Kuta Software - Infinite Calculus

Name____________________

Related Rates

Date________________ Period____

Solve each related rate problem.

1) Water leaking onto a floor forms a circular pool. The radius of the pool increases at a rate of 4 cm/min. How fast is the area of the pool increasing when the radius is 5 cm?

A = area of circle r = radius t = time

Equation: $A = \pi r^2$ Given rate: $\frac{dr}{dt} = 4$ Find: $\left.\frac{dA}{dt}\right|_{r=5}$

$$\left.\frac{dA}{dt}\right|_{r=5} = 2\pi r \cdot \frac{dr}{dt} = 40\pi \text{ cm}^2\text{/min}$$

2) Oil spilling from a ruptured tanker spreads in a circle on the surface of the ocean. The area of the spill increases at a rate of 9π m²/min. How fast is the radius of the spill increasing when the radius is 10 m?

A = area of circle r = radius t = time

Equation: $A = \pi r^2$ Given rate: $\frac{dA}{dt} = 9\pi$ Find: $\left.\frac{dr}{dt}\right|_{r=10}$

$$\left.\frac{dr}{dt}\right|_{r=10} = \frac{1}{2\pi r} \cdot \frac{dA}{dt} = \frac{9}{20} \text{ m/min}$$

3) A conical paper cup is 10 cm tall with a radius of 10 cm. The cup is being filled with water so that the water level rises at a rate of 2 cm/sec. At what rate is water being poured into the cup when the water level is 8 cm?

V = volume of material in cone h = height t = time

Equation: $V = \frac{\pi h^3}{3}$ Given rate: $\frac{dh}{dt} = 2$ Find: $\left.\frac{dV}{dt}\right|_{h=8}$

$$\left.\frac{dV}{dt}\right|_{h=8} = \pi h^2 \cdot \frac{dh}{dt} = 128\pi \text{ cm}^3\text{/sec}$$

4) A spherical balloon is inflated so that its radius (r) increases at a rate of $\frac{2}{r}$ cm/sec. How fast is the volume of the balloon increasing when the radius is 4 cm?

V = volume of sphere r = radius t = time

Equation: $V = \frac{4}{3}\pi r^3$ Given rate: $\frac{dr}{dt} = \frac{2}{r}$ Find: $\left.\frac{dV}{dt}\right|_{r=4}$

$$\left.\frac{dV}{dt}\right|_{r=4} = 4\pi r^2 \cdot \frac{dr}{dt} = 32\pi \text{ cm}^3\text{/sec}$$

5) A 7 ft tall person is walking away from a 20 ft tall lamppost at a rate of 5 ft/sec. Assume the scenario can be modeled with right triangles. At what rate is the length of the person's shadow changing when the person is 16 ft from the lamppost?

x = distance from person to lamppost y = length of shadow t = time

Equation: $\frac{x+y}{20} = \frac{y}{7}$ Given rate: $\frac{dx}{dt} = 5$ Find: $\left.\frac{dy}{dt}\right|_{x=16}$

$$\left.\frac{dy}{dt}\right|_{x=16} = \frac{7}{13} \cdot \frac{dx}{dt} = \frac{35}{13} \text{ ft/sec}$$

6) An observer stands 700 ft away from a launch pad to observe a rocket launch. The rocket blasts off and maintains a velocity of 900 ft/sec. Assume the scenario can be modeled as a right triangle. How fast is the observer to rocket distance changing when the rocket is 2400 ft from the ground?

a = altitute of rocket z = distance from observer to rocket t = time

Equation: $a^2 + 490000 = z^2$ Given rate: $\frac{da}{dt} = 900$ Find: $\left.\frac{dz}{dt}\right|_{a=2400}$

$$\left.\frac{dz}{dt}\right|_{a=2400} = \frac{a}{z} \cdot \frac{da}{dt} = 864 \text{ ft/sec}$$