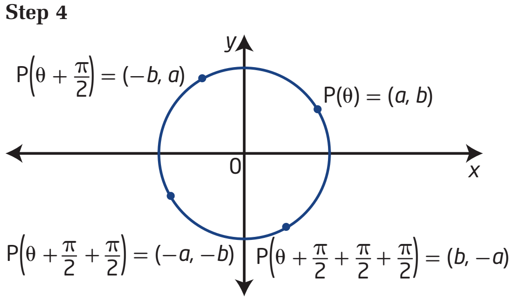
Point	$+\frac{1}{4}$ rotation	$-\frac{1}{4}$ rotation	Step 4: Description
$P(0) = (1, 0)$	$P\left(\frac{\pi}{2}\right) = (0, 1)$	$P\left(-\frac{\pi}{2}\right) = (0, -1)$	*x*- and *y*-values change places and take signs of new quadrant
$P\left(\frac{\pi}{3}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$	$P\left(\frac{\pi}{3} + \frac{\pi}{2}\right) = P\left(\frac{5\pi}{6}\right) = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$	$P\left(\frac{\pi}{3} - \frac{\pi}{2}\right) = P\left(-\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$	*x*- and *y*-values change places and take signs of new quadrant
$P\left(\frac{5\pi}{3}\right) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$	$P\left(\frac{5\pi}{3} + \frac{\pi}{2}\right) = P\left(\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$	$P\left(\frac{5\pi}{3} - \frac{\pi}{2}\right) = P\left(\frac{7\pi}{6}\right) = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$	*x*- and *y*-values change places and take signs of new quadrant

Diagrams:

Steps 1–3



Step 4



9. a) $x^2 + y^2 = 1$ **b)** $\left(\frac{\sqrt{5}}{3}, \frac{2}{3}\right)$

c) $\theta + \frac{\pi}{2}$ **d)** quadrant IV

e) maximum value is +1, minimum value is − 1

10. a) Yes. In quadrant I the values of $\cos\theta$ decrease from 1 at $\theta = 0°$ to 0 at $\theta = 90°$, since the *x*-coordinate on the unit circle represents $\cos\theta$, in the first quadrant the values of *x* will range from 1 to 0.

b) Substitute the values of *x* and *y* into the equation $x^2 + y^2 = 1$, Mya was not correct, the correct answer is $y = \sqrt{1 - (0.807)^2}$
$= \sqrt{0.348\ 751}$
$\approx 0.590\ 551$

c) $x = 0.9664$

11. b) All denominators are 2.

c) The numerators of the *x*-coordinates decrease from $\sqrt{3}$, $\sqrt{2}$, $\sqrt{1} = 1$, the numerators of the *y*-coordinates increase from $\sqrt{1}$, $\sqrt{2}$, $\sqrt{3}$. The *x*-coordinates are moving closer to the *y*-axis and therefore decrease in value, whereas the *y*-coordinates are moving further away from the *x*-axis and therefore increase in value.

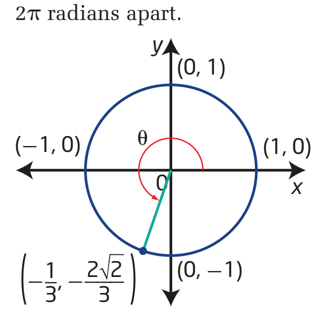
d) Since $x^2 + y^2 = 1$ then $x = \sqrt{1 - y^2}$ and $y = \sqrt{1 - x^2}$, all solutions involve taking square roots.

12. a) $-2\pi \le \theta < 4\pi$ represents three rotations around the unit circle and includes three coterminal angles for each point on the unit circle.

b) If $P(\theta) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, then $\theta = -\frac{4\pi}{3}$ when $-2\pi \le \theta \le 0$, $\theta = \frac{2\pi}{3}$ when $0 \le \theta \le 2\pi$, and $\theta = \frac{8\pi}{3}$ when $2\pi \le \theta < 4\pi$.

c) All these angles are coterminal since they are all 2π radians apart.

13. a)

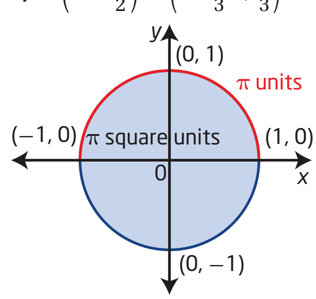


This point represents the terminal point of an angular rotation on the unit circle.

b) quadrant III **c)** $P\left(\theta + \frac{\pi}{2}\right) = \left(\frac{2\sqrt{2}}{3}, -\frac{1}{3}\right)$

d) $P\left(\theta - \frac{\pi}{2}\right) = \left(-\frac{2\sqrt{2}}{3}, \frac{1}{3}\right)$

14.



π units is the perimeter of half of a unit circle since $a = r\theta = (1)\pi = \pi$ units. π square units is the area of a unit circle since $A = \pi r^2 = \pi(1)^2$
$= \pi$ square units.

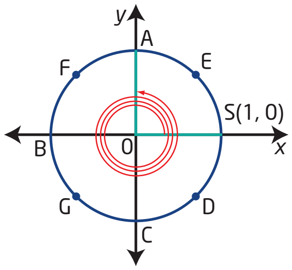
15. a) B($-a$, b), C($-a$, $-b$), D(a, $-b$)

b) **i)** $\theta + \pi$ = C($-a$, $-b$) **ii)** $\theta - \pi$ = C($-a$, $-b$)
iii) $-\theta + \pi$ = B($-a$, b) **iv)** $-\theta - \pi$ = B($-a$, b)

c) They do not differ.

16. a) $\theta = \frac{5\pi}{4}$; $a = r\theta = (1)\left(\frac{5\pi}{4}\right) = \frac{5\pi}{4}$

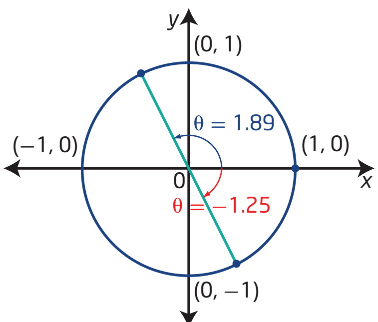
b) $P\left(\frac{13\pi}{2}\right)$ represents the ordered pair of the point where the terminal arm of the angle $\frac{13\pi}{2}$ intersects the unit circle. Since one rotation of the unit circle is 2π, then $\frac{13\pi}{2}$ represents three complete rotations with an extra $\frac{\pi}{2}$ or quarter rotation, therefore ending at point A.



c) Point C = $P\left(\frac{3\pi}{2}\right) \approx P(4.71)$ and point D = $P\left(\frac{7\pi}{4}\right) \approx P(5.50)$. Therefore P(5), lies between points C and D.

17. a) $\left(-\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right)$ and $\left(\frac{1}{\sqrt{10}}, -\frac{3}{\sqrt{10}}\right)$

b)

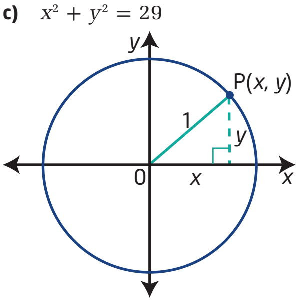


θ represents the angle in standard position.

18. a) $\left(\frac{5}{\sqrt{29}}, \frac{2}{\sqrt{29}}\right)$ **b)** $\sqrt{29}$

c) $x^2 + y^2 = 29$

19.

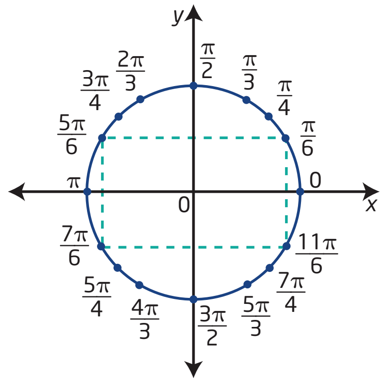


From the diagram: opposite side = y, adjacent side = x and hypotenuse = 1.
Since $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ then $\sin\theta = \frac{y}{1} = y$ or $y = \sin\theta$. Similarly, $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$, so $\cos\theta = \frac{x}{1} = x$ or $x = \cos\theta$. Therefore any point on the unit circle can be represented by the coordinates ($\cos\theta$, $\sin\theta$).

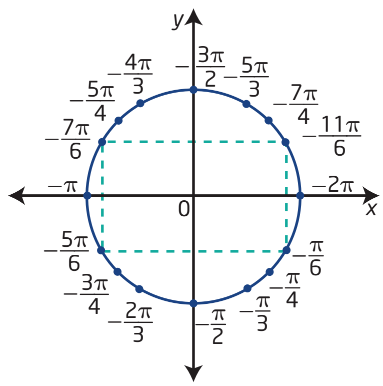
20. a) $\left(1, \frac{\pi}{4}\right)$ **b)** $\left(\frac{\sqrt{31}}{6}, 3.509\right)$

c) $\left(2\sqrt{2}, \frac{\pi}{4}\right)$ **d)** (5, 5.640)

C1 a)



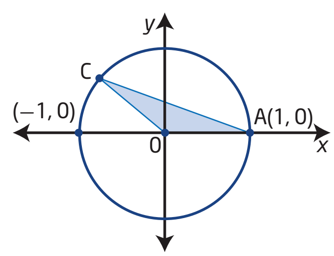
b)



c) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$, $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$, $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$, $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

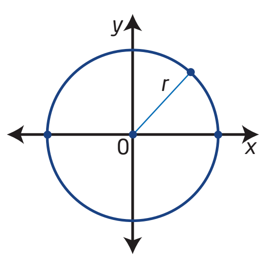
d) Example: The circumference is divided into eighths by successive quarter rotations, each eighth of the circumference measures $\frac{\pi}{4}$. The exact coordinates of the points can be determined using the special right triangles ($1:1:\sqrt{2}$ and $1:\sqrt{3}:2$) with signs adjusted according to the quadrant.

C2 a) $\frac{\pi}{5}$ **b)**



$\frac{3\pi}{20}$

C3 a)



$x^2 + y^2 = r^2$

b) Compare with a quadratic function. When $y = x^2$ is translated so its vertex moves from (0, 0) to (h, k), its equation becomes $y = (x - h)^2 + k$. So, a reasonable conjecture for the circle centre (0, 0) moving its centre to (h, k) is $(x - h)^2 + (y - k)^2 = r^2$. Test some key points on the circle centre (0, 0) such as $(r, 0)$. When the centre moves to (h, k) the test point moves to $(r + h, k)$. Substitute into the left side of the equation.
$(r + h - h)^2 + (k - k)^2 = r^2 + 0 =$ right side.

C4 a) 21.5% **b)** $\pi : 4$

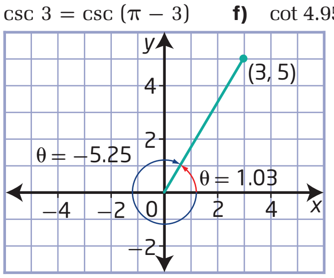
4.3 Trigonometric Ratios, pages 201 to 205

1. a) $\frac{\sqrt{2}}{2}$ **b)** $\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$ **c)** $-\frac{\sqrt{2}}{2}$
d) $\sqrt{3}$ **e)** -2 **f)** -2
g) undefined **h)** -1 **i)** $\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$
j) $\frac{\sqrt{3}}{2}$ **k)** $-\frac{\sqrt{3}}{2}$ **l)** $\sqrt{2}$

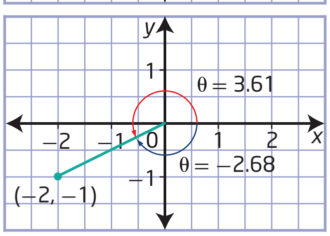
2. a) 0.68 **b)** −2.75 **c)** 1.04
d) −1.00 **e)** −0.96 **f)** 1.37
g) 0.78 **h)** 0.71 **i)** 0.53
j) −0.97 **k)** −3.44 **l)** undefined

3. a) I or IV **b)** II or IV **c)** III or IV
d) II **e)** II **f)** I

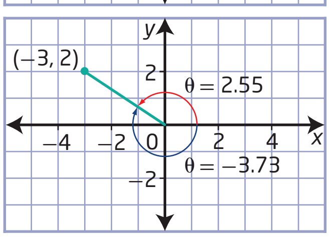
4. a) $\sin 250° = -\sin 70°$ **b)** $\tan 290° = -\tan 70°$
c) $\sec 135° = -\sec 45°$ **d)** $\cos 4 = -\cos (4 - \pi)$
e) $\csc 3 = \csc (\pi - 3)$ **f)** $\cot 4.95 = \cot (4.95 - \pi)$

5. a)


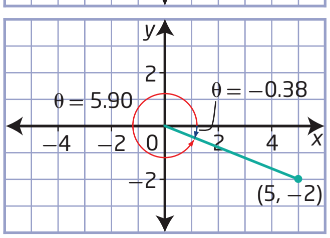
1.03, −5.25

b)


3.61, −2.68

c)


2.55, −3.73

d)


5.90, −0.38

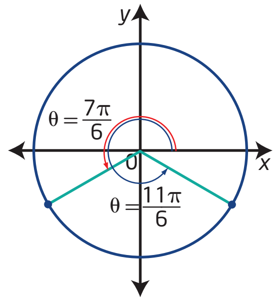
6. a) positive **b)** negative **c)** negative
d) positive **e)** positive **f)** positive

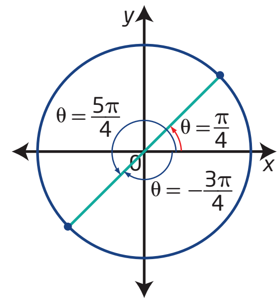
7. a) $\sin^{-1} 0.2014 = 0.2$; an angle of 0.2 radians has a sine ratio of 0.2014
b) $\tan^{-1} 1.429 = 7$; an angle of 7 radians has a tangent ratio of 1.429
c) sec 450° is undefined; an angle of 450° has a secant ratio that is undefined
d) cot (−180°) is undefined; an angle of −180° has a cotangent ratio that is undefined

8. a) $-\frac{4}{5}$ **b)** $-\frac{4}{3}$ **c)** −1.25

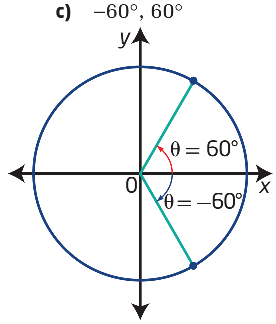
9. a) 1 **b)** 2 **c)** 1
d) −1 **e)** 1 **f)** 3

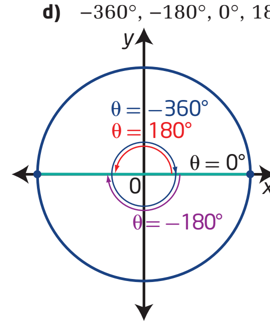
10. a) $\frac{7\pi}{6}, \frac{11\pi}{6}$ **b)** $-\frac{3\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4}$



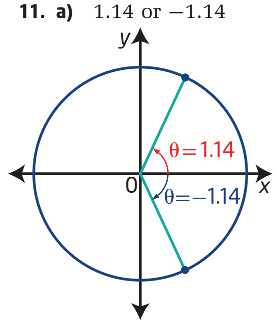


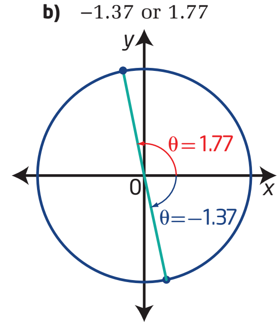
c) −60°, 60° **d)** −360°, −180°, 0°, 180°



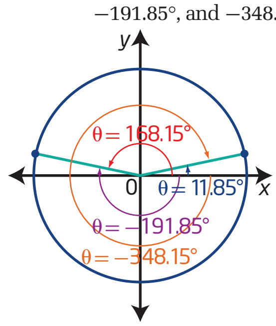


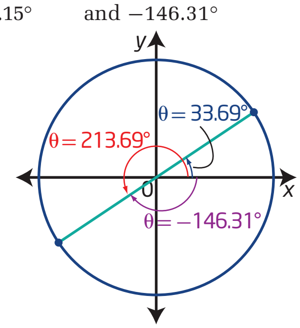
11. a) 1.14 or −1.14 **b)** −1.37 or 1.77





c) 11.85°, 168.15°, −191.85°, and −348.15° **d)** 33.69°, 213.69° and −146.31°





12. a) $\cos\theta = -\frac{4}{5}$, $\tan\theta = -\frac{3}{4}$, $\csc\theta = \frac{5}{3}$, $\sec\theta = -\frac{5}{4}$, $\cot\theta = -\frac{4}{3}$

b) $\sin\theta = \pm\frac{1}{3}$, $\tan\theta = \pm\frac{\sqrt{2}}{4}$, $\csc\theta = \pm 3$, $\sec\theta = -\frac{3\sqrt{2}}{4}$, $\cot\theta = \pm 2\sqrt{2}$

c) $\sin\theta = \pm\frac{2}{\sqrt{13}}$, $\cos\theta = \pm\frac{3}{\sqrt{13}}$, $\csc\theta = \pm\frac{\sqrt{13}}{2}$, $\sec\theta = \pm\frac{\sqrt{13}}{3}$, $\cot\theta = \frac{3}{2}$

d) $\sin\theta = \pm\frac{\sqrt{39}}{4\sqrt{3}}$ or $\pm\frac{\sqrt{13}}{4}$, $\cos\theta = \frac{3}{4\sqrt{3}}$ or $\frac{\sqrt{3}}{4}$, $\csc\theta = \pm\frac{4\sqrt{3}}{\sqrt{39}}$ or $\pm\frac{4\sqrt{13}}{13}$, $\tan\theta = \pm\frac{\sqrt{39}}{3}$, $\cot\theta = \pm\frac{3}{\sqrt{39}}$ or $\pm\frac{\sqrt{39}}{13}$

13. Sketch the point and angle in standard position. Draw the reference triangle. Find the missing value of the hypotenuse by using the equation $x^2 + y^2 = r^2$. Use $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ to find the exact value. Therefore, $\cos\theta = -\frac{2}{\sqrt{13}}$ or $-\frac{2\sqrt{13}}{13}$.

14. a) $\frac{4900°}{360°} = 13\frac{11}{18}$ revolutions counterclockwise

b) quadrant III **c)** 40°

d) $\sin 4900° = -0.643$, $\cos 4900° = -0.766$, $\tan 4900° = 0.839$, $\csc 4900° = -1.556$, $\sec 4900° = -1.305$, $\cot 4900° = 1.192$

15. a) 0.8; For an angle whose cosine is 0.6, think of a 3-4-5 right triangle, or in this case a 0.6-0.8-1 right triangle. The x-coordinate is the same as the cosine or 0.6, the sine is the y-coordinate which will be 0.8.

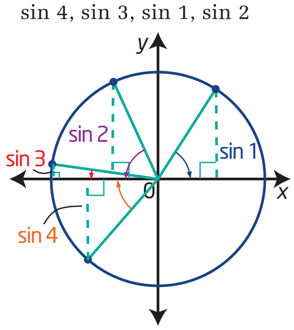
b) 0.8; Since $\cos^{-1} 0.6 = 90° - \sin^{-1} 0.6$ and $\sin^{-1} 0.6 = 90° - \cos^{-1} 0.6$, then $\cos(\sin^{-1} 0.6) = \sin(\cos^{-1} 0.6)$. Alternatively use similar reasoning as in part a) except the x- and y-coordinates are switched.

16. a) He is not correct. His calculator was in degree measure but the angle is expressed in radians.

b) Set calculator to radian mode and find the value of $\cos\left(\frac{40\pi}{7}\right)$. Since $\sec\theta = \frac{1}{\cos\theta}$, take the reciprocal of $\cos\left(\frac{40\pi}{7}\right)$ to get $\sec\left(\frac{40\pi}{7}\right) \approx 1.603\,875\,472$.

17. a) sin 4, sin 3, sin 1, sin 2

b)

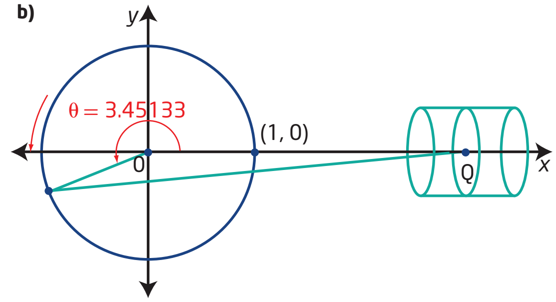


Sin 4 is in quadrant III and has a negative value, therefore it has the least value. Sin 3 is in quadrant II but has the smallest reference angle and is therefore the second smallest. Sin 1 has a smaller reference angle than sin 2.

c) cos 3, cos 4, cos 2, cos 1

18. a) 2 units

b)

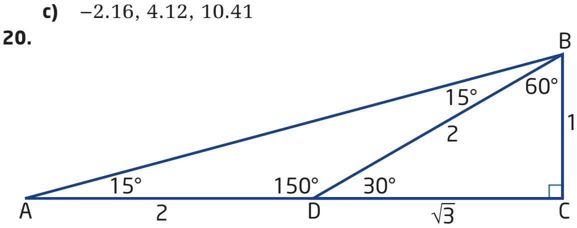


c) 0.46 units

19. a) 2.21, 8.50 **b)** −11.31°, 348.69°

c) −2.16, 4.12, 10.41

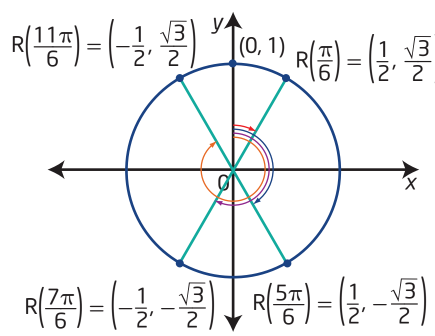
20.



△BCD is a 30°-60°-90° triangle, so DC = $\sqrt{3}$ units and BD = 2 units. △ABD has two equal angles of 15°, so AD = BD = 2. Then $\tan 15° = \frac{BC}{AC} = \frac{BC}{CD + DA} = \frac{1}{\sqrt{3} + 2}$.

21. Since $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{2.5}{5.0} = \frac{1}{2}$ then $\theta = 60°$. Since 60° is $\frac{2}{3}$ of 90° then the point is $\frac{1}{3}$ the distance on the arc from (0, 5) to (5, 0).

22. a)



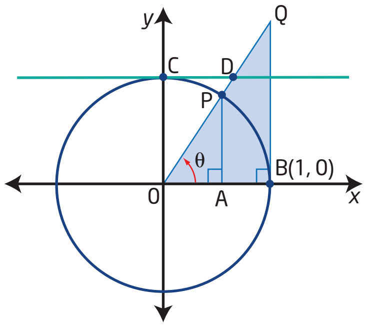
b) $R\left(\frac{\pi}{6}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and $R\left(\frac{5\pi}{6}\right) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

c) $R\left(\frac{\pi}{6}\right) = P\left(\frac{\pi}{3}\right)$, $R\left(\frac{5\pi}{6}\right) = P\left(\frac{5\pi}{3}\right)$, $R\left(\frac{7\pi}{6}\right) = P\left(\frac{4\pi}{3}\right)$, $R\left(\frac{11\pi}{6}\right) = P\left(\frac{2\pi}{3}\right)$, where R(θ) represents the new angle and P(θ) represents the conventional angle in standard position.

d) The new system is the same as bearings in navigation, except bearings are measured in degrees, not radians.

23. a) In △OBQ, $\cos\theta = \frac{OB}{OQ} = \frac{1}{OQ}$. So, $\sec\theta = \frac{1}{\cos\theta} = OQ$.

b)



In $\triangle OCD$, $\angle ODC = \theta$ (alternate angles). Then, $\sin \theta = \frac{OC}{OD} = \frac{1}{OD}$. So, $\csc \theta = \frac{1}{\sin \theta} = OD$.
Similarly, $\cot \theta = CD$.

C1 a) Paula is correct. Examples: $\sin 0° = 0$, $\sin 10° \approx 0.1736$, $\sin 25° \approx 0.4226$, $\sin 30° = 0.5$, $\sin 45° \approx 0.7071$, $\sin 60° \approx 0.8660$, $\sin 90° = 1$.

b) In quadrant II, sine decreases from $\sin 90° = 1$ to $\sin 180° = 0$. This happens because the y-value of points on the unit circle are decreasing toward the horizontal axis as the value of the angle moves from 90° to 180°.

c) Yes, the sine ratio increases in quadrant IV, from its minimum value of −1 at 270° up to 0 at 0°.

C2 When you draw its diagonals, the hexagon is composed of six equilateral triangles. On the diagram shown, each vertex will be 60° from the previous one. So, the coordinates, going in a positive direction from (1, 0) are $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, $(-1, 0)$, $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$, and $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.

C3 a) $\text{slope}_{OP} = \frac{\sin \theta}{\cos \theta}$ or $\tan \theta$

b) Yes, this formula applies in each quadrant. In quadrant II, $\sin \theta$ is negative, which makes the slope negative, as expected. Similar reasoning applies in the other quadrants.

c) $y = \left(\frac{\sin \theta}{\cos \theta}\right)x$ or $y = (\tan \theta)x$

d) Any line whose slope is defined can be translated vertically by adding the value of the y-intercept b. The equation will be $y = \left(\frac{\sin \theta}{\cos \theta}\right)x + b$ or $y = (\tan \theta)x + b$.

C4 a) $\frac{4}{5}$ **b)** $\frac{3}{5}$ **c)** $\frac{5}{4}$ **d)** $-\frac{4}{5}$

4.4 Introduction to Trigonometric Equations, pages 211 to 214

1. a) two solutions; $\sin \theta$ is positive in quadrants I and II

b) four solutions; $\cos \theta$ is positive in quadrants I and IV, giving two solutions for each of the two complete rotations

c) three solutions; $\tan \theta$ is negative in quadrants II and IV, and the angle rotates through these quadrants three times from −360° to 180°

d) two solutions; $\sec \theta$ is positive in quadrants I and IV and the angle is in each quadrant once from −180° to 180°

2. a) $\theta = \frac{\pi}{3} + 2\pi n, n \in I$ **b)** $\theta = \frac{5\pi}{3} + 2\pi n, n \in I$

3. a) $\theta = \frac{\pi}{6}, \frac{11\pi}{6}$ **b)** $\theta = 0°, 180°$

c) $\theta = -135°, -45°, 45°, 135°, 225°, 315°$

d) $\theta = -\frac{3\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$

4. a) $\theta = 1.35, 4.49$ **b)** $\theta = 1.76, 4.52$

c) $\theta = 1.14, 2.00$ **d)** $\theta = 0.08, 3.22$

e) 1.20 and 5.08 **f)** 3.83 and 5.59

5. a) $\theta = \pi$ **b)** $\theta = -\frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$

c) $x = -315°, -225°, 45°, 135°$

d) $x = -150°, -30°$

e) $x = -45°, 135°, 315°$

f) $\theta = -\frac{5\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{17\pi}{6}$

6. a) $\theta \in [-2\pi, 2\pi]$ **b)** $\theta \in \left[-\frac{\pi}{3}, \frac{7\pi}{3}\right]$

c) $\theta \in [0°, 270°]$ **d)** $0 \le \theta < \pi$

e) $0° < \theta < 450°$ **f)** $-2\pi < \theta \le 4\pi$

7. a) $\theta = 0, \frac{\pi}{3}, \frac{5\pi}{3}$

b) $\theta = 63.435°, 243.435°, 135°, 315°$

c) $\theta = 0, \frac{\pi}{2}, \pi$

d) $\theta = -180°, -70.529°, 70.529°$

8. Check for $\theta = 180°$.
Left Side $= 5(\cos 180°)^2 = 5(-1)^2 = 5$
Right Side $= -4 \cos 180° = -4(-1) = 4$
Since Left Side ≠ Right Side, $\theta = 180°$ is not a solution.
Check for $\theta = 270°$.
Left Side $= 5(\cos 270°)^2 = 5(0)^2 = 0$
Right Side $= -4 \cos 270° = -4(0) = 0$
Since Left Side = Right Side, $\theta = 270°$ is a solution.

9. a) They should not have divided both sides of the equation by $\sin \theta$. This will eliminate one of the possible solutions.

b)
$$2 \sin^2 \theta = \sin \theta$$
$$2 \sin^2 \theta - \sin \theta = 0$$
$$\sin \theta(2 \sin \theta - 1) = 0$$
$$\sin \theta = 0 \quad \text{and} \quad 2 \sin \theta - 1 = 0$$
$$\sin \theta = \frac{1}{2}$$
$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \pi$$

10. $\sin \theta = 0$ when $\theta = 0, \pi$, and 2π but none of these values are in the interval $(\pi, 2\pi)$.

11. $\sin \theta$ is only defined for the values $-1 \le \sin \theta \le 1$, and 2 is outside this range, so $\sin \theta = 2$ has no solution.

12. Yes, the general solutions are $\theta = \frac{\pi}{3} + 2\pi n, n \in I$ and $\theta = \frac{5\pi}{3} + 2\pi n, n \in I$. Since there are an infinite number of integers, there will be an infinite number of solutions coterminal with $\frac{\pi}{3}$ and $\frac{5\pi}{3}$.

13. a) Helene can check her work by substituting π for θ in the original equation.
Left Side $= 3(\sin \pi)^2 - 2 \sin \pi$
$= 3(0)^2 - 2(0)$
$= 0$
$=$ Right Side

b) $\theta = 0, 0.7297, 2.4119, \pi$

14. 25.56°

15. a) June **b)** December

c) Yes. Greatest sales of air conditioners be expected to happen before the hottest months (June) and the least sales before the coldest months (December).

16. The solution is correct as far as the statement "Sine is negative in quadrants II and III." Sine is actually negative in quadrants III and IV. Quadrant III solution is $180° + 41.8° = 221.8°$ and quadrant IV solution is $360° - 41.8° = 318.2°$.

17. Examples: Tan 90° has no solution since division by 0 is undefined. $\sin\theta = 2$ does not have a solution. The range of $y = \sin\theta$ is $-1 \le y \le 1$ and 2 is beyond this range.

18. $\sec\theta = -\frac{5}{3}$

19. a) 0 s, 3 s, 6 s, 9 s **b)** 1.5 s, $1.5 + 6n$, $n \in W$
c) 1.4 m below sea level

20. a) Substitute I = 0, then $0 = 4.3 \sin 120\pi t$
$0 = \sin 120\pi t$
$\sin\theta = 0$ at $\theta = 0, \pi, 2\pi, \ldots$
$0 = 120\pi t \rightarrow t = 0$
$\pi = 120\pi t \rightarrow t = \frac{1}{120}$
$2\pi = 120\pi t \rightarrow t = \frac{1}{60}$
Since the current must alternate from 0 to positive back to 0 and then negative back to 0, it will take $\frac{1}{60}$ s for one complete cycle or 60 cycles in one second.
b) $t = 0.004\ 167 + \frac{1}{60}n$, $n \in W$ seconds
c) $t = 0.0125 + \frac{1}{60}n$, $n \in W$ seconds
d) 4.3 amps

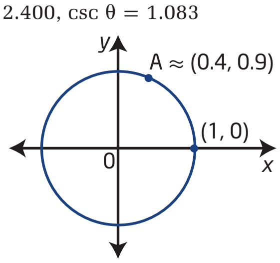
21. $x = \frac{\pi}{3}, \frac{2\pi}{3}$

22. a) No. **b)** $\sin\theta = \frac{-1 + \sqrt{5}}{2}$ and $\frac{-1 - \sqrt{5}}{2}$
c) 0.67, 2.48

23. a) The height of the trapezoid is $4 \sin\theta$ and its base is $4 + 2(4\cos\theta)$. Use the formula for the area of a trapezoid:
$A = \frac{\text{sum of parallel sides}}{2} \times \text{height}$
$A = \left(\frac{4 + 4 + 8\cos\theta}{2}\right)(4\sin\theta)$
$A = 8(1 + \cos\theta)(2\sin\theta)$
$A = 16\sin\theta(1 + \cos\theta)$
b) $\frac{\pi}{3}$
c) Example: Graph $y = 16\sin\theta(1 + \cos\theta)$ and find the maximum for domain in the first quadrant.

C1 The principles involved are the same up to the point where you need to solve for a trigonometric ratio.

C2 a) Check if $x^2 + y^2 = 1$. Yes, A is on the unit circle.
b) $\cos\theta = 0.385$, $\tan\theta = 2.400$, $\csc\theta = 1.083$
c) 67.4°; this angle measure seems reasonable as shown on the diagram.



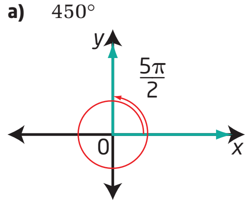
C3 a) Non-permissible values are values that the variable can never be because the expression is not defined in that case. For a rational expression, this occurs when the denominator is zero.
Example: $\frac{3}{x}$, $x \neq 0$
b) Example: $\tan\frac{\pi}{2}$ **c)** $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$
d) $\frac{\pi}{2} + \pi n$, $n \in I$

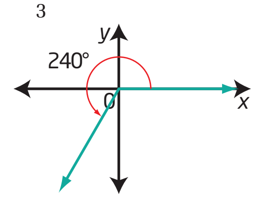
C4 a) 30°, 150°, 270°
b) Exact, because $\sin^{-1}(0.5)$ and $\sin^{-1}(-1)$ correspond to exact angle measures.
c) Example: Substitute $\theta = 30°$ in each side. Left side $= 2\sin^2 30° = 2(0.5)^2 = 0.5$. Right side $= 1 - \sin 30° = 1 - 0.5 = 0.5$. The value checks.

Chapter 4 Review, pages 215 to 217

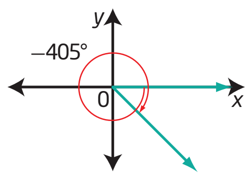
1. a) quadrant II **b)** quadrant II
c) quadrant III **d)** quadrant II

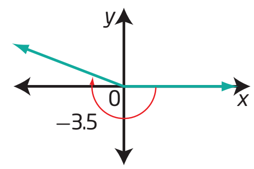
2. a) 450° **b)** $\frac{4\pi}{3}$





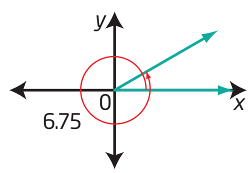
c) $-\frac{9\pi}{4}$ **d)** $-\frac{630°}{\pi}$

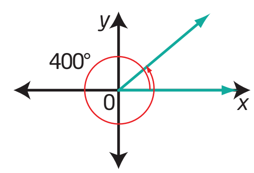




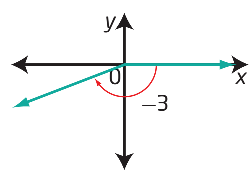
3. a) 0.35 **b)** −3.23 **c)** −100.27° **d)** 75°

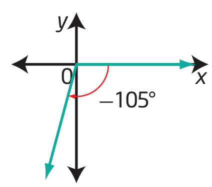
4. a) 0.467 **b)** 40°





c) 3.28 **d)** 255°





5. a) $250° \pm (360°)n$, $n \in N$ **b)** $\frac{5\pi}{2} \pm 2\pi n$, $n \in N$
c) $-300° \pm (360°)n$, $n \in N$ **d)** $6 \pm 2\pi n$, $n \in N$

6. a) 160 000π radians/minute **b)** 480 000°/s

7. a) $\left(\frac{-\sqrt{3}}{2}, \frac{1}{2}\right)$ **b)** $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ **c)** (0, 1)
d) $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ **e)** $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ **f)** $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

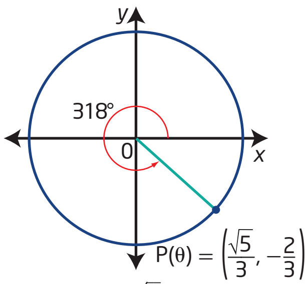
8. a) Reflect $P\left(\frac{\pi}{3}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ in the y-axis to give $P\left(\frac{2\pi}{3}\right) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$; then reflect this point in the x-axis to give $P\left(\frac{4\pi}{3}\right) = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$. Reflect about the original point in the x-axis to give $P\left(\frac{5\pi}{3}\right) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.
b) $\left(-\frac{1}{3}, -\frac{2\sqrt{2}}{3}\right)$

c) quadrant IV; $P\left(\frac{5\pi}{6}\right)$ lies in quadrant II and $P\left(\frac{5\pi}{6} + \pi\right)$ is a half circle away, so it lies in quadrant IV. $\theta = \frac{11\pi}{6}$ $P\left(\frac{11\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

9. a) $P\left(\frac{\pi}{2}\right)$ and $P\left(-\frac{3\pi}{2}\right)$ **b)** $P\left(\frac{11\pi}{6}\right)$ and $P\left(-\frac{\pi}{6}\right)$
c) $P\left(\frac{3\pi}{4}\right)$ and $P\left(-\frac{5\pi}{4}\right)$ **d)** $P\left(\frac{2\pi}{3}\right)$ and $P\left(-\frac{4\pi}{3}\right)$

10. a) P(−150°) and P(210°) **b)** P(180°)
c) P(135°) **d)** P(−60°) and P(300°)

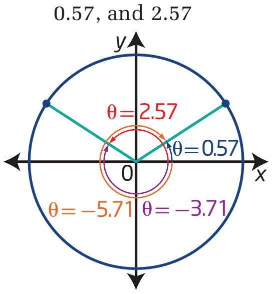
11. a) $\theta = 318°$ or 5.55 **b)** IV
c) $\left(-\frac{\sqrt{5}}{3}, \frac{2}{3}\right)$
d) $\left(\frac{2}{3}, \frac{\sqrt{5}}{3}\right)$
e) $\left(-\frac{2}{3}, -\frac{\sqrt{5}}{3}\right)$

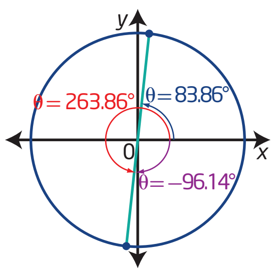


12. $\sin\theta = \frac{2\sqrt{2}}{3}$, $\tan\theta = 2\sqrt{2}$, $\sec\theta = 3$, $\csc\theta = \frac{3}{2\sqrt{2}}$ or $\frac{3\sqrt{2}}{4}$, $\cot\theta = \frac{1}{2\sqrt{2}}$ or $\frac{\sqrt{2}}{4}$

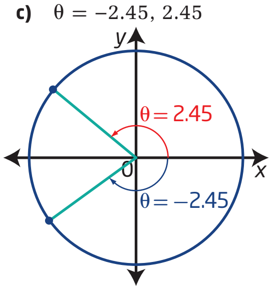
13. a) 1 **b)** $-\frac{\sqrt{2}}{2}$ **c)** $\sqrt{3}$
d) $-\frac{2\sqrt{3}}{3}$ **e)** 0 **f)** $-\frac{2\sqrt{3}}{3}$

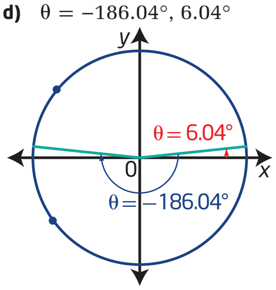
14. a) $\theta = -5.71, -3.71, 0.57,$ and 2.57 **b)** $\theta = -96.14°, 83.86°, 263.86°$





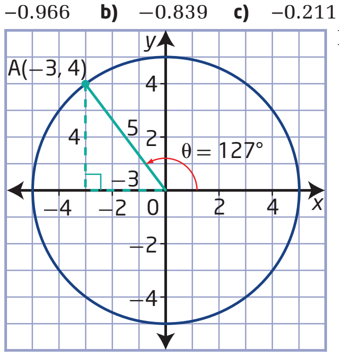
c) $\theta = -2.45, 2.45$ **d)** $\theta = -186.04°, 6.04°$





15. a) −0.966 **b)** −0.839 **c)** −0.211 **d)** 2.191

16. a) Example: 127°



b) $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = -\frac{3}{5}$ **c)** $-\frac{1}{12}$
d) 126.9° or 2.2

17. a) $\cos\theta(\cos\theta + 1)$ **b)** $(\sin\theta - 4)(\sin\theta + 1)$
c) $(\cot\theta + 3)(\cot\theta - 3)$ **d)** $(2\tan\theta - 5)(\tan\theta - 2)$

18. a) 2 is not a possible value for $\sin\theta$, $|\sin\theta| \leq 1$
b) $\tan 90° = \frac{\sin 90°}{\cos 90°} = \frac{1}{0}$, but division by 0 is undefined, so tan 90° has no solutions

19. a) 2 solutions **b)** 2 solutions
c) 1 solution **d)** 6 solutions

20. a) $\theta = 45°, 135°$ **b)** $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$
c) $\theta = -150°, 30°, 210°$ **d)** $\theta = -\frac{\pi}{4}, \frac{3\pi}{4}$

21. a) $\theta = \frac{\pi}{2}$
b) $\theta = 108.435°, 180°, 288.435°, 360°$
c) $\theta = 70.529°, 120°, 240°,$ and $289.471°$
d) $\theta = -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$

22. Examples:
a) $0 \leq \theta < 2\pi$ **b)** $-2\pi \leq \theta < \frac{\pi}{2}$
c) $-720° \leq \theta < 0°$ **d)** $-270° \leq \theta < 450°$

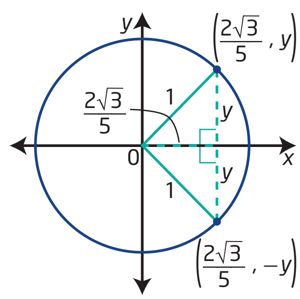
23. a) $x = \frac{7\pi}{6} + 2\pi n, n \in I$ and $x = \frac{11\pi}{6} + 2\pi n, n \in I$
b) $x = 90° + (360°)n, n \in I$ and $x = (180°)n, n \in I$
c) $x = 120° + (360°)n, n \in I$ and $x = 240° + (360°)n, n \in I$
d) $x = \frac{\pi}{4} + \pi n, n \in I$ and $x = \frac{\pi}{3} + \pi n, n \in I$

Chapter 4 Practice Test, pages 218 to 219

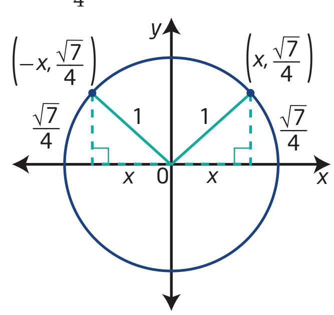
1. D **2.** C **3.** A **4.** B **5.** B

6. a) 4668.5° or 81.5
b) 92.6 Yes; a smaller tire requires more rotations to travel the same distance so it will experience greater tire wear.

7. a) $x^2 + y^2 = 1$
b) **i)** $y = \pm\frac{\sqrt{13}}{5}$



ii) $x = -\frac{3}{4}$



c) In the expression $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$, substitute the y-value for the opposite side and 1 for the hypotenuse. Since $x^2 + y^2 = 1$ then $\cos^2 \theta + \sin^2 \theta = 1$. Substitute the value you determined for $\sin \theta$ into $\cos^2 \theta + \sin^2 \theta = 1$ and solve for $\cos \theta$.

8. a) Cosine is negative in quadrants II and III. Find the reference angle by subtracting π from the given angle in quadrant III. To find the solution in quadrant II, subtract the reference angle from π.

b) Given each solution θ, add $2\pi n$, $n \in I$ to obtain each general solution $\theta + 2\pi n$, $n \in I$.

9. $\theta = \frac{3\pi}{4} + 2\pi n, n \in I$ or $\theta = \frac{5\pi}{4} + 2\pi n, n \in I$

10. Since $1° = \frac{\pi}{180}$, then $3° = \frac{3\pi}{180}$ or $\frac{\pi}{60}$.

$3 = \frac{3(180°)}{\pi} \approx 172°$.

11. a) quadrant III **b)** 40°

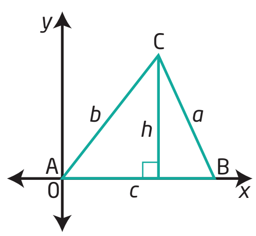
c) $\sin(-500°) = -0.6$, $\cos(-500°) = -0.8$, $\tan(-500°) = 0.8$, $\csc(-500°) = -1.6$, $\sec(-500°) = -1.3$, $\cot(-500°) = 1.2$

12. a) $\frac{5\pi}{4}, -\frac{3\pi}{4}; \frac{5\pi}{4} \pm 2\pi n, n \in N$

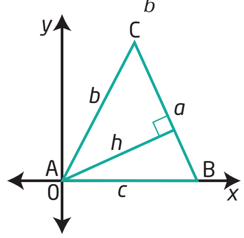
b) 145°, –215°, 145° ± (360°)n, $n \in N$

13. 7.7 km

14.



Given $A = \frac{1}{2}bh$, b = side c, since $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$ then $\sin A = \frac{h}{b}$ or $h = b \sin A$ and $A = \frac{1}{2}bc \sin A$ or



Given $A = \frac{1}{2}bh$, b = side a, since $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$ then $\sin B = \frac{h}{c}$ or $h = c \sin B$, therefore $A = \frac{1}{2}ac \sin B$.

15. a) $\theta = -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}, -2.21, 0.93, 4.07$

b) 0.67, 2.48 **c)** 0, π, 2π, 4.47, 1.33

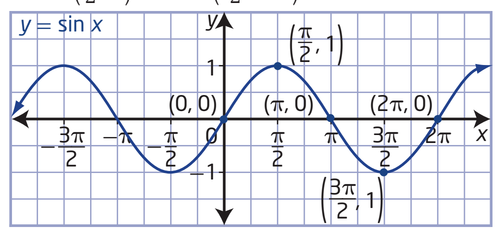
16. $\frac{28\pi}{3}$ m or 29.32 m

Chapter 5 Trigonometric Functions and Graphs

5.1 Graphing Sine and Cosine Functions, pages 233 to 237

1. a) $(0, 0), \left(\frac{\pi}{2}, 1\right), (\pi, 0), \left(\frac{3\pi}{2}, -1\right), (2\pi, 0)$

b)



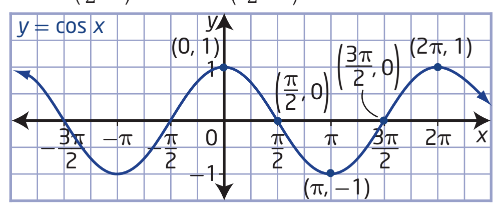
c) x-intercepts: $-2\pi, -\pi, 0, \pi, 2\pi$

d) y-intercept: 0

e) The maximum value is 1, and the minimum value is -1.

2. a) $(0, 1), \left(\frac{\pi}{2}, 0\right), (\pi, -1), \left(\frac{3\pi}{2}, 0\right), (2\pi, 1)$

b)



c) x-intercepts: $-\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$

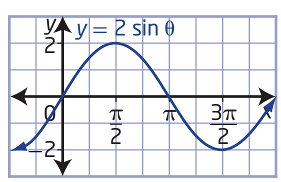
d) y-intercept: 1

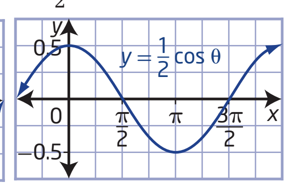
e) The maximum value is 1, and the minimum value is -1.

3.

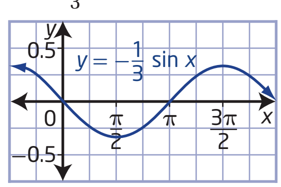
Property	$y = \sin x$	$y = \cos x$
maximum	1	1
minimum	–1	– 1
amplitude	1	1
period	2π	2π
domain	$\{x \mid x \in R\}$	$\{x \mid x \in R\}$
range	$\{y \mid -1 \le y \le 1, y \in R\}$	$\{y \mid -1 \le y \le 1, y \in R\}$
y-intercept	0	1
x-intercepts	$\pi n, n \in I$	$\frac{\pi}{2} + \pi n, n \in I$

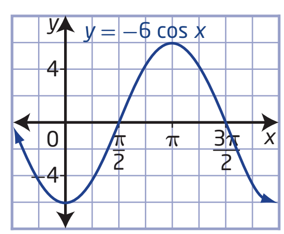
4. a) 2 **b)** $\frac{1}{2}$



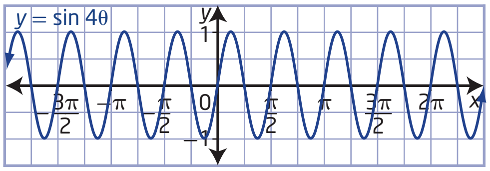


c) $\frac{1}{3}$ **d)** 6

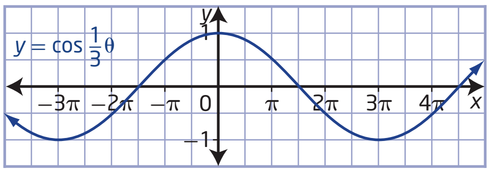




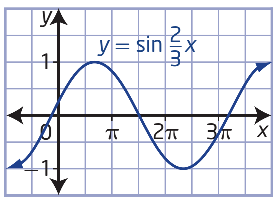
5. a) $\frac{\pi}{2}$ or 90°



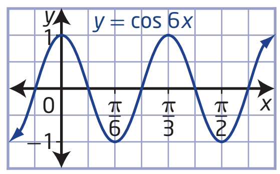
b) 6π or 1080°



c) 3π or 540°



d) $\frac{\pi}{3}$ or 60°



6. a) A **b)** D **c)** C **d)** B

7. a) Amplitude is 3; stretched vertically by a factor of 3 about the *x*-axis.

b) Amplitude is 5; stretched vertically by a factor of 5 about the *x*-axis and reflected in the *x*-axis.

c) Amplitude is 0.15; stretched vertically by a factor of 0.15 about the *x*-axis.

d) Amplitude is $\frac{2}{3}$; stretched vertically by a factor of $\frac{2}{3}$ about the *x*-axis and reflected in the *x*-axis.

8. a) Period is 180°; stretched horizontally by a factor of $\frac{1}{2}$ about the *y*-axis.

b) Period is 120°; stretched horizontally by a factor of $\frac{1}{3}$ about the *y*-axis and reflected in the *y*-axis.

c) Period is 1440°; stretched horizontally by a factor of 4 about the *y*-axis.

d) Period is 540°; stretched horizontally by a factor of $\frac{3}{2}$ about the *y*-axis.

9. a) Amplitude is 2; period is 360° or 2π.

b) Amplitude is 4; period is 180° or π.

c) Amplitude is $\frac{5}{3}$; period is 540° or 3π.

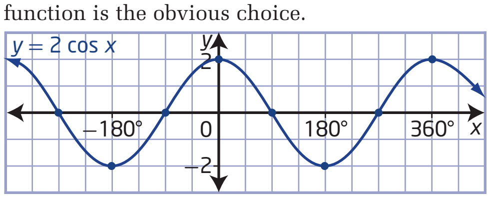
d) Amplitude is 3; period is 720° or 4π.

10. a) Graph A: Amplitude is 2 and period is 4π. Graph B: Amplitude is 0.5 and period is π.

b) Graph A: $y = 2 \sin \frac{1}{2}x$; Graph B: $y = 0.5 \cos 2x$

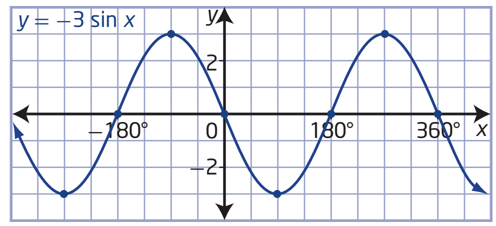
c) Graph A starts at 0, so the sine function is the obvious choice. Graph B starts at 1, so the cosine function is the obvious choice.

11. a)



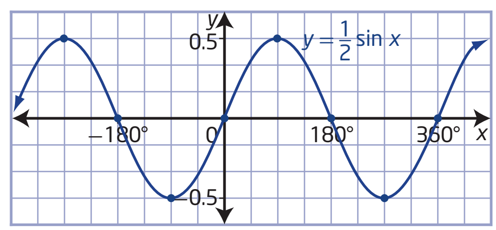
Property	Points on the Graph of $y = 2 \cos x$
maximum	(−360°, 2), (0°, 2), (360°, 2)
minimum	(−180°, −2), (180°, −2)
x-intercepts	(−270°, 0), (−90°, 0), (90°, 0), (270°, 0)
y-intercept	(0, 2)
period	360°
range	$\{y \mid -2 \le y \le 2, y \in R\}$

b)



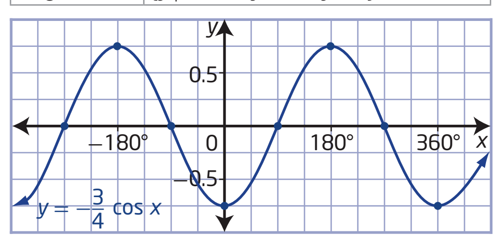
Property	Points on the Graph of $y = -3 \sin x$
maximum	(−90°, 3), (270°, 3)
minimum	(−270°, −3), (90°, −3)
x-intercepts	(−360°, 0), (−180°, 0), (0°, 0), (180°, 0), (360°, 0)
y-intercept	(0, 0)
period	360°
range	$\{y \mid -3 \le y \le 3, y \in R\}$

c)



Property	Points on the Graph of $y = \frac{1}{2} \sin x$
maximum	(−270°, 0.5), (90°, 0.5)
minimum	(−90°, −0.5), (270°, −0.5)
x-intercepts	(−360°, 0), (−180°, 0), (0°, 0), (180°, 0), (360°, 0)
y-intercept	(0, 0)
period	360°
range	$\{y \mid -0.5 \le y \le 0.5, y \in R\}$

d)



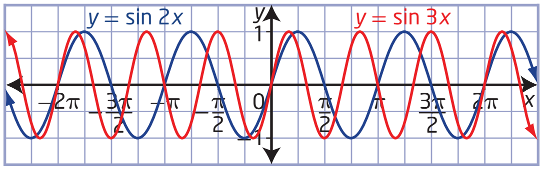
Property	Points on the Graph of $y = -\frac{3}{4} \cos x$
maximum	(−180°, 0.75), (180°, 0.75)
minimum	(−360°, −0.75), (0°, −0.75), (360°, −0.75)
x-intercepts	(−270°, 0), (−90°, 0), (90°, 0), (270°, 0)
y-intercept	(0, −0.75)
period	360°
range	$\{y \mid -0.75 \le y \le 0.75, y \in R\}$

12. a) $B\left(\frac{\pi}{4}, 3\right)$, $C\left(\frac{\pi}{2}, 0\right)$, $D\left(\frac{3\pi}{4}, -3\right)$, $E(\pi, 0)$

b) $C\left(\frac{\pi}{2}, 0\right)$, $D(\pi, -2)$, $E\left(\frac{3\pi}{2}, 0\right)$, $F(2\pi, 2)$

c) $B(-3\pi, 1)$, $C(-2\pi, 0)$, $D(-\pi, -1)$, $E(0, 0)$

13.



The amplitude, maximum, minimum, y-intercepts, domain, and range are the same for both graphs. The period and x-intercepts are different.

14. a) Amplitude is 5; period is $\frac{4\pi}{3}$.

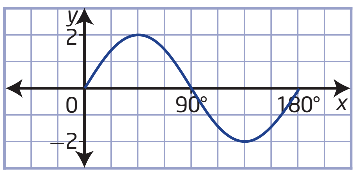
b) Amplitude is 4; Period is $\frac{2\pi}{3}$.

15. a) Amplitude is 20 mm Hg; Period is 0.8 s.

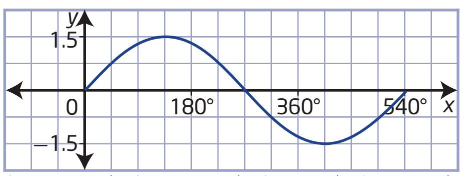
b) 75 bpm

16. Answers may vary.

17. a)



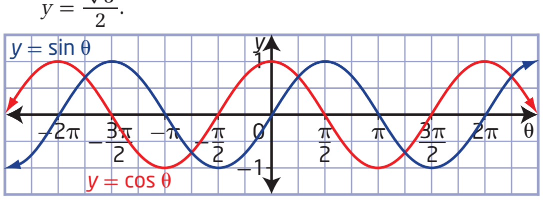
b)



18. a) $\left(-\frac{7\pi}{4}, \frac{\sqrt{2}}{2}\right)$, $\left(-\frac{5\pi}{4}, \frac{\sqrt{2}}{2}\right)$, $\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right)$, $\left(\frac{9\pi}{4}, \frac{\sqrt{2}}{2}\right)$;
Find the points of intersection of $y = \sin \theta$ and $y = \frac{\sqrt{2}}{2}$.

b) $\left(-\frac{11\pi}{6}, \frac{\sqrt{3}}{2}\right)$, $\left(-\frac{\pi}{6}, \frac{\sqrt{3}}{2}\right)$, $\left(\frac{11\pi}{6}, \frac{\sqrt{3}}{2}\right)$, $\left(\frac{13\pi}{6}, \frac{\sqrt{3}}{2}\right)$;
Find the points of intersection of $y = \cos \theta$ and $y = \frac{\sqrt{3}}{2}$.

19.



a) The graphs have the same maximum and minimum values, the same period, and the same domain and range.

b) The graphs have different x- and y-intercepts.

c) A horizontal translation could make them the same graph.

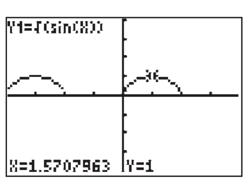
20. 12

21. a) $\frac{2\pi}{3}$ **b)** 12

22. 0.9

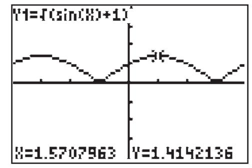
23. a) Example: The graph of $y = \sqrt{\sin x}$ will contain the portions of the graph of $y = \sin x$ that lie on or above the x-axis.

b)



c) Example: The function $y = \sqrt{\sin x + 1}$ is defined for all values of x, while the function $y = \sqrt{\sin x}$ is not.

d)



24. It is sinusoidal and the period is 2π.

C1 Step 5

a) The x-coordinate of each point on the unit circle represents $\cos \theta$. The y-coordinate of each point on the unit circle represents the $\sin \theta$.

b) The y-coordinates of the points on the sine graph are the same as the y-coordinates of the points on the unit circle. The y-coordinates of the points on the cosine graph are the same as the x-coordinates of the points on the unit circle.

C2 The constant is 1. The sum of the squares of the legs of each right triangle is equal to the radius of the unit circle, which is always 1.

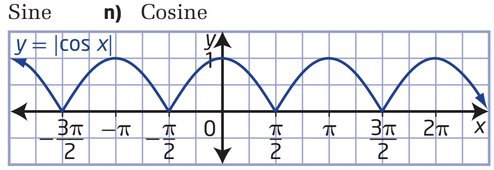
C3 a) Cannot determine because the amplitude is not given.

b) $f(4) = 0$; given in the question.

c) $f(84) = 0$; the period is 40° so it returns to 0 every 40°.

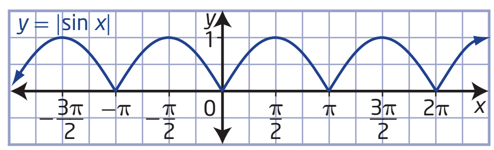
C4 a) Sine and Cosine **b)** Sine and Cosine
c) Sine and Cosine **d)** Sine and Cosine
e) Sine **f)** Cosine **g)** Cosine **h)** Sine
i) Cosine **j)** Sine **k)** Cosine **l)** Sine
m) Sine **n)** Cosine

C5 a)



The parts of the graph below the x-axis have been reflected across the x-axis.

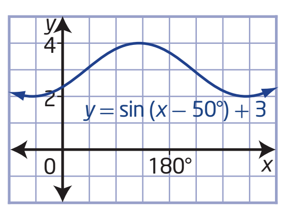
b)



The parts of the graph below the x-axis have been reflected across the x-axis.

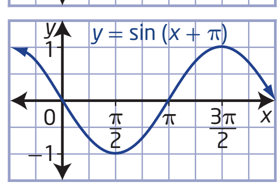
5.2 Transformations of Sinusoidal Functions, pages 250 to 255

1. a)



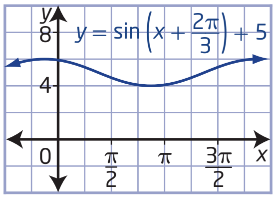
The phase shift is 50° right. The vertical displacement is 3 units up.

b)



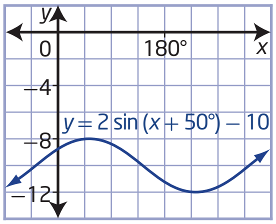
The phase shift is π units left. There is no vertical displacement.

c)



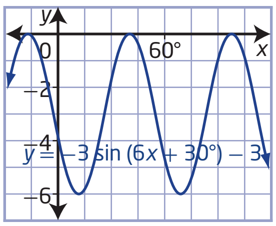
The phase shift is $\frac{2\pi}{3}$ units left. The vertical displacement is 5 units up.

d)



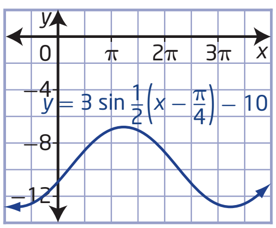
The phase shift is 50° left. The vertical displacement is 10 units down.

e)



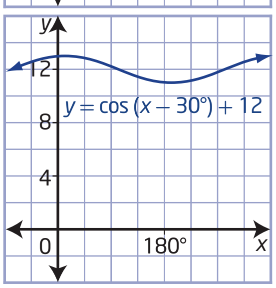
The phase shift is 5° left. The vertical displacement is 3 units down.

f)



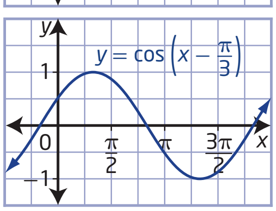
The phase shift is $\frac{\pi}{4}$ units right. The vertical displacement is 10 units down.

2. a)



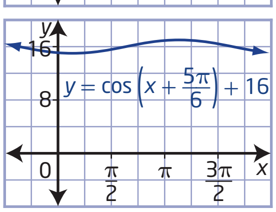
The phase shift is 30° right. The vertical displacement is 12 units up.

b)



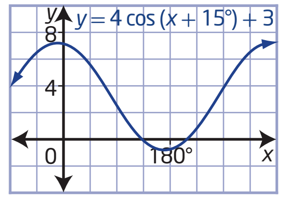
The phase shift is $\frac{\pi}{3}$ units right. There is no vertical displacement.

c)



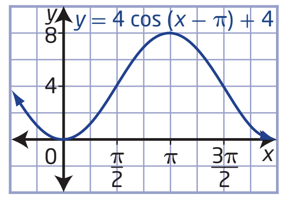
The phase shift is $\frac{5\pi}{6}$ units left. The vertical displacement is 16 units up.

d)



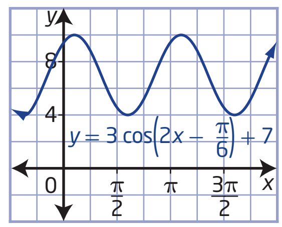
The phase shift is 15° left. The vertical displacement is 3 units up.

e)



The phase shift π units right. The vertical displacement is 4 units up.

f)



The phase shift is $\frac{\pi}{12}$ units right. The vertical displacement is 7 units up.

3. a) **i)** $\{y \mid 2 \le y \le 8, y \in \mathrm{R}\}$
ii) $\{y \mid -5 \le y \le -1, y \in \mathrm{R}\}$
iii) $\{y \mid 2.5 \le y \le 5.5, y \in \mathrm{R}\}$
iv) $\left\{y \mid \frac{1}{12} \le y \le \frac{17}{12}, y \in \mathrm{R}\right\}$

b) Take the vertical displacement and add and subtract the amplitude to it. The region in between these points is the range.

4. a) D **b)** C **c)** B **d)** A **e)** E

5. a) D **b)** B **c)** C **d)** A

6. a) $y = 4\sin 2\left(x - \frac{\pi}{2}\right) - 6$

b) $y = 0.5\sin\frac{1}{2}\left(x + \frac{\pi}{6}\right) + 1$

c) $y = \frac{3}{4}\sin\frac{1}{2}x - 5$

7. a) $a = 3, b = \frac{1}{2}, c = -2, d = 3; y = 3\cos\frac{1}{2}(x + 2) + 3$

b) $a = \frac{1}{2}, b = 4, c = 3, d = -5;$
$y = \frac{1}{2}\cos 4(x - 3) - 5$

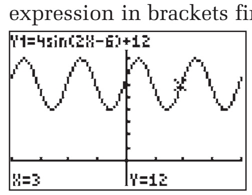
c) $a = -\frac{3}{2}, b = \frac{1}{3}, c = \frac{\pi}{4}, d = -1;$
$y = -\frac{3}{2}\cos\frac{1}{3}\left(x - \frac{\pi}{4}\right) - 1$

8. red, orange, yellow, green, blue, indigo, violet

9. *b*

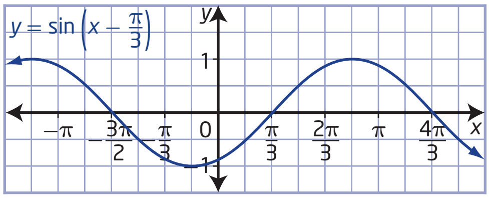
10. a) Stewart is correct. He remembered to factor the expression in brackets first.

b)

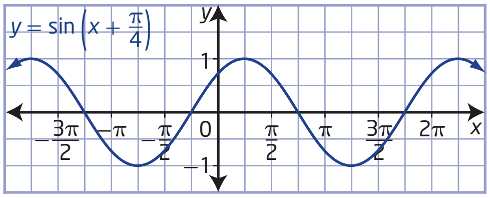


11. a) $\{y \mid -1 \le y \le 5, y \in \mathrm{R}\}$ **b)** $\{y \mid -6 \le y \le 0, y \in \mathrm{R}\}$
c) $\{y \mid -13 \le y \le -7, y \in \mathrm{R}\}$
d) $\{y \mid 5 \le y \le 11, y \in \mathrm{R}\}$

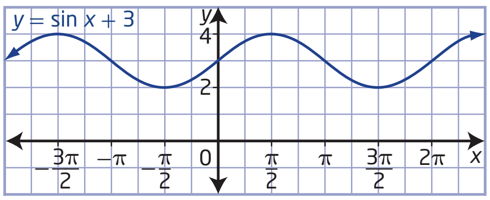
12. a)



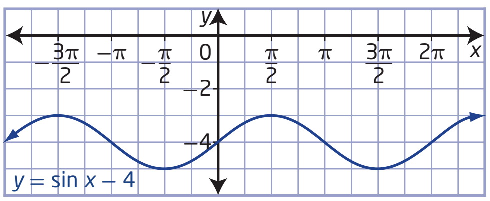
b)



c)



d)



13. $a = 9$, $d = -4$

14. a) **i)** 3 **ii)** 2π
iii) $\frac{\pi}{4}$ units right **iv)** none
v) domain $\{x \mid x \in R\}$, range $\{y \mid -3 \le y \le 3, y \in R\}$
vi) The maximum value of 3 occurs at $x = \frac{3\pi}{4}$.
vii) The minimum value of −3 occurs at $x = \frac{7\pi}{4}$.

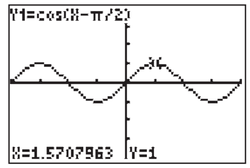
b) **i)** 2 **ii)** 2π
iii) $\frac{\pi}{2}$ units right **iv)** 2 units down
v) domain $\{x \mid x \in R\}$, range $\{y \mid -4 \le y \le 0, y \in R\}$
vi) The maximum value of 0 occurs at $x = \frac{\pi}{2}$.
vii) The minimum value of −4 occurs at $x = \frac{3\pi}{2}$.

c) **i)** 2 **ii)** π
iii) $\frac{\pi}{4}$ units right **iv)** 1 unit up
v) domain $\{x \mid x \in R\}$, range $\{y \mid -1 \le y \le 3, y \in R\}$
vi) The maximum value of 3 occurs at $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$.
vii) The minimum value of −1 occurs at $x = 0$, $x = \pi$, and $x = 2\pi$.

15. a) $y = 2 \sin x - 1$ **b)** $y = 3 \sin 2x + 1$
c) $y = 2 \sin 4\left(x - \frac{\pi}{4}\right) + 2$

16. a) $y = 2 \cos 2\left(x - \frac{\pi}{4}\right) + 1$
b) $y = 2 \cos\left(x + \frac{\pi}{2}\right) - 1$ **c)** $y = \cos(x - \pi) + 1$

17. a)



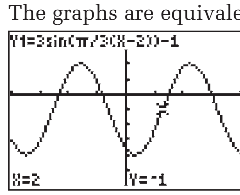
b) $y = \sin x$
c) The graph of the cosine function shifted $\frac{\pi}{2}$ units right is equivalent to the graph of the sine function.

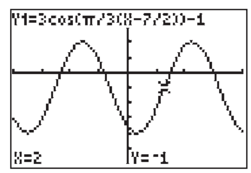
18. phase shift of $\frac{\pi}{2}$ units left

19. a) **i)** Phase shift is 30° right; period is 360°; x-intercepts are at 120° and 300°.
ii) Maximums occur at (30°, 3) and (390°, 3); minimum occurs at (210°, −3).
b) **i)** Phase shift is $\frac{\pi}{4}$ units right; period is π; x-intercepts are at $\frac{\pi}{2}$ and π
ii) Maximums occur at $\left(\frac{\pi}{4}, 3\right)$ and $\left(\frac{5\pi}{4}, 3\right)$; minimum occurs at $\left(\frac{3\pi}{4}, -3\right)$.

20. $y = 50 \cos \frac{\pi}{2640}(x - 9240) + 5050$

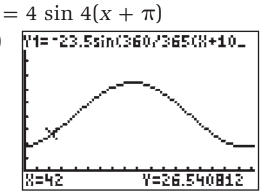
21. The graphs are equivalent.





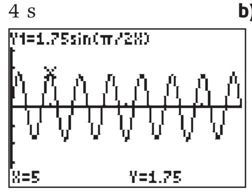
22. $y = 4 \sin 4(x + \pi)$

23. a)



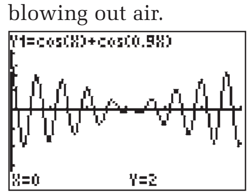
b) approximately 26.5°
c) day 171 or June 21

24. a) 4 s **b)** 15 cycles per minute
c)



d) The air flow velocity is 0 L/s. This corresponds to when the lungs are either completely full or completely empty.
e) The air flow velocity is −1.237 L/s. This corresponds to part of a cycle when the lungs are blowing out air.

25. a)



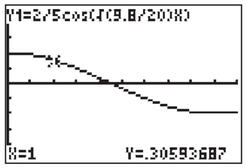
b) The amplitude is 2. The period is 20π.

26. a) **i)** 120° **ii)** $\frac{3\pi}{4}$ **iii)** π **iv)** $\frac{\pi}{4}$
b) Example: When graphed, a cosine function is ahead of the graph of a sine function by 90°. So, adding 90° to the phase shift in part a) works.

27. a) $y = 3 \sin (x + \pi) + 2$ **b)** $y = 3 \sin 2\left(x - \frac{\pi}{2}\right) + 1$

c) $y = 2 \sin \left(x + \frac{\pi}{2}\right) + 5$ **d)** $y = 5 \sin 3(x - 120°) - 1$

28. a) $P = \frac{2}{5} \cos \sqrt{\frac{9.8}{20}}t$



b) approximately −0.20 radians or 3.9 cm along the arc to the left of the vertical

C1 a changes the amplitude, b changes the period, c changes the phase shift, d changes the vertical translation; Answers may vary.

C2 a) They are exactly same.

b) This is because the sine of a negative number is the same as the negative sine of the number.

c) They are mirror images reflected in the x-axis.

d) It is correct.

C3 $\frac{5\pi}{4}$ square units

C4 a) $0 < b < 1$ **b)** $a > 1$

c) Example: $c = 0$, $d = 0$ **d)** $d > a$

e) Example: $c = -\frac{\pi}{2}$, $b = 1$, $d = 0$ **f)** $b = 3$

5.3 The Tangent Function, pages 262 to 265

1. a) 1, 45° **b)** −1.7, 120.5°

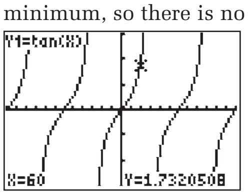
c) −1.7, 300.5° **d)** 1, 225°

2. a) undefined **b)** −1 **c)** 1

d) 0 **e)** 0 **f)** 1

3. No. The tangent function has no maximum or minimum, so there is no amplitude.

4.



−300°, −120°, 240°

5. $\frac{\tan \theta}{\sin \theta} = \frac{1}{\cos \theta}$; $\tan \theta = \frac{\sin \theta}{\cos \theta}$

6. a) slope $= \frac{y}{x}$

b) Since y is equal to $\sin \theta$ and x is equal to $\cos \theta$, then $\tan \theta = \frac{y}{x}$.

c) slope $= \frac{\sin \theta}{\cos \theta}$ **d)** $\tan \theta = \frac{y}{x}$

7. a) $\tan \theta = \frac{y}{x}$ **b)** $\tan \theta = \frac{\sin \theta}{\cos \theta}$

c) $\sin \theta$ and $\cos \theta$ are equal to y and x, respectively.

8. a)

θ	tan θ
89.5°	114.59
89.9°	572.96
89.999°	57 295.78
89.999 999°	57 295 779.51

b) The value of tan θ increases to infinity.

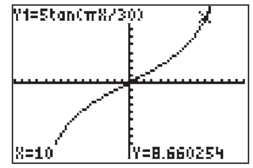
c)

θ	tan θ
90.5°	−114.59
90.01°	−5729.58
90.001°	−57 295.78
90.000 001°	−57 295 779.51

The value of tan θ approaches negative infinity.

9. a) $d = 5 \tan \frac{\pi}{30}t$

b)

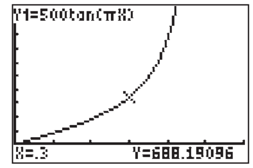


c) 8.7 m

d) At $t = 15$ s, the camera is pointing along a line parallel to the wall and is turning away from the wall.

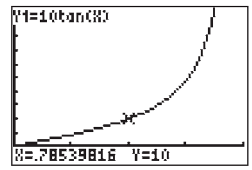
10. a) $d = 500 \tan \pi t$

b)



c) The asymptote represents the moment when the ray of light shines along a line that is parallel to the shore.

11. $d = 10 \tan x$



12. a) a tangent function

b) The slope would be undefined. It represents the place on the graph where the asymptote is.

13. Example:

a) (4, 3) **b)** 0.75

c) tan θ is the slope of the graph.

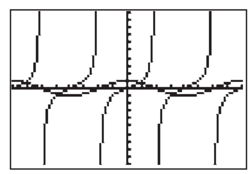
14. a) tan 0.5 ≈ 0.5463, power series ≈ 0.5463

b) sin 0.5 ≈ 0.4794, power series ≈ 0.4794

c) cos 0.5 ≈ 0.8776, power series ≈ 0.8776

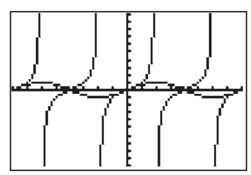
C1 The domain of $y = \sin x$ and $y = \cos x$ is all real numbers. The tangent function is not defined at $x = \frac{\pi}{2} + n\pi$, $n \in I$. Thus, these numbers must be excluded from the domain of $y = \tan x$.

C2 a)



Example: The tangent function has asymptotes at the same x-values where zeros occur on the cosine function.

b)



Example: The tangent function has zeros at the same x-values where zeros occur on the sine function.

C3 Example: A circular or periodic function repeats its values over a specific period. In the case of $y = \tan x$, the period is π. So, the equation $\tan (x + \pi) = \tan x$ is true for all x in the domain of $\tan x$.

5.4 Equations and Graphs of Trigonometric Functions, pages 275 to 281

1. a) $x = 0, \pi, 2\pi$ **b)** $x = \pi n$ where n is an integer

c) $x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}, 2\pi$

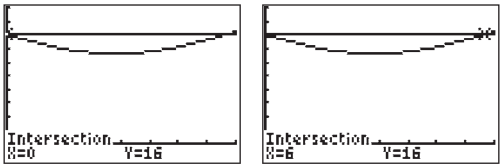
2. Examples:

a) 1.25, 4.5

b) −3, −1.9, 0.1, 1.2, 3.2, 4.1, 6.3, 7.2

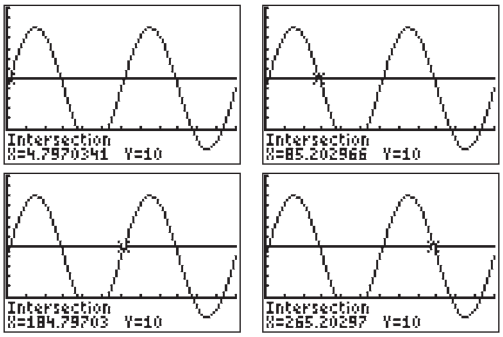
3. Examples: −50°, −10°, 130°, 170°, 310°, 350°

4. a)



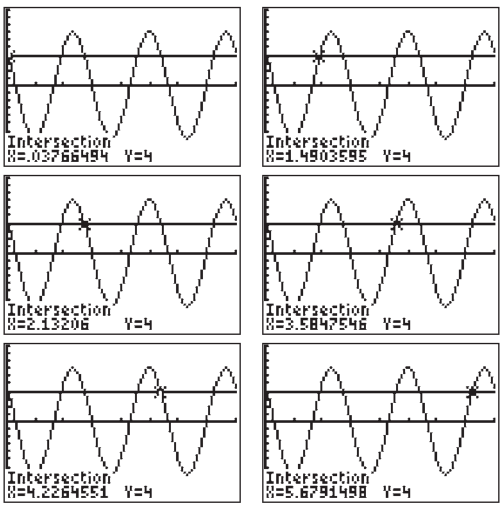
$x = 0$ and $x = 6$

b)



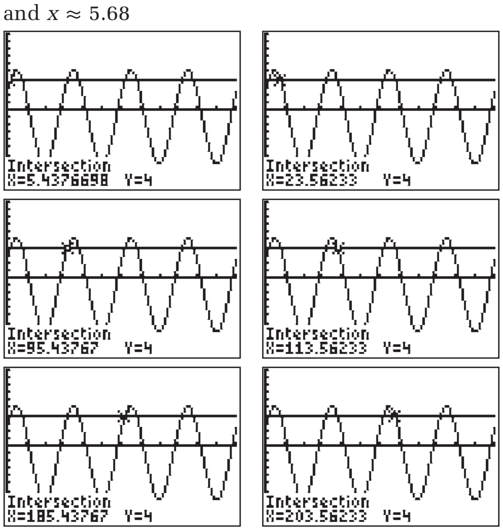
$x \approx 4.80°$, $x \approx 85.20°$, $x \approx 184.80°$, $x \approx 265.20°$

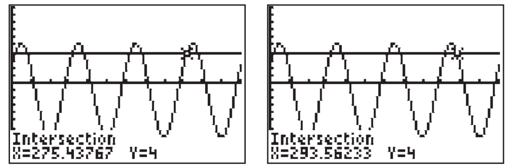
c)



$x \approx 0.04$, $x \approx 1.49$, $x \approx 2.13$, $x \approx 3.58$, $x \approx 4.23$, and $x \approx 5.68$

d)





$x \approx 5.44°$, $x \approx 23.56°$, $x \approx 95.44°$, $x \approx 113.56°$, $x \approx 185.44°$, $x \approx 203.56°$, $x \approx 275.44°$, and $x \approx 293.56°$

5. a) $x \approx 1.33$
b) $x \approx 3.59°$ and $x \approx 86.41°$
c) $x \approx 1.91 + \pi n$ and $x \approx 3.09 + \pi n$, where n is an integer
d) $x \approx 4.50° + (8°)n$ and $x \approx 7.50° + (8°)n$, where n is an integer

6. a) domain $\{t \mid t \geq 0, t \in R\}$, range $\{P \mid 2000 \leq P \leq 14\,000, P \in N\}$
b) domain $\{t \mid t \geq 0, t \in R\}$, range $\{h \mid 1 \leq h \leq 13, h \in R\}$
c) domain $\{t \mid t \geq 0, t \in R\}$, range $\{h \mid 6 \leq h \leq 18, h \in R\}$
d) domain $\{t \mid t \geq 0, t \in R\}$, range $\{h \mid 5 \leq h \leq 23, h \in R\}$

7. $\frac{1}{200}$ s or 5 ms

8. a) Period is 100°; sinusoidal axis is at $y = 15$; amplitude is 9.
b) Period is $\frac{4\pi}{3}$; sinusoidal axis is at $y = -6$; amplitude is 10.
c) Period is $\frac{1}{50}$ s or 20 ms; sinusoidal axis is at $y = 0$; amplitude is 10.

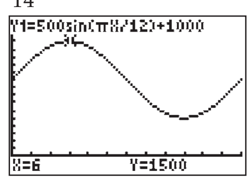
9. a) 28 m **b)** 0 min, 0.7 min, 1.4 min, …
c) 2 m **d)** 0.35 min, 1.05 min, 1.75 min, …
e) 0.18 min **f)** approximately 23.1 m

10. 78.5 cm

11. $V = 155 \sin 120\pi t$

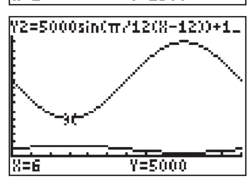
12. a) $\frac{1}{14}$ days **b)** 102.9 min **c)** 14 revolutions

13. a)



It takes approximately 15 months for the fox population to drop to 650.

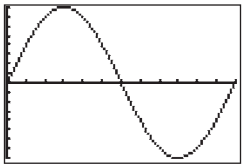
b)



c)

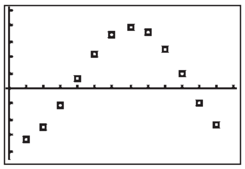
	Arctic Fox	Lemming
Maximum Population	1500	15 000
Month	6	18
Minimum Population	500	5000
Month	18	6

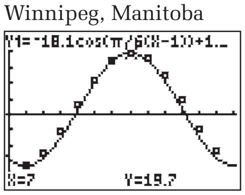
d) Example: The maximum for the predator occurs at a minimum for the prey and vice versa. The predators population depends on the prey, so every time the lemming's population changes the arctic fox population changes in accordance.

14. a) 

b) 35.1 cm
c) 1 s

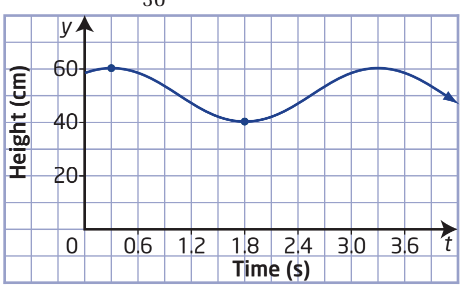
15. a) Maximum is 7.5 Sun widths; minimum is 1 Sun width.
b) 24 h
c) $y = -3.25 \sin \frac{\pi}{12}x + 4.25$, where x represents the time, in hours, and y represents the number of Sun widths

16. a) 

b) 1.6 °C
c) $y = -18.1 \cos \frac{\pi}{6}(x - 1) + 1.6$, where x represents the time, in months, and y represents the average monthly temperature, in degrees Celsius, for Winnipeg, Manitoba
d) 

e) about 2.5 months

17. a) $T = -4.5 \cos \frac{\pi}{30}t + 38.5$
b) 36.25 °C

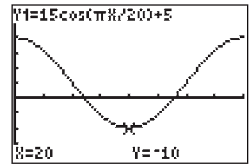
18. a) 

b) $y = 10 \sin \frac{2\pi}{3}(t + 0.45) + 50$, where t represents the time, in seconds, and y represents the height of the mass, in centimetres, above the floor
c) 43.3 cm
d) 0.0847 s

19. a) $h = -10 \cos \frac{\pi}{30}t + 12$, where t represents the time, in seconds, and h represents the height of a passenger, in metres, above the ground
b) 15.1 m
c) approximately 21.1 s, 38.9 s

20. a) $h = 7 \sin \frac{2\pi}{5}(t + 1.75) + 15$ or
$h = 7 \cos \frac{2\pi}{5}(t + 0.5) + 15$, where t represents the time, in seconds, and h represents the height of the tip of the blade, in metres, above the ground
b) 20.66 m
c) 4.078 s

21. a) $y = -9.7 \cos \frac{\pi}{183}(t - 26) + 13.9$, where t represents the time, in days, and y represents the average daily maximum temperature, in degrees Celsius
b) 18.6 °C
c) 88 days

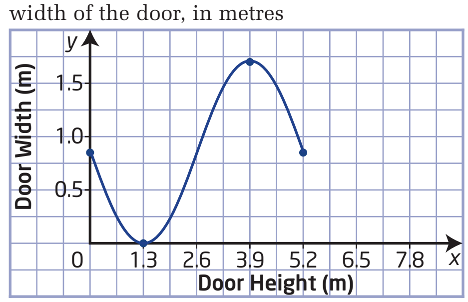
22. a) $y = 15 \cos \frac{\pi}{20}t + 5$
b) 

c) approximately +9.6% of the total assets
d) Example: No, because it fluctuates too much.

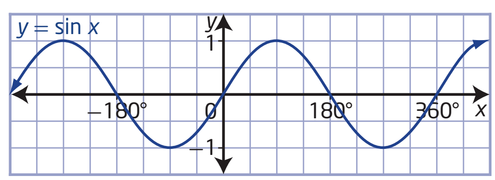
23. a) $y = 1.2 \sin \frac{\pi}{2}t$, where t represents the time, in seconds, and y represents the distance for a turn, in metres, from the midline
b) $y = 1.2 \sin \frac{2\pi}{5}t$; The period increases.

C1 Examples:
a) Use a sine function as a model when the curve or data begins at or near the intersection of the vertical axis and the sinusoidal axis.
b) Use a cosine function as a model when the curve or data has a maximum or minimum near or at the vertical axis.

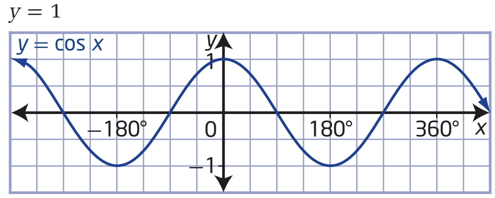
C2 Example:
a)–b) The parameter b has the greatest influence on the graph of the function. It changes the period of the function. Parameters c and d change the location of the curve, but not the shape. Parameter a changes the maximum and minimum values.

C3 Examples:
a) $y = -0.85 \sin \frac{2\pi}{5.2}x + 0.85$, where x represents the height of the door, in metres, and y represents the width of the door, in metres
b) 

Chapter 5 Review, pages 282 to 285

1. a) 

x-intercepts: −360°, −180°, 0°, 180°, 360°
b) y-intercept: 0
c) domain $\{x \mid x \in \mathrm{R}\}$,
range $\{y \mid -1 \le y \le 1, y \in \mathrm{R}\}$, period is 2π
d) $y = 1$

2. a) 

x-intercepts: −270°, −90°, 90°, 270°
b) y-intercept: 1

c) domain $\{x \mid x \in R\}$, range $\{y \mid -1 \le y \le 1, y \in R\}$, period is 2π

d) $y = 1$

3. a) A **b)** D **c)** B **d)** C

4. a) Amplitude is 3; period is π or 180°.

b) Amplitude is 4; period is 4π or 720°.

c) Amplitude is $\frac{1}{3}$; period is $\frac{12\pi}{5}$ or 432°.

d) Amplitude is 5; period is $\frac{4\pi}{3}$ or 240°.

5. a) Compared to the graph of $y = \sin x$, the graph of $y = \sin 2x$ completes two cycles in $0° \le x \le 360°$ and the graph of $y = 2 \sin x$ has an amplitude of 2.

b) Compared to the graph of $y = \sin x$, the graph of $y = -\sin x$ is reflected in the x-axis and the graph of $y = \sin(-x)$ is reflected in the y-axis. The graphs of $y = -\sin x$ and $y = \sin(-x)$ are the same.

c) Compared to the graph of $y = \cos x$, the graph of $y = -\cos x$ is reflected in the x-axis and the graph of $y = \cos(-x)$ is reflected in the y-axis. The graph of $y = \cos(-x)$ is the same as $y = \cos x$.

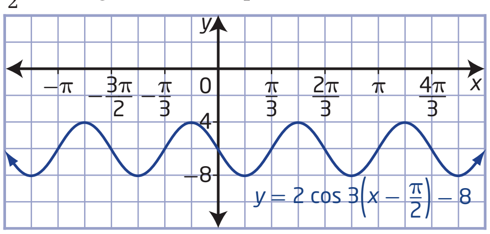
6. a) $y = 3 \cos 2x$ **b)** $y = 4 \cos \frac{12}{5}x$

c) $y = \frac{1}{2} \cos \frac{1}{2}x$ **d)** $y = \frac{3}{4} \cos 12x$

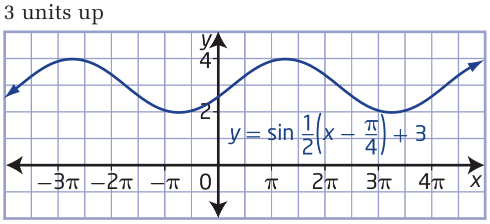
7. a) $y = 8 \sin 2x$ **b)** $y = 0.4 \sin 6x$

c) $y = \frac{3}{2} \sin \frac{1}{2}x$ **d)** $y = 2 \sin 3x$

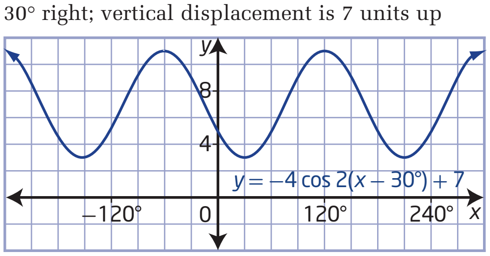
8. a) Amplitude is 2; period is $\frac{2\pi}{3}$; phase shift is $\frac{\pi}{2}$ units right; vertical displacement is 8 units down



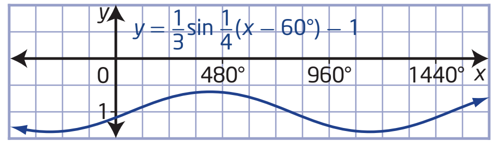
b) Amplitude is 1; period is 4π; phase shift is $\frac{\pi}{4}$ units right; vertical displacement is 3 units up



c) Amplitude is 4; period is 180°; phase shift is 30° right; vertical displacement is 7 units up



d) Amplitude is $\frac{1}{3}$; period is 1440°; phase shift is 60° right; vertical displacement is 1 unit down



9. a) They both have periods of π.

b) $f(x)$ has a phase shift of $\frac{\pi}{2}$ units right; $g(x)$ has a phase shift of $\frac{\pi}{4}$ units right

c) π units right **d)** $\frac{\pi}{b}$ units right

10. a) $y = 3 \sin 2(x - 45°) + 1$, $y = -3 \cos 2x + 1$

b) $y = 2 \sin 2x - 1$, $y = 2 \cos 2(x - 45°) - 1$

c) $y = 2 \sin 2\left(x - \frac{\pi}{4}\right) - 1$, $y = -2 \cos 2x - 1$

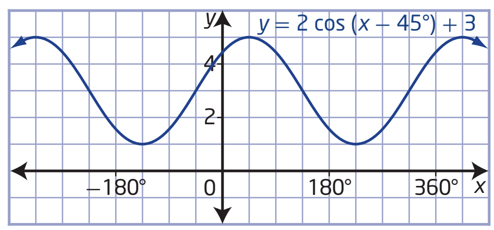
d) $y = 3 \sin \frac{1}{2}\left(x - \frac{\pi}{2}\right) + 1$, $y = 3 \cos \frac{1}{2}\left(x - \frac{3\pi}{2}\right) + 1$

11. a) $y = 4 \sin 2\left(x - \frac{\pi}{3}\right) - 5$

b) $y = \frac{1}{2} \cos \frac{1}{2}\left(x + \frac{\pi}{6}\right) + 1$

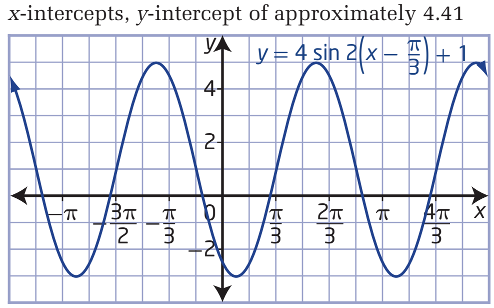
c) $y = \frac{2}{3} \sin \frac{2}{3}x - 5$

12. a)



domain $\{x \mid x \in R\}$, range $\{y \mid 1 \le y \le 5, y \in R\}$, maximum value is 5, minimum value is 1, no x-intercepts, y-intercept of approximately 4.41

b)



domain $\{x \mid x \in R\}$, range $\{y \mid -3 \le y \le 5, y \in R\}$, maximum value is 5, minimum value is −3, x-intercepts: approximately $0.92 + n\pi$, $2.74 + n\pi$, $n \in I$, y-intercept: approximately −2.5

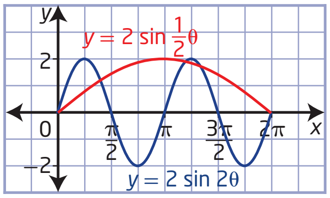
13. a) vertically stretched by a factor of 3 about the x-axis, horizontally stretched by a factor of $\frac{1}{2}$ about the y-axis, translated $\frac{\pi}{3}$ units right and 6 units up

b) vertically stretched by a factor of 2 about the x-axis, reflected in the x-axis, horizontally stretched by a factor of 2 about the y-axis, translated $\frac{\pi}{4}$ units left and 3 units down

c) vertically stretched by a factor of $\frac{3}{4}$ about the x-axis, horizontally stretched by a factor of $\frac{1}{2}$ about the y-axis, translated 30° right and 10 units up

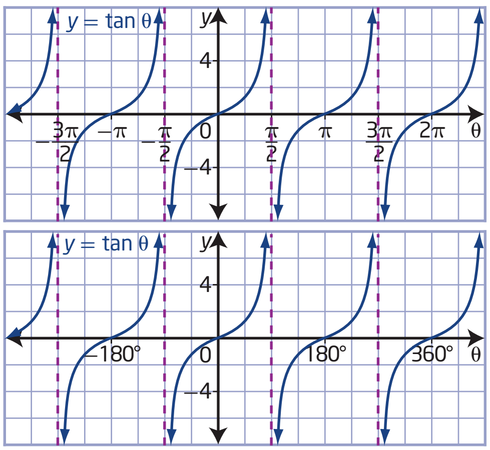
d) reflected in the x-axis, horizontally stretched by a factor of $\frac{1}{2}$ about the y-axis, translated 45° left and 8 units down

14. a)



b) Compared to the graph of $y = \sin \theta$, the graph of $y = 2 \sin 2\theta$ is vertically stretched by a factor of 2 about the x-axis and half the period. Compared to the graph of $y = \sin \theta$, the graph of $y = 2 \sin \frac{1}{2}\theta$ is vertically stretched by a factor of 2 about the x-axis and double the period.

15. a)



b) **i)** domain $\{x \mid -2\pi \le x \le 2\pi, x \ne -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, x \in \mathrm{R}\}$ or $\{x \mid -360° \le x \le 360°, x \ne -270°, -90°, 90°, 270°, x \in \mathrm{R}\}$

ii) range $\{y \mid y \in \mathrm{R}\}$ **iii)** y-intercept: 0

iv) x-intercepts: $-2\pi, -\pi, 0, \pi, 2\pi$ or $-360°, -180°, 0°, 180°, 360°$

v) asymptotes: $x = -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$ or $x = -270°, -90°, 90°, 270°$

16. a) $\left(1, \frac{1}{\sqrt{3}}\right)$ **b)** $\tan \theta = \frac{\sin \theta}{\cos \theta}$

c) As θ approaches 90°, $\tan \theta$ approaches infinity.

d) tan 90° is not defined.

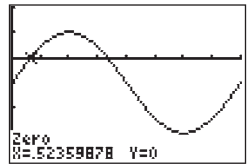
17. a) Since $\cos \theta$ is the denominator, when it is zero $\tan \theta$ becomes undefined.

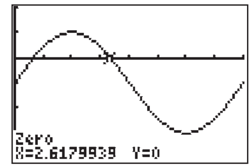
b) Since $\sin \theta$ is the numerator, when it is zero $\tan \theta$ becomes zero.

18. The shadow has no length which makes the slope infinite. This relates to the asymptotes on the graph of $y = \tan \theta$.

19. A vertical asymptote is an imaginary line that the graph comes very close to touching but in fact never does. If a trigonometric function is represented by a quotient, such as the tangent function, asymptotes generally occur at values for which the function is not defined; that is, when the function in the denominator is equal to zero.

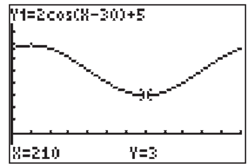
20. a)





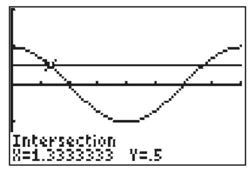
$x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$ or $x \approx 0.52$ and $x \approx 2.62$

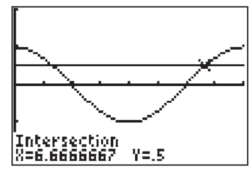
b)



no solution

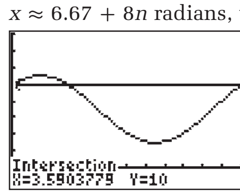
c)

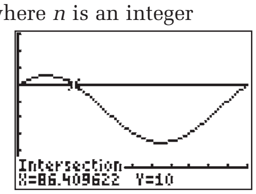




$x \approx 1.33 + 8n$ radians and $x \approx 6.67 + 8n$ radians, where n is an integer

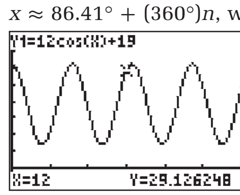
d)





$x \approx 3.59° + (360°)n$ and $x \approx 86.41° + (360°)n$, where n is an integer

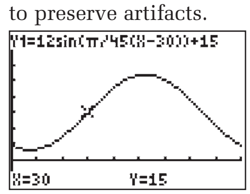
21. a)



b) 9.4 h

c) Example: A model for temperature variance is important for maintaining constant temperatures to preserve artifacts.

22. a)



b) maximum height: 27 m, minimum height: 3 m

c) 90 s **d)** approximately 25.4 m

23. a) $L = -3.7 \cos \frac{2\pi}{365}(t + 10) + 12$

b) approximately 12.8 h of daylight

24. a) approximately 53 sunspots

b) around the year 2007

c) around the year 2003

Chapter 5 Practice Test, pages 286 to 287

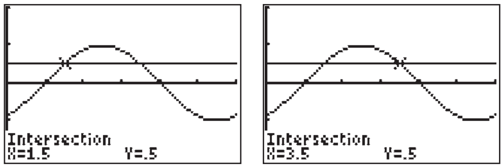
1. A **2.** D **3.** C **4.** D **5.** B **6.** A **7.** C **8.** $\frac{\pi}{2}$

9. asymptotes: $x = \frac{\pi}{2} + n\pi, n \in \mathrm{I}$,

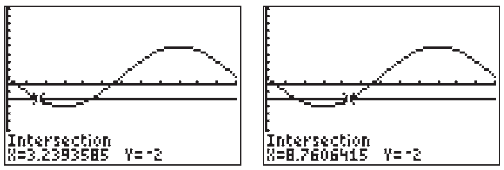
domain $\left\{x \mid x \ne \frac{\pi}{2} + n\pi, x \in \mathrm{R}, n \in \mathrm{I}\right\}$,

range $\{y \mid y \in \mathrm{R}\}$, period is π

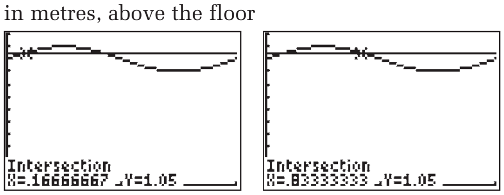
10. Example: They have the same maximum and minimum values. Neither function has a horizontal or vertical translation.

11. Amplitude is 120; period is 0.0025 s or 2.5 ms.
12. The minimum depth of 2 m occurs at 0 h, 12 h, and 24 hour. The maximum depth of 8 m occurs at 6 h and 18 h.
13. a) 

$x = 1.5 + 6n$ radians and $x = 3.5 + 6n$ radians, where n is an integer

b) 

$x \approx 3.24° + (24°)n$ and
$x \approx 8.76° + (24°)n$, where n is an integer
14. Example: Graph II has half the period of graph I. Graph I represents a cosine curve with no phase shift. Graph II represents a sine curve with no phase shift. Graph I and II have the same amplitude and both graphs have no vertical translations.
15. a) $h = 0.1 \sin \pi t + 1$, where t represents the time, in seconds, and h represents the height of the mass, in metres, above the floor

b) 

approximately 0.17 s and 0.83 s

c) $t = \frac{1}{6}$ or 0.1666… and $t = \frac{5}{6}$ or 0.8333
16. a) $y = 3 \sin 2\left(x - \frac{\pi}{4}\right) - 1$ b) $y = -3 \cos 2x - 1$
17. a) A, B b) A, B or C, D, E c) B

Chapter 6 Trigonometric Identities

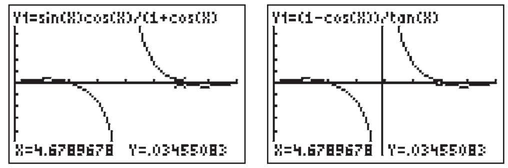
6.1 Reciprocal, Quotient, and Pythagorean Identities, pages 296 to 298

1. a) $x \neq \pi n$; $n \in \mathrm{I}$ b) $x \neq \left(\frac{\pi}{2}\right)n$, $n \in \mathrm{I}$

c) $x \neq \frac{\pi}{2} + 2\pi n$ and $x \neq \pi n$, $n \in \mathrm{I}$

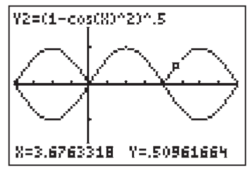
d) $x \neq \frac{\pi}{2} + \pi n$ and $x \neq \pi + 2\pi n$, $n \in \mathrm{I}$
2. Some identities will have non-permissible values because they involve trigonometric functions that have non-permissible values themselves or a function occurs in a denominator. For example, an identity involving sec θ has non-permissible values $\theta \neq 90° + 180°n$, where $n \in \mathrm{I}$, because these are the non-permissible values for the function.
3. a) tan x b) sin x c) sin x
4. a) cot x b) csc x c) sec x
5. a) When substituted, both values satisfied the equation.

b) $x \neq 0°, 90°, 180°, 270°$
6. a) $x \neq \pi + 2\pi n$, $n \in \mathrm{I}$; $x \neq \frac{\pi}{2} + \pi n$, $n \in \mathrm{I}$

b) 

Yes, it appears to be an identity.

c) The equation is verified for $x = \frac{\pi}{4}$.
7. a) $\cos^2 \theta$ b) 0.75 c) 25%
8. a) All three values check when substituted.

b) 

c) The equation is not an identity since taking the square then the square root removes the negative sign and sin x is negative from π to 2π.
9. a) $\mathrm{E} = \frac{I \cos \theta}{R^2}$ b) $\mathrm{E} = \frac{I \cot \theta}{R^2 \csc \theta}$

$$\mathrm{E} = \frac{I\left(\frac{\cos \theta}{\sin \theta}\right)}{R^2\left(\frac{1}{\sin \theta}\right)}$$

$$\mathrm{E} = \left(\frac{I \cos \theta}{\sin \theta}\right)\left(\frac{\sin \theta}{R^2}\right)$$

$$\mathrm{E} = \frac{I \cos \theta}{R^2}$$
10. $\cos x$, $x \neq 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$
11. a) It appears to be equivalent to sec x.

b) $x \neq \frac{\pi}{2} + \pi n$, $n \in \mathrm{I}$

c)
$$\frac{\csc^2 x - \cot^2 x}{\cos x} = \frac{\frac{1}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x}}{\cos x} = \frac{\frac{1 - \cos^2 x}{\sin^2 x}}{\cos x} = \frac{\frac{\sin^2 x}{\sin^2 x}}{\cos x} = \frac{1}{\cos x} = \sec x$$
12. a) Yes, it could be an identity.

b)
$$\frac{\cot x}{\sec x} + \sin x = \frac{\cos x}{\sin x} \div \frac{1}{\cos x} + \sin x = \frac{\cos^2 x}{\sin x} + \sin x = \frac{\cos^2 x + \sin^2 x}{\sin x} = \csc x$$
13. a) 1 = 1

b) The left side = 1, but the right side is undefined.

c) The chosen value is not permissible for the tan x function.

d) The left side $= \frac{2}{\sqrt{2}}$, but the right side = 2.

e) Giselle has found a permissible value for which the equation is not true, so they can conclude that it is not an identity.
14. 2
15. 7.89

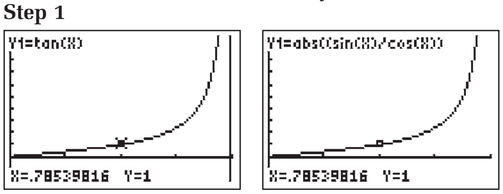
16. $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = \frac{1-\sin\theta+1+\sin\theta}{(1-\sin\theta)(1+\sin\theta)}$
$= \frac{2}{(1-\sin^2\theta)}$
$= 2\sec^2\theta$

17. $m = \csc x$

C1 $\cot^2 x + 1$
$= \frac{\cos^2 x}{\sin^2 x} + \frac{\sin^2 x}{\sin^2 x}$
$= \frac{\cos^2 x + \sin^2 x}{\sin^2 x}$
$= \frac{1}{\sin^2 x}$
$= \csc^2 x$

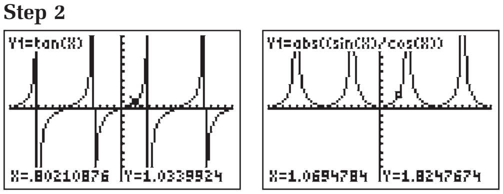
C2 $\left(\frac{\sin\theta}{1+\cos\theta}\right)\left(\frac{1-\cos\theta}{1-\cos\theta}\right)$
$= \frac{\sin\theta - \sin\theta\cos\theta}{1-\cos^2\theta}$
$= \frac{\sin\theta - \sin\theta\cos\theta}{\sin^2\theta}$
$= \frac{1-\cos\theta}{\sin\theta}$
It helps to simplify by creating an opportunity to use the Pythagorean identity.

C3 **Step 1**



Yes, over this domain it is an identity.

Step 2



The equation is not an identity since the graphs of the two sides are not the same.

Step 3 Example: $y = \cot\theta$ and $y = \left|\frac{\cos\theta}{\sin\theta}\right|$ are identities over the domain $0 \le \theta \le \frac{\pi}{2}$ but not over the domain $-2\pi \le \theta \le 2\pi$

Step 4 The weakness with this approach is that for some more complicated identities you may think it is an identity when really it is only an identity over that domain.

6.2 Sum, Difference, and Double-angle Identities, pages 306 to 308

1. a) $\cos 70°$ b) $\sin 35°$ c) $\cos 38°$
d) $\sin\frac{\pi}{4}$ e) $4\sin\frac{2\pi}{3}$

2. a) $\cos 60° = 0.5$ b) $\sin 45° = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$
c) $\cos\frac{\pi}{3} = 0.5$ d) $\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$

3. $\cos 2x = 1 - 2\sin^2 x$;
$1 - \cos 2x = 1 - 1 + 2\sin^2 x = 2\sin^2 x$

4. a) $\sin\frac{\pi}{2}$ b) $6\sin 48°$ c) $\tan 152°$ d) $\cos\frac{\pi}{3}$
e) $-\cos\frac{\pi}{6}$

5. a) $\sin\theta$ b) $\cos x$ c) $\cos\theta$ d) $\cos x$

6. Example: When $x = 60°$ and $y = 30°$, then left side $= 0.5$, but right side ≈ 0.366.

7. $\cos(90° - x) = \cos 90°\cos x + \sin 90°\sin x$
$= \sin x$

8. a) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ or $\frac{\sqrt{6}-\sqrt{2}}{4}$ b) $\frac{-\sqrt{3}+1}{\sqrt{3}+1}$ or $\sqrt{3}-2$
c) $\frac{1+\sqrt{3}}{2\sqrt{2}}$ or $\frac{\sqrt{2}+\sqrt{6}}{4}$ d) $\frac{-\sqrt{3}-1}{2\sqrt{2}}$ or $\frac{-\sqrt{6}-\sqrt{2}}{4}$
e) $\sqrt{2}(1+\sqrt{3})$ f) $\frac{1-\sqrt{3}}{2\sqrt{2}}$ or $\frac{\sqrt{2}-\sqrt{6}}{4}$

9. a) $P = 1000\sin(x + 113.5°)$
b) i) 101.056 W/m² ii) 310.676 W/m²
iii) −50.593 W/m²
c) The answer in part iii) is negative which means that there is no sunlight reaching Igloolik. At latitude 66.5°, the power received is 0 W/m².

10. $-2\cos x$

11. a) $\frac{119}{169}$ b) $-\frac{120}{169}$ c) $-\frac{12}{13}$

12. a) Both sides are equal for this value.
b) Both sides are equal for this value.
c) $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$
$= \frac{2\tan x}{1-\tan^2 x}\left(\frac{\cos^2 x}{\cos^2 x}\right)$
$= \frac{2\left(\frac{\sin x}{\cos x}\right)(\cos^2 x)}{\left(1-\frac{\sin^2 x}{\cos^2 x}\right)\cos^2 x}$
$= \frac{2\sin x\cos x}{\cos^2 x - \sin^2 x}$

13. a) $d = \frac{v_o^2\sin 2\theta}{g}$ b) 45°
c) It is easier after applying the double-angle identity since there is only one trigonometric function whose value has to be found.

14. $k - 1$

15. a) $\cos^4 x - \sin^4 x = (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$
$= \cos^2 x - \sin^2 x$
$= \cos 2x$
b) $\frac{\csc^2 x - 2}{\csc^2 x} = 1 - \frac{2}{\csc^2 x}$
$= 1 - 2\sin^2 x$
$= \cos 2x$

16. a) $\frac{1-\cos 2x}{2} = \frac{1-1+2\sin^2 x}{2} = \sin^2 x$
b) $\frac{4-8\sin^2 x}{2\sin x\cos x} = \frac{4\cos 2x}{\sin 2x} = \frac{4}{\tan 2x}$

17. $-\frac{2}{\sqrt{29}}$

18. $k = 3$

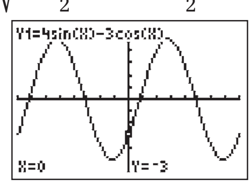
19. a) $0.9928, -0.392\ 82$ or $\frac{\pm 4\sqrt{3}+3}{10}$
b) 0.9500 or $\frac{\sqrt{5}+2\sqrt{3}}{6}$

20. a) $\frac{56}{65}$ b) $\frac{63}{65}$ c) $\frac{-7}{25}$ d) $\frac{24}{25}$

21. a) $\sin x$ b) $\tan x$

22. $\cos x = 2\cos^2\left(\frac{x}{2}\right) - 1$
$\frac{\cos x + 1}{2} = \cos^2\left(\frac{x}{2}\right)$
$\pm\sqrt{\frac{\cos x + 1}{2}} = \cos\frac{x}{2}$

23. a)



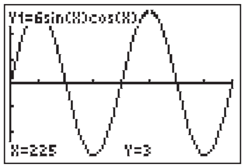
b) $a = 5$, $c = 37°$
c) $y = 5\sin(x - 36.87°)$

24. $y = 3 \sin 2x - 3$

C1 a) **i)** $\frac{120}{169}$ or 0.7101 **ii)** $\frac{120}{169}$ or 0.7101

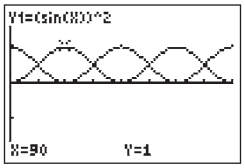
b) Using identities is more straightforward.

C2 a)

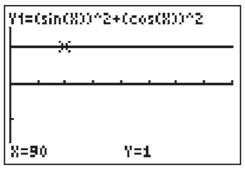


b) To find the sine function from the graph, compare the amplitude and the period to that of a base sine curve. The alternative equation is $y = 3 \sin 2x$.

C3 a)

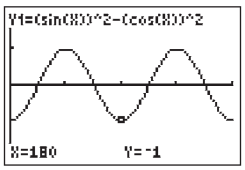


b)



The graph will be the horizontal line $y = 1$.

c)



The resultant graph is a cosine function reflected over the x-axis and the period becomes π.

d) $f(x) = -\cos 2x$. Using trigonometric identities, $\sin^2 x - \cos^2 x = 1 - \cos^2 x - \cos^2 x$.
$= 1 - 2\cos^2 x$
$= -\cos 2x$

6.3 Proving Identities, pages 314 to 315

1. a) $\sin x$ **b)** $\frac{\cos x + 1}{6}$

c) $\frac{\sin x}{\cos x + 1}$ **d)** $\sec x - 4 \csc x$

2. a)
$$\cos x + \cos x \tan^2 x = \cos x + \frac{\sin^2 x}{\cos x}$$
$$= \frac{\cos^2 x}{\cos x} + \frac{\sin^2 x}{\cos x}$$
$$= \frac{1}{\cos x}$$
$$= \sec x$$

b)
$$\frac{\sin^2 x - \cos^2 x}{\sin x + \cos x} = \frac{(\sin x - \cos x)(\sin x + \cos x)}{\sin x + \cos x}$$
$$= \sin x - \cos x$$

c)
$$\frac{\sin x \cos x - \sin x}{\cos^2 x - 1} = \frac{\sin x \cos x - \sin x}{-\sin^2 x}$$
$$= \frac{-\sin x(1 - \cos x)}{-\sin^2 x}$$
$$= \frac{1 - \cos x}{\sin x}$$

d)
$$\frac{1 - \sin^2 x}{1 + 2\sin x - 3\sin^2 x} = \frac{(1 - \sin x)(1 + \sin x)}{(1 - \sin x)(1 + 3\sin x)}$$
$$= \frac{1 + \sin x}{1 + 3\sin x}$$

3. a) $\frac{\sin x + 1}{\cos x}$ **b)** $\frac{-2\tan x}{\cos x}$

c) $\csc x$ **d)** $2\cot^2 x$

4. a) $\frac{1}{\sin x} - \frac{\cos^2 x}{\sin x}$ **b)** $\sin x$

5. $\frac{\sin 2x}{2 \sin x} = \frac{2 \sin x \cos}{2 \sin x} = \cos x,\ x \neq \pi n;\ n \in I$

6. $\cos x$

7. a)
$$\frac{\csc x}{2 \cos x} = \frac{1}{2 \sin x \cos x}$$
$$= \frac{1}{\sin 2x}$$
$$= \csc 2x$$

b)
$$\sin x + \cos x \cot x = \sin x + \frac{\cos^2 x}{\sin x}$$
$$= \frac{1}{\sin x}$$
$$= \csc x$$

8. Hannah's choice takes fewer steps.

9. a) 42.3 m

b)
$$\frac{v_o^2 \sin 2\theta}{g} = \frac{v_o^2 2 \sin\theta \cos\theta}{g}$$
$$= \frac{2v_o^2 \sin^2\theta \cos\theta}{g \sin\theta}$$
$$= \frac{2v_o^2 \sin^2\theta}{g \tan\theta}$$
$$= \frac{2v_o^2(1 - \cos^2\theta)}{g \tan\theta}$$

10. a) Left Side
$$= \frac{\csc x}{2 \cos x}$$
$$= \frac{1}{2 \sin x \cos x}$$
$$= \frac{1}{\sin 2x}$$
$$= \csc 2x$$
= Right Side

b) Left Side
$$= \frac{\sin x \cos x}{1 + \cos x}$$
$$= \frac{(\sin x \cos x)(1 - \cos x)}{(1 + \cos x)(1 - \cos x)}$$
$$= \frac{\sin x \cos x - \sin x \cos^2 x}{\sin^2 x}$$
$$= \frac{\cos x - \cos^2 x}{\sin x}$$
$$= \frac{1 - \cos x}{\tan x}$$
= Right Side

c) Left Side $= \frac{\sin x + \tan x}{1 + \cos x}$
$$= \left(\frac{\sin x}{1} + \frac{\sin x}{\cos x}\right) \div (1 + \cos x)$$
$$= \left(\frac{\sin x \cos x + \sin x}{\cos x}\right) \times \frac{1}{1 + \cos x}$$
$$= \left(\frac{\sin x(1 + \cos x)}{\cos x}\right) \times \frac{1}{1 + \cos x}$$
$$= \frac{\sin x}{\cos x}$$

Right Side $= \frac{\sin 2x}{2\cos^2 x}$
$$= \frac{2 \sin x \cos x}{2 \cos^2 x}$$
$$= \frac{\sin x}{\cos x}$$

Left Side = Right Side

11. a) Left Side $= \frac{\sin 2x}{\cos x} + \frac{\cos 2x}{\sin x}$
$$= \frac{2 \sin x \cos x}{\cos x} + \frac{1 - 2\sin^2 x}{\sin x}$$
$$= 2 \sin x + \csc x - 2 \sin x$$
$$= \csc x$$
= Right Side

b) Left Side
$= \csc^2 x + \sec^2 x$
$= \dfrac{1}{\sin^2 x} + \dfrac{1}{\cos^2 x}$
$= \dfrac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}$
$= \dfrac{1}{\sin^2 x \cos^2 x}$
$= \csc^2 x \sec^2 x$
= Right Side

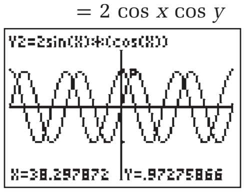
c) Left Side
$= \dfrac{\cot x - 1}{1 - \tan x}$
$= \dfrac{\dfrac{1 - \tan x}{\tan x}}{1 - \tan x}$
$= \dfrac{1 - \tan x}{\tan x(1 - \tan x)}$
$= \dfrac{1}{\tan x}$
$= \dfrac{\csc x}{\sec x}$
= Right Side

12. a) Left Side $= \sin (90° + \theta)$
$= \sin 90° \cos \theta + \cos 90° \sin \theta$
$= \cos \theta$
Right Side $= \sin (90° - \theta)$
$= \sin 90° \cos \theta - \cos 90° \sin \theta$
$= \cos \theta$

b) Left Side $= \sin (2\pi - \theta)$
$= \sin (2\pi) \cos (\theta) - \cos (2\pi) \sin (\theta)$
$= -\sin \theta$
= Right Side

13. Left Side $= 2 \cos x \cos y$
Right Side $= \cos(x + y) + \cos(x - y)$
$= \cos x \cos y - \sin x \sin y + \cos x \cos y +$
$\sin x \sin y$
$= 2 \cos x \cos y$

14. a)



No, this is not an identity.

b) Replacing the variable with 0 is a counter example.

15. a) $x \neq \pi n;\ n \in I$

b) Left Side $= \dfrac{\sin 2x}{1 - \cos 2x}$
$= \dfrac{2 \sin x \cos x}{1 - 1 + 2 \sin^2 x}$
$= \dfrac{\cos x}{\sin x}$
$= \cot x$
= Right Side

16. Right Side
$= \dfrac{\sin 4x - \sin 2x}{\cos 4x + \cos 2x}$
$= \dfrac{2 \sin 2x \cos 2x - 2 \sin x \cos x}{\cos 4x + 2 \cos^2 x - 1}$
$= \dfrac{2(2 \sin x \cos x)(2 \cos^2 x - 1) - 2 \sin x \cos x}{2 \cos^2 2x - 1 + 2 \cos^2 x - 1}$
$= \dfrac{(2 \sin x \cos x)(2(2 \cos^2 x - 1) - 1)}{2(2 \cos^2 x - 1)^2 + 2 \cos^2 x - 2}$
$= \dfrac{(2 \sin x \cos x)(4 \cos^2 x - 3)}{2(4 \cos^4 x - 4 \cos^2 x + 1) + 2 \cos^2 x - 2}$
$= \dfrac{(2 \sin x \cos x)(4 \cos^2 x - 3)}{8 \cos^4 x - 6 \cos^2 x}$
$= \dfrac{(2 \sin x \cos x)(4 \cos^2 x - 3)}{2 \cos^2 x(4 \cos^2 x - 3)}$
$= \dfrac{2 \sin x \cos x}{2 \cos^2 x}$
$= \tan x$
= Left Side

17. Left Side $= \dfrac{\sin 2x}{1 - \cos 2x}$
$= \dfrac{\sin 2x}{1 - \cos 2x}\left(\dfrac{1 + \cos 2x}{1 + \cos 2x}\right)$
$= \dfrac{\sin 2x + \sin 2x \cos 2x}{1 - \cos^2 2x}$
$= \dfrac{\sin 2x + \sin 2x \cos 2x}{\sin^2 2x}$
$= \dfrac{1}{\sin 2x} + \dfrac{\cos 2x}{\sin 2x}$
$= \dfrac{1}{\sin 2x} + \dfrac{1 - 2 \sin^2 x}{\sin 2x}$
$= \dfrac{2}{\sin 2x} - \dfrac{2 \sin^2 x}{\sin 2x}$
$= 2 \csc 2x - \dfrac{2 \sin^2 x}{2 \sin x \cos x}$
$= 2 \csc 2x - \tan x$
= Right Side

18. Left Side $= \dfrac{1 - \sin^2 x - 2 \cos x}{\cos^2 x - \cos x - 2}$
$= \dfrac{\cos^2 x - 2 \cos x}{\cos^2 x - \cos x - 2}$
$= \dfrac{\cos x(\cos x - 2)}{(\cos x - 2)(\cos x + 1)}$
$= \dfrac{\cos x}{\cos x + 1}$
$= \dfrac{\dfrac{\cos x}{\cos x}}{\dfrac{\cos x + 1}{\cos x}}$
$= \dfrac{1}{1 + \sec x}$
= Right Side

19. a) $\sin \theta_t = \dfrac{n_1 \sin \theta_i}{n_2}$

b) Using $\sin^2 x + \cos^2 x = 1$, $\cos x = \sqrt{1 - \sin^2 x}$
Then, replace this in the equation.

c) Substitute $\sin \theta_t = \dfrac{n_1 \sin \theta_i}{n_2}$.

C1 Graphing gives a visual approximation, so some functions may look the same but actually are not. Verifying numerically is not enough since it may not hold for other values.

C2 Left Side $= \cos\left(\dfrac{\pi}{2} - x\right)$
$= \cos\left(\dfrac{\pi}{2}\right) \cos x + \sin\left(\dfrac{\pi}{2}\right) \sin x$
$= \sin x$
= Right Side

C3 a) $\cos x \geq 0$, $\dfrac{\pi}{2} + 2\pi n < x < \dfrac{3\pi}{2} + 2\pi n$, $n \in I$

b) $x = 1$

c) $x = \pi$, $\cos x$ will give a negative answer and radical functions always give a positive answer, so the equation is not an identity.

d) An identity is always true whereas an equation is true for certain values or a restricted domain.

6.4 Solving Trigonometric Equations Using Identities, pages 320 to 321

1. a) $0, \dfrac{\pi}{4}, \pi, \dfrac{5\pi}{4}$ **b)** $0, \dfrac{\pi}{3}, \pi, \dfrac{5\pi}{3}$

c) $\dfrac{3\pi}{2}$ **d)** $\dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{3\pi}{2}$

2. a) 0°, 120°, 240° **b)** 270°

c) no solution **d)** 0°, 120°, 180°, 300°

3. a) $2\sin^2 x + 3\sin x + 1 = 0; \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

b) $2\sin^2 x + 3\sin x + 1 = 0; \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

c) $\sin^2 x + 2\sin x - 3 = 0, \frac{\pi}{2}$

d) $2 - \sin^2 x = 0$; no solution

4. $-150°, -30°, 30°, 150°$

5. 0.464, 2.034, 3.605, 5.176

6. There are two more solutions that Sanesh did not find since she divided by cos (*x*). The extra solutions are $x = 90° + 360°n$ and $x = 270° + 360°n$.

7. a) $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$ **b)** $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

8. $x = \frac{\pi}{2} + \pi n, n \in I$

9. $x = \frac{\pi}{2} + 2\pi n, n \in I$

10. 7. Inspection of each factor shows that there are 2 + 1 + 4 solutions, which gives a total of 7 solutions over the interval $0° < x \leq 360°$.

11. $\frac{\pi}{2}, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}$

12. $B = -3, C = -2$

13. Example: $\sin 2x - \sin 2x \cos^2 x = 0$; $x = \left(\frac{\pi}{2}\right)n, n \in I$

14. $x = \left(\frac{\pi}{2}\right)(2n + 1), n \in I, x = \frac{\pi}{6} + 2\pi n, n \in I,$
$x = \frac{5\pi}{6} + 2\pi n, n \in I$

15. 12 solutions

16. $x = \pi + 2\pi n, n \in I, x = \pm 0.955\ 32 + n\pi, n \in I$

17. $x = \frac{\pi}{4} + \pi n, n \in I, x = -\frac{\pi}{4} + \pi n, n \in I$

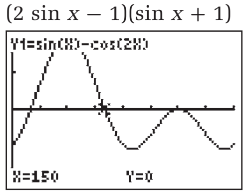
18. $-1.8235, 1.8235$

19. $x = 2\pi n, n \in I, x = \pm\frac{\pi}{3} + 2\pi n, n \in I$

20. 1 and -2

C1 a) $\cos 2x = 1 - 2\sin^2 x$ **b)** $(2\sin x - 1)(\sin x + 1)$

c) 30°, 150°, 270° **d)**



C2 a) You cannot factor the left side of the equation because there are no two integers whose product is -3 and whose sum is 1.

b) $-0.7676, 0.4343$

c) 64.26°, 140.14°, 219.86°, 295.74°, 424.26°, 500.14°, 579.86°, 655.74°

C3 Example: $\sin 2x \cos x + \cos x = 0$; The reason this is not an identity is that it is not true for all replacement values of the variable. For example, if $x = 30°$, the two sides are not equal. The solutions are $90° + 180°n, n \in I$ and $135° + 180°n, n \in I$.

Chapter 6 Review, pages 322 to 323

1. a) $x \neq \frac{\pi}{2} + n\pi, n \in I$ **b)** $x \neq \left(\frac{\pi}{2}\right)n, n \in I$

c) $x = \pm\frac{\pi}{3} + 2\pi n, n \in I$ **d)** $x \neq \frac{\pi}{2} + n\pi, n \in I$

2. a) cos *x* **b)** tan *x* **c)** tan *x* **d)** cos *x*

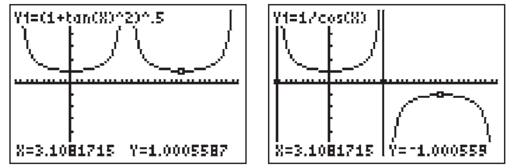
3. a) 1 **b)** 1 **c)** 1

4. a) Both sides have the same value so the equation is true for those values.

b) $x \neq 90°, 270°$

5. a) Example: $x = 0, 1$

b)



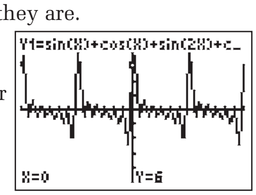
c) The graphs are the same for part of the domain. Outside of this interval they are not the same.

6. a) $f(0) = 2, f\left(\frac{\pi}{6}\right) = 1 + \sqrt{3}$

b) $\sin x + \cos x + \sin 2x + \cos 2x$
$= \sin x + \cos x + 2\sin x\cos x + 1 - \sin^2 x$

c) No, because you cannot write the first two terms as anything but the way they are.

d) You cannot get a perfect saw tooth graph but the approximation gets closer as you increase the amount of iterations. Six terms give a reasonable approximation.



7. a) $\sin 90° = 1$ **b)** $\sin 30° = 0.5$

c) $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ **d)** $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

8. a) $\frac{\sqrt{3} - 1}{2\sqrt{2}}$ or $\frac{\sqrt{6} - \sqrt{2}}{4}$ **b)** $\frac{\sqrt{3} + 1}{2\sqrt{2}}$ or $\frac{\sqrt{6} + \sqrt{2}}{4}$

c) $\sqrt{3} - 2$ **d)** $\frac{\sqrt{3} + 1}{2\sqrt{2}}$ or $\frac{\sqrt{6} + \sqrt{2}}{4}$

9. a) $\frac{7}{13\sqrt{2}}$ or $\frac{7\sqrt{2}}{26}$ **b)** $\frac{12 - 5\sqrt{3}}{26}$

c) $-\frac{120}{169}$

10. $1 + \frac{1}{\sqrt{2}}$

11. tan *x*

12. a) $\frac{\cos x}{\sin x - 1}$ or $\frac{-1 - \sin x}{\cos x}$

b) $\tan^2 x \sin^2 x$

13. a) Left Side
$= 1 + \cot^2 x$
$= 1 + \frac{\cos^2 x}{\sin^2 x}$
$= \frac{\sin^2 x + \cos^2 x}{\sin^2 x}$
$= \frac{1}{\sin^2 x}$
$= \csc^2 x$
= Right Side

b) Right Side
$= \csc 2x - \cot 2x$
$= \frac{1}{\sin 2x} - \frac{\cos 2x}{\sin 2x}$
$= \frac{1 - (2\cos^2 x - 1)}{2\sin x\cos x}$
$= \frac{2\sin^2 x}{2\sin x\cos x}$
$= \tan x$
= Left Side

c) Left Side
$= \sec x + \tan x$
$= \frac{1}{\cos x} + \frac{\sin x}{\cos x}$
$= \frac{1 + \sin x}{\cos x}$
$= \frac{1 - \sin^2 x}{(1 - \sin x)\cos x}$
$= \frac{\cos x}{1 - \sin x}$
= Right Side

d) Left Side
$= \frac{1}{1 + \cos x} + \frac{1}{1 - \cos x}$
$= \frac{1 - \cos x}{1 - \cos^2 x} + \frac{1 + \cos x}{1 - \cos^2 x}$
$= \frac{2}{\sin^2 x}$
$= 2\csc^2 x$
= Right Side

14. a) It is true when $x = \frac{\pi}{4}$. The equation is not necessarily an identity. Sometimes equations can be true for a small domain of *x*.

b) $x = \frac{\pi}{2} + n\pi, n \in I$

c) Left Side $= \sin 2x$
$= 2\sin x\cos x$
$= \dfrac{2\sin x\cos^2 x}{\cos x}$
$= \dfrac{2\tan x}{\sec^2 x}$
$= \dfrac{2\tan x}{1+\tan^2 x}$
$=$ Right Side

15. a) Left Side
$= \dfrac{\cos x + \cot x}{\sec x + \tan x}$
$= \dfrac{\cos x + \dfrac{\cos x}{\sin x}}{\dfrac{1}{\cos x} + \dfrac{\sin x}{\cos x}}$
$= \dfrac{\dfrac{\dfrac{\sin x\cos^2 x}{\sin x} + \dfrac{\cos^2 x}{\sin x}}{1+\sin x}}{}$
$= \dfrac{\dfrac{(\sin x + 1)\cos^2 x}{\sin x}}{1+\sin x}$
$= \dfrac{\cos x\cos x}{\sin x}$
$= \cos x\cot x$
$=$ Right Side

b) Left Side
$= \sec x + \tan x$
$= \dfrac{1}{\cos x} + \dfrac{\sin x}{\cos x}$
$= \dfrac{1+\sin x}{\cos x}$
$= \dfrac{1-\sin^2 x}{(1-\sin x)\cos x}$
$= \dfrac{\cos x}{1-\sin x}$
$=$ Right Side

16. a) You can disprove it by trying a value of x or by graphing.
b) Substituting $x = 0$ makes the equation fail.

17. a) $x = 0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$ **b)** $x = \frac{5\pi}{6}, \frac{11\pi}{6}$
c) $x = \frac{7\pi}{6}, \frac{11\pi}{6}$ **d)** $x = 0, \frac{\pi}{2}, \frac{3\pi}{2}$

18. a) $x = 15°, 75°, 195°, 255°$ **b)** $x = 90°, 270°$
c) $x = 30°, 150°, 270°$ **d)** $x = 0°, 180°$

19. $x = \pm\frac{\pi}{3} + n\pi, n \in I$

20. $\cos x = \pm\frac{4}{5}$

21. $x = -2\pi, -\pi, 0, \pi, 2\pi$

Chapter 6 Practice Test, page 324

1. A **2.** A **3.** D **4.** D **5.** A **6.** D

7. a) $\dfrac{1-\sqrt{3}}{2\sqrt{2}}$ or $\dfrac{\sqrt{2}-\sqrt{6}}{4}$
b) $\dfrac{\sqrt{3}+1}{2\sqrt{2}}$ or $\dfrac{\sqrt{6}+\sqrt{2}}{4}$

8. Left Side $= \cot\theta - \tan\theta$
$= \dfrac{1}{\tan\theta} - \tan\theta$
$= \dfrac{1-\tan^2\theta}{\tan\theta}$
$= 2\left(\dfrac{1-\tan^2\theta}{2\tan\theta}\right)$
$= 2\cot 2\theta$
$=$ Right Side
$\theta = \left(\frac{\pi}{2}\right)n, n \in I$

9. Theo's Formula $= I_0\cos^2\theta$
$= I_0 - I_0\sin^2\theta$
$= I_0 - \dfrac{I_0}{\csc^2\theta}$
$=$ Sany's Formula

10. a) $A = \frac{2\pi}{3} + 2\pi n, n \in I, A = \frac{4\pi}{3} + 2\pi n, n \in I$
b) $B = \pi n, n \in I, B = \frac{\pi}{6} + 2\pi n, n \in I,$
$B = \frac{5\pi}{6} + 2\pi n, n \in I$
c) $\theta = \pi n, n \in I, \theta = \pm\frac{\pi}{3} + 2\pi n, n \in I$

11. $x = \frac{\pi}{2} + n\pi, n \in I$

12. $\dfrac{-4-3\sqrt{3}}{10}$

13. $x = \frac{\pi}{4}, \frac{5\pi}{4}$

14. $x = 0°, 90°, 270°$

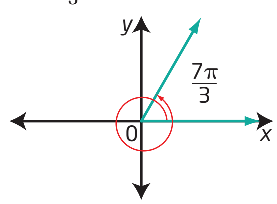
15. a) Left Side $= \dfrac{\cot x}{\csc x - 1}$
$= \dfrac{\cot x(\csc x + 1)}{\csc^2 x - 1}$
$= \dfrac{\cot x(\csc x + 1)}{1 + \cot^2 x - 1}$
$= \dfrac{(\csc x + 1)}{\cot x}$
$=$ Right Side

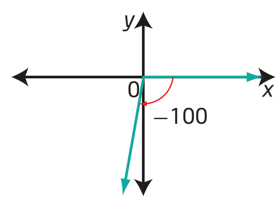
b) Left Side $= \sin(x+y)\sin(x-y)$
$= (\sin x\cos y + \sin y\cos x) \times (\sin x\cos y - \sin y\cos x)$
$= \sin^2 x\cos^2 y - \sin^2 y\cos^2 x$
$= \sin^2 x(1-\sin^2 y) - \sin^2 y(1-\sin^2 x)$
$= \sin^2 x - \sin^2 y$
$=$ Right Side

16. $x = \frac{\pi}{2} + 2\pi n, n \in I, x = \frac{\pi}{6} + 2\pi n, n \in I,$
$x = \frac{5\pi}{6} + 2\pi n, n \in I$

Cumulative Review, Chapters 4–6, pages 326 to 327

1. a) $\frac{7\pi}{3} \pm 2\pi n, n \in N$ **b)** $-100° \pm (360°)n, n \in N$





2. a) $229°$ **b)** $-300°$
3. a) $\frac{7\pi}{6}$ **b)** $-\frac{25\pi}{9}$
4. a) 13.1 ft **b)** 106.9 ft
5. a) $x^2 + y^2 = 25$ **b)** $x^2 + y^2 = 16$
6. a) quadrant III **b)** $-\frac{2\pi}{3}, \frac{4\pi}{3}$
c) $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$; when the given quadrant III angle is rotated through $\frac{\pi}{2}$, its terminal arm is in quadrant IV and its coordinates are switched and the signs adjusted.
d) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$; when the given quadrant III angle is rotated through $-\pi$, its terminal arm is in quadrant I and its coordinates are the same but the signs adjusted.

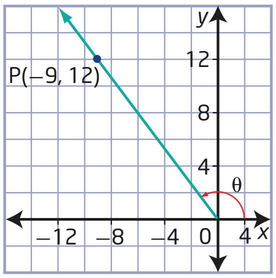
7. a) $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right), \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$; the points have the same x-coordinates but opposite y-coordinates.

b) $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right), \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$; the points have the same x-coordinates but opposite y-coordinates.

8. a) $-\frac{\sqrt{3}}{2}$ **b)** $\frac{1}{2}$ **c)** $-\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$

d) $\sqrt{2}$ **e)** undefined **f)** $-\sqrt{3}$

9. a)



b) $\sin\theta = \frac{4}{5}$, $\cos\theta = -\frac{3}{5}$, $\tan\theta = -\frac{4}{3}$,

$\csc\theta = \frac{5}{4}$, $\sec\theta = -\frac{5}{3}$, $\cot\theta = -\frac{3}{4}$

c) $\theta = 126.87° + (360°)n, n \in I$

10. a) $-\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **b)** $-30°, 30°$

c) $\frac{3\pi}{4}, \frac{7\pi}{4}$

11. a) $\theta = \frac{3\pi}{4} + 2\pi n, n \in I$; $\frac{5\pi}{4} + 2\pi n, n \in I$

b) $\theta = \frac{\pi}{2} + 2\pi n, n \in I$ **c)** $\theta = \frac{\pi}{2} + \pi n, n \in I$

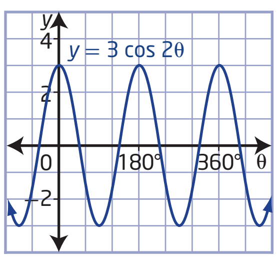
12. a) $\theta = 0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}, 2\pi$ **b)** $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

13. a) $\theta = 27°, 153°, 207°, 333°$

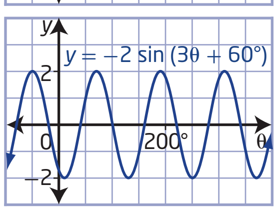
b) $\theta = 90°, 199°, 341°$

14. $y = 3\sin\frac{1}{2}\left(x + \frac{\pi}{4}\right)$

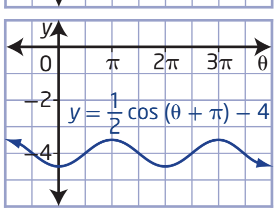
15. a) amplitude 3, period 180°, phase shift 0, vertical displacement 0



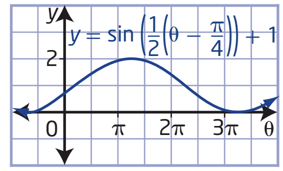
b) amplitude 2, period 120°, phase shift 20° left, vertical displacement 0



c) amplitude $\frac{1}{2}$, period 2π, phase shift π units left, vertical displacement 4 units down



d) amplitude 1, period 4π, phase shift $\frac{\pi}{4}$ units right, vertical displacement 1 unit up

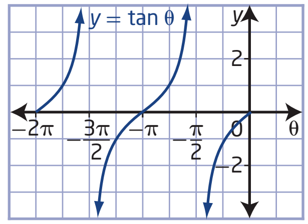


16. a) $y = 2\sin(x - 30°) + 3$, $y = 2\cos(x - 120°) + 3$

b) $y = \sin 2\left(x + \frac{\pi}{3}\right) - 1$, $y = \cos 2\left(x + \frac{\pi}{12}\right) - 1$

17. $y = 4\cos 1.2(x + 30°) - 3$

18. a)



b) $x = -\frac{\pi}{2}$, $x = -\frac{3\pi}{2}$

19. a) $h(x) = -25\cos\frac{2\pi}{11}x + 26$ **b)** $x = 3.0$ min

20. a) $\theta \neq \frac{\pi}{2} + \pi n, n \in I$, $\tan^2\theta$

b) $x \neq \left(\frac{\pi}{2}\right)n, n \in I$, $\sec^2 x$

21. a) $-\frac{\sqrt{3} - 1}{2\sqrt{2}}$ or $-\frac{\sqrt{6} - \sqrt{2}}{4}$

b) $\frac{\sqrt{3} - 1}{2\sqrt{2}}$ or $\frac{\sqrt{6} - \sqrt{2}}{4}$

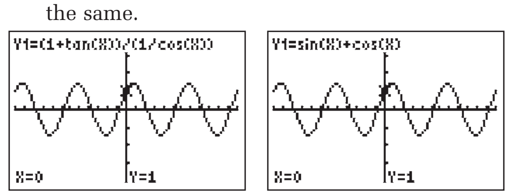
22. a) $\cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$ **b)** $\sin 90° = 1$

c) $\tan\frac{7\pi}{3} = \sqrt{3}$

23. a) Both sides have the same value for A = 30°.

b) Left Side $= \sin^2 A + \cos^2 A + \tan^2 A$
$= 1 + \tan^2 A$
$= \sec^2 A$
$=$ Right Side

24. a) It could be an identity as the graphs look the same.



b) Left Side $= \frac{1 + \tan x}{\sec x}$
$= \frac{1}{\sec x} + \frac{\tan x}{\sec x}$
$= \cos x + \frac{\sin x}{\cos x} \div \frac{1}{\cos x}$
$= \cos x + \sin x$
$=$ Right Side

25. Right Side $= \frac{\cos 2\theta}{1 + \sin 2\theta}$
$= \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta}$
$= \frac{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}{(\cos\theta + \sin\theta)(\cos\theta + \sin\theta)}$
$= \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$
$=$ Left Side

26. a) $x = \frac{5\pi}{6} + \pi n, n \in I$, $x = \frac{\pi}{6} + \pi n, n \in I$

b) $x = \frac{\pi}{2} + \pi n, n \in I, x = \frac{7\pi}{6} + 2\pi n, n \in I,$
$x = \frac{11\pi}{6} + 2\pi n, n \in I$

27. a) This is an identity so all θ are a solution.
b) Yes, because the left side can be simplified to 1.

Unit 2 Test, pages 328 to 329

1. B **2.** D **3.** C **4.** C **5.** B **6.** D **7.** C **8.** A

9. $-\frac{\sqrt{3}}{2}$

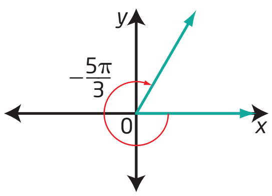
10. $-\frac{2}{3}, \frac{2}{3}$

11. $\frac{7}{13\sqrt{2}}$ or $\frac{7\sqrt{2}}{26}$

12. 1.5, 85.9°

13. $-\frac{11\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{11\pi}{6}$

14. a)

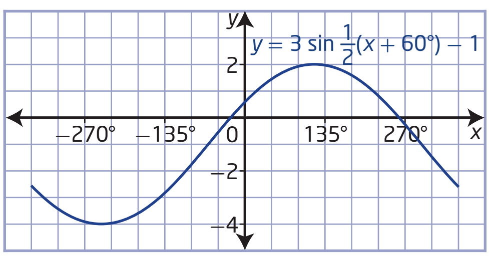


b) −300°
c) $-\frac{5\pi}{3} \pm 2\pi n, n \in N$
d) No, following the equation above it is impossible to obtain $\frac{10\pi}{3}$.

15. $x = 0.412, 2.730, 4.712$

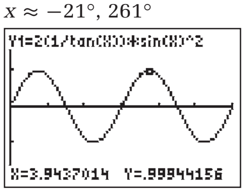
16. Sam is correct, there are four solutions in the given domain. Pat made an error when finding the square root. Pat forgot to solve for the positive and negative solutions.

17. a)



b) $-4 \leq y \leq 2$
c) amplitude 3, period 720°, phase shift 60° left, vertical displacement 1 unit down
d) $x \approx -21°, 261°$

18. a)



b) $g(\theta) = \sin 2\theta$
c) $f(\theta) = 2\cot\theta\sin^2\theta$
$= \frac{2\cos\theta\sin^2\theta}{\sin\theta}$
$= 2\cos\theta\sin\theta$
$= \sin 2\theta$
$= g(\theta)$

19. a) It is true: both sides have the same value.
b) $x \neq \frac{\pi n}{2}, n \in I$
c) Left Side
$= \tan x + \frac{1}{\tan x}$
$= \frac{\tan^2 x + 1}{\tan x}$
$= \frac{\sec^2 x}{\tan x}$
$= \sec x\left(\frac{1}{\cos x}\right)\left(\frac{\cos x}{\sin x}\right)$
$= \frac{\sec x}{\sin x}$
= Right Side

20. a) 6.838 m **b)** 12.37 h **c)** 3.017 m

Chapter 7 Exponential Functions

7.1 Characteristics of Exponential Functions, pages 342 to 345

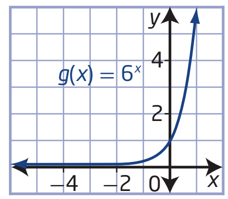
1. a) No, the variable is not the exponent.
b) Yes, the base is greater than 0 and the variable is the exponent.
c) No, the variable is not the exponent.
d) Yes, the base is greater than 0 and the variable is the exponent.

2. a) $f(x) = 4^x$ **b)** $g(x) = \left(\frac{1}{4}\right)^x$
c) $x = 0$, which is the y-intercept

3. a) B **b)** C **c)** A

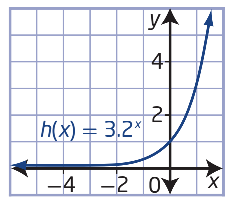
4. a) $f(x) = 3^x$ **b)** $f(x) = \left(\frac{1}{5}\right)^x$

5. a)



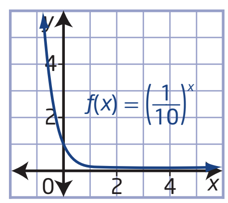
domain $\{x \mid x \in R\}$, range $\{y \mid y > 0, y \in R\}$, y-intercept 1, function increasing, horizontal asymptote $y = 0$

b)



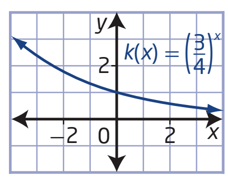
domain $\{x \mid x \in R\}$, range $\{y \mid y > 0, y \in R\}$, y-intercept 1, function increasing, horizontal asymptote $y = 0$

c)



domain $\{x \mid x \in R\}$, range $\{y \mid y > 0, y \in R\}$, y-intercept 1, function decreasing, horizontal asymptote $y = 0$

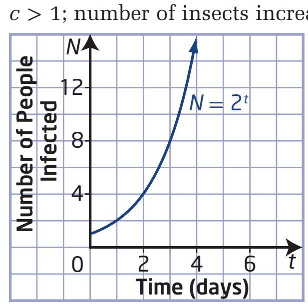
d)



domain $\{x \mid x \in R\}$, range $\{y \mid y > 0, y \in R\}$, y-intercept 1, function decreasing, horizontal asymptote $y = 0$

6. a) $c > 1$; number of bacteria increases over time
b) $0 < c < 1$; amount of actinium-225 decreases over time
c) $0 < c < 1$; amount of light decreases with depth
d) $c > 1$; number of insects increases over time

7. a)



The function $N = 2^t$ is exponential since the base is greater than zero and the variable t is an exponent.

b) **i)** 1 person **ii)** 2 people
iii) 16 people **iv)** 1024 people