Chapter 1
Prerequisites for Calculus

Section 1.1 Lines (pp. 2–11)

Quick Review 1.1

1. $y = -2 + 4(3-3) = -2 + 4(0) = -2 + 0 = -2$

2. $$\begin{aligned} 3 &= 3 - 2(x+1) \\ 3 &= 3 - 2x - 2 \\ 2x &= -2 \\ x &= -1 \end{aligned}$$

3. $m = \dfrac{2-3}{5-4} = \dfrac{-1}{1} = -1$

4. $m = \dfrac{2-(-3)}{3-(-1)} = \dfrac{5}{4}$

5. (a) $3(2) - 4\left(\dfrac{1}{4}\right) \overset{?}{=} 5$
   $6 - 1 = 5$ Yes

   (b) $3(3) - 4(-1) \overset{?}{=} 5$
   $13 \neq 5$ No

6. (a) $7 \overset{?}{=} -2(-1) + 5$
   $7 = 2 + 5$ Yes

   (b) $1 = -2(-2) + 5$
   $1 \neq 9$ No

7. $$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(0-1)^2 + (1-0)^2} \\ &= \sqrt{2} \end{aligned}$$

8. $$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(1-2)^2 + \left(-\frac{1}{3} - 1\right)^2} \\ &= \sqrt{(-1)^2 + \left(-\frac{4}{3}\right)^2} \\ &= \sqrt{1 + \frac{16}{9}} \\ &= \sqrt{\frac{25}{9}} \\ &= \frac{5}{3} \end{aligned}$$

9. $$\begin{aligned} 4x - 3y &= 7 \\ -3y &= -4x + 7 \\ y &= \frac{4}{3}x - \frac{7}{3} \end{aligned}$$

10. $$\begin{aligned} -2x + 5y &= -3 \\ 5y &= 2x - 3 \\ y &= \frac{2}{5}x - \frac{3}{5} \end{aligned}$$

Section 1.1 Exercises

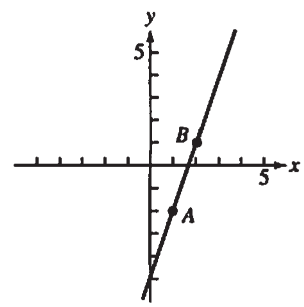
1. $\Delta x = -1 - 1 = -2$
   $\Delta y = -1 - 2 = -3$

2. $\Delta x = -1 - (-3) = 2$
   $\Delta y = -2 - 2 = -4$

3. $\Delta x = -8 - (-3) = -5$
   $\Delta y = 1 - 1 = 0$

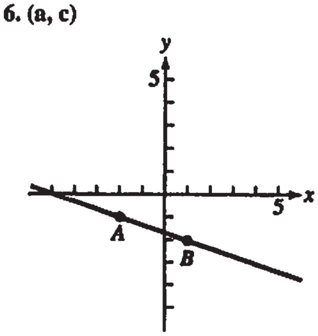
4. $\Delta x = 0 - 0 = 0$
   $\Delta y = -2 - 4 = -6$

5. (a, c)



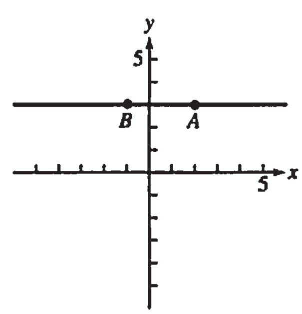
   (b) $m = \dfrac{1-(-2)}{2-1} = \dfrac{3}{1} = 3$

6. (a, c)



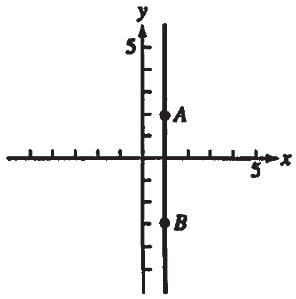
   (b) $m = \dfrac{-2-(-1)}{1-(-2)} = \dfrac{-1}{3} = -\dfrac{1}{3}$

7. (a, c)



   (b) $m = \dfrac{3-3}{-1-2} = \dfrac{0}{-3} = 0$

8. **(a, c)**



(b) $m = \frac{-3-2}{1-1} = \frac{-5}{0}$ (undefined)

This line has no slope.

9. **(a)** $x = 3$

(b) $y = 2$

10. **(a)** $x = -1$

(b) $y = \frac{4}{3}$

11. **(a)** $x = 0$

(b) $y = -\sqrt{2}$

12. **(a)** $x = -\pi$

(b) $y = 0$

13. $y = 1(x-1)+1$

14. $y = -1[x-(-1)]+1$
$y = -1(x+1)+1$

15. $y = 2(x-0)+3$

16. $y = -2[x-(-4)]+0$
$y = -2(x+4)+0$

17. $y = 3x-2$

18. $y = -1x+2$ or $y = -x+2$

19. $y = -\frac{1}{2}x-3$

20. $y = \frac{1}{3}x-1$

21. $m = \frac{3-0}{2-0} = \frac{3}{2}$

$y = \frac{3}{2}(x-0)+0$

$y = \frac{3}{2}x$

$2y = 3x$

$3x-2y = 0$

22. $m = \frac{1-1}{2-1} = \frac{0}{1} = 0$

$y = 0(x-1)+1$

$y = 1$

23. $m = \frac{-2-0}{-2-(-2)} = \frac{-2}{0}$ (undefined)

Vertical line: $x = -2$

24. $m = \frac{-2-1}{2-(-2)} = \frac{-3}{4} = -\frac{3}{4}$

$y = -\frac{3}{4}[x-(-2)]+1$

$4y = -3(x+2)+4$

$4y = -3x-2$

$3x+4y = -2$

25. The line contains (0, 0) and (10, 25).

$m = \frac{25-0}{10-0} = \frac{25}{10} = \frac{5}{2}$

$y = \frac{5}{2}x$

26. The line contains (0, 0) and (5, 2).

$m = \frac{2-0}{5-0} = \frac{2}{5}$

$y = \frac{2}{5}x$

27. $3x+4y = 12$

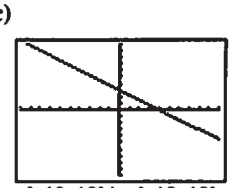
$4y = -3x+12$

$y = -\frac{3}{4}x+3$

(a) Slope: $-\frac{3}{4}$

(b) y-intercept: 3

(c)



[−10, 10] by [−10, 10]

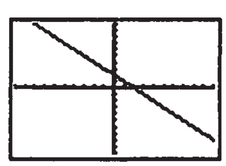
28. $x+y = 2$

$y = -x+2$

(a) Slope: −1

(b) y-intercept: 2

(c)



[−10, 10] by [−10, 10]

29. $\frac{x}{3}+\frac{y}{4}=1$

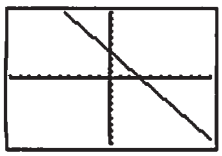
$\frac{y}{4}=-\frac{x}{3}+1$

$y=-\frac{4}{3}x+4$

(a) Slope: $-\frac{4}{3}$

(b) y-intercept: 4

(c)



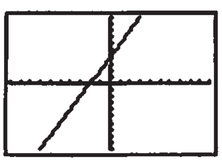
[−10, 10] by [−10, 10]

30. $y=2x+4$

(a) Slope: 2

(b) y-intercept: 4

(c)



[−10, 10] by [−10, 10]

31. (a) The desired line has slope −1 and passes through (0, 0):

$y=-1(x-0)+0$ or $y=-x$.

(b) The desired line has slope $\frac{-1}{-1}=1$ and passes through (0, 0):

$y=1(x-0)+0$ or $y=x$.

32. (a) The given equation is equivalent to $y=-2x+4$. The desired line has slope −2 and passes through (−2, 2):

$y=-2(x+2)+2$ or $y=-2x-2$.

(b) The desired line has slope $\frac{-1}{-2}=\frac{1}{2}$ and passes through (−2, 2):

$y=\frac{1}{2}(x+2)+2$ or $y=\frac{1}{2}x+3$.

33. (a) The given line is vertical, so we seek a vertical line through (−2, 4): $x=-2$.

(b) We seek a horizontal line through (−2, 4): $y=4$.

34. (a) The given line is horizontal, so we seek a horizontal line through $\left(-1,\ \frac{1}{2}\right)$: $y=\frac{1}{2}$.

(b) We seek a vertical line through $\left(-1,\ \frac{1}{2}\right)$: $x=-1$.

35. $m=\frac{9-2}{3-1}=\frac{7}{2}$

$f(x)=\frac{7}{2}(x-1)+2=\frac{7}{2}x-\frac{3}{2}$

Check: $f(5)=\frac{7}{2}(5)-\frac{3}{2}=16$, as expected.

Since $f(x)=\frac{7}{2}x-\frac{3}{2}$, we have $m=\frac{7}{2}$ and $b=-\frac{3}{2}$.

36. $m=\frac{-4-(-1)}{4-2}=\frac{-3}{2}=-\frac{3}{2}$

$f(x)=-\frac{3}{2}(x-2)+(-1)=-\frac{3}{2}x+2$

Check: $f(6)=-\frac{3}{2}(6)+2=-7$, as expected.

Since $f(x)=-\frac{3}{2}x+2$, we have $m=-\frac{3}{2}$ and $b=2$.

37. $-\frac{2}{3}=\frac{y-3}{4-(-2)}$

$-\frac{2}{3}(6)=y-3$

$-4=y-3$

$-1=y$

38. $2=\frac{2-(-2)}{x-(-8)}$

$2(x+8)=4$

$x+8=2$

$x=-6$

39. $y=1\cdot(x-3)+4$

$y=x-3+4$

$y=x+1$

This is the same as the equation obtained in Example 5.

40. (a) When $y=0$, we have $\frac{x}{c}=1$, so $x=c$.

When $x=0$, we have $\frac{y}{d}=1$, so $y=d$.

(b) When $y=0$, we have $\frac{x}{c}=2$, so $x=2c$.

When $x=0$, we have $\frac{y}{d}=2$, so $y=2d$.

The x-intercept is $2c$ and the y-intercept is $2d$.

41. **(a)** The given equations are equivalent to $y = -\frac{2}{k}x + \frac{3}{k}$ and $y = -x + 1$, respectively, so the slopes are $-\frac{2}{k}$ and -1. The lines are parallel when $-\frac{2}{k} = -1$, so $k = 2$.

(b) The lines are perpendicular when $-\frac{2}{k} = \frac{-1}{-1}$, so $k = -2$.

42. **(a)** $m \approx \frac{68-69.5}{0.4-0} = \frac{-1.5}{0.4} = -3.75$ degrees/inch

(b) $m \approx \frac{9-68}{4-0.4} = \frac{-59}{3.6} \approx -16.1$ degrees/inch

(c) $m \approx \frac{4-9}{4.7-4} = \frac{-5}{0.7} \approx -7.1$ degrees/inch

(d) Best insulator: Fiberglass insulation

Poorest insulator: Gypsum wallboard

The best insulator will have the largest temperature change per inch, because that will allow larger temperature differences on opposite sides of thinner layers.

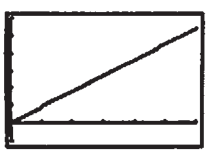
43. Slope: $k = \frac{\Delta p}{\Delta d} = \frac{10.94-1}{100-0} = \frac{9.94}{100}$
$= 0.0994$ atmospheres per meter

At 50 meters, the pressure is

$p = 0.0994(50) + 1 = 5.97$ atmospheres.

44. **(a)** $d(t) = 45t$

(b)



[0, 6] by [−50, 300]

(c) The slope is 45, which is the speed in miles per hour.

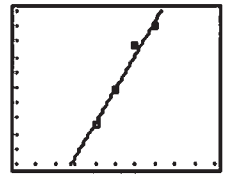
(d) Suppose the car has been traveling 45 mph for several hours when it is first observed at point P at time $t = 0$.

(e) The car starts at time $t = 0$ at a point 30 miles past P.

45. **(a)** $y = 1{,}060.4233x - 2{,}077{,}548.669$

(b) The slope is 1,060.4233. It represents the approximate rate of increase in earnings in dollars per year.

(c)



[1995, 2005] by [40000, 50000]

(d) When $x = 2000$,

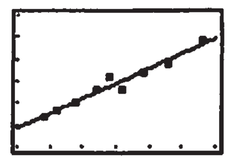
$y \approx 1{,}060.4233(2000) - 2{,}077{,}548.669 \approx 43{,}298$.

In 2000, the construction workers' average annual compensation will be about $43,298.

46. **(a)** $y = 0.680x + 9.013$

(b) The slope is 0.68. It represents the approximate average weight gain in pounds per month.

(c)



[15, 45] by [15, 45]

(d) When $x = 30$, $y \approx 0.680(30) + 9.013 = 29.413$.

She weighs about 29 pounds.

47. False: $m = \frac{\Delta y}{\Delta x}$ and $\Delta x = 0$, so it is undefined, or has no slope.

48. False: perpendicular lines satisfy the equation

$m_1 m_2 = -1$, or $m_1 = -\frac{1}{m_2}$

49. A: $y = m(x - x_1) + y_1$

$y = \frac{1}{2}(x+3) + 4$

or $y - 4 = \frac{1}{2}(x+3)$

50. E.

51. D: $y = 2x - 5$

$0 = 2x - 5$

$5 = 2x$

$x = \frac{5}{2}$

52. B: $y = -3x - 7$

$-1 = -3(-2) - 7$

$-1 = 6 - 7$

$-1 = -1$

53. (a) $y = 5632x - 11{,}080{,}280$

(b) The rate at which the median price is increasing in dollars per year

(c) $y = 2732x - 5{,}362{,}360$

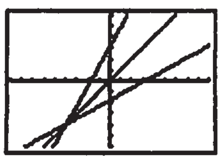
(d) The median price is increasing at a rate of about \$5632 per year in the Northeast, and about \$2732 per year in the Midwest. It is increasing more rapidly in the Northeast.

54. (a) Suppose x°F is the same as x°C.

$$x = \frac{9}{5}x + 32$$
$$\left(1 - \frac{9}{5}\right)x = 32$$
$$-\frac{4}{5}x = 32$$
$$x = -40$$

Yes, −40°F is the same as −40°C.

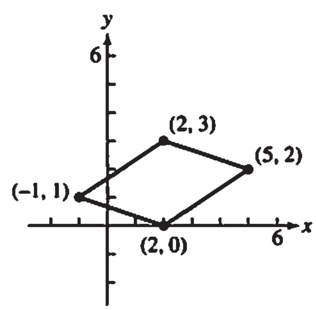
(b)

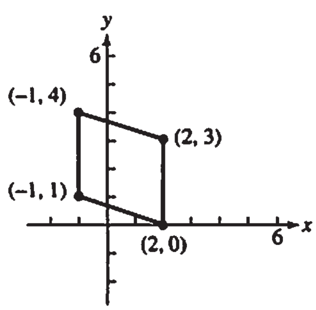


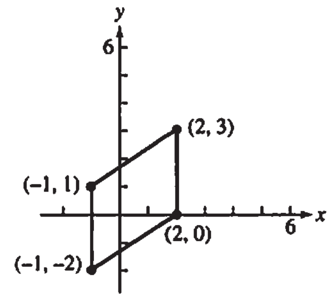
[−90, 90] by [−60, 60]

It is related because all three lines pass through the point(−40, −40) where the Fahrenheit and Celsius temperatures are the same.

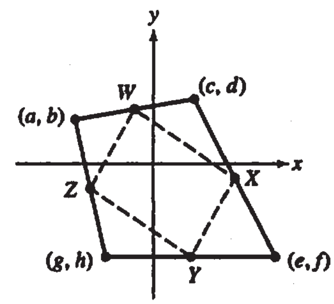
55. The coordinates of the three missing vertices are (5, 2), (−1, 4) and (−1, −2), as shown below.







56.



Suppose that the vertices of the given quadrilateral are (a, b), (c, d), (e, f), and (g, h). Then the midpoints of the consecutive sides are

$W\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$, $X\left(\frac{c+e}{2}, \frac{d+f}{2}\right)$, $Y\left(\frac{e+g}{2}, \frac{f+h}{2}\right)$, and $Z\left(\frac{g+a}{2}, \frac{h+b}{2}\right)$. When these four points are connected, the slopes of the sides of the resulting figure are:

$$WX: \frac{\frac{d+f}{2} - \frac{b+d}{2}}{\frac{c+e}{2} - \frac{a+c}{2}} = \frac{f-b}{e-a}$$

$$XY: \frac{\frac{f+h}{2} - \frac{d+f}{2}}{\frac{e+g}{2} - \frac{c+e}{2}} = \frac{h-d}{g-c}$$

$$ZY: \frac{\frac{f+h}{2} - \frac{h+b}{2}}{\frac{e+g}{2} - \frac{g+a}{2}} = \frac{f-b}{e-a}$$

$$WZ: \frac{\frac{h+b}{2} - \frac{b+d}{2}}{\frac{g+a}{2} - \frac{a+c}{2}} = \frac{h-d}{g-c}$$

Opposite sides have the same slope and are parallel.

57. The radius through (3, 4) has slope $\frac{4-0}{3-0}=\frac{4}{3}$.

The tangent line is tangent to this radius, so its slope is $\frac{-1}{4/3}=-\frac{3}{4}$. We seek the line of slope $-\frac{3}{4}$ that passes through (3, 4).

$$y=-\frac{3}{4}(x-3)+4$$
$$y=-\frac{3}{4}x+\frac{9}{4}+4$$
$$y=-\frac{3}{4}x+\frac{25}{4}$$

58. **(a)** The equation for line L can be written as $y=-\frac{A}{B}x+\frac{C}{B}$, so its slope is $-\frac{A}{B}$. The perpendicular line has slope $\frac{-1}{-A/B}=\frac{B}{A}$ and passes through (a, b), so its equation is $y=\frac{B}{A}(x-a)+b$.

(b) Substituting $\frac{B}{A}(x-a)+b$ for y in the equation for line L gives:

$$Ax+B\left[\frac{B}{A}(x-a)+b\right]=C$$
$$A^2x+B^2(x-a)+ABb=AC$$
$$(A^2+B^2)x=B^2a+AC-ABb$$
$$x=\frac{B^2a+AC-ABb}{A^2+B^2}$$

Substituting the expression for x in the equation for line L gives:

$$A\left(\frac{B^2a+AC-ABb}{A^2+B^2}\right)+By=C$$
$$By=\frac{-A(B^2a+AC-ABb)}{A^2+B^2}+\frac{C(A^2+B^2)}{A^2+B^2}$$
$$By=\frac{-AB^2a-A^2C+A^2Bb+A^2C+B^2C}{A^2+B^2}$$
$$By=\frac{A^2Bb+B^2C-AB^2a}{A^2+B^2}$$
$$y=\frac{A^2b+BC-ABa}{A^2+B^2}$$

The coordinates of Q are

$$\left(\frac{B^2a+AC-ABb}{A^2+B^2},\frac{A^2b+BC-ABa}{A^2+B^2}\right).$$

(c) Distance

$$=\sqrt{(x-a)^2+(y-b)^2}$$
$$=\sqrt{\left(\frac{B^2a+AC-ABb}{A^2+B^2}-a\right)^2+\left(\frac{A^2b+BC-ABa}{A^2+B^2}-b\right)^2}$$
$$=\sqrt{\left(\frac{B^2a+AC-ABb-a(A^2+B^2)}{A^2+B^2}\right)^2+\left(\frac{A^2b+BC-ABa-b(A^2+B^2)}{A^2+B^2}\right)^2}$$
$$=\sqrt{\left(\frac{AC-ABb-A^2a}{A^2+B^2}\right)^2+\left(\frac{BC-ABa-B^2b}{A^2+B^2}\right)^2}$$
$$=\sqrt{\left(\frac{A(C-Bb-Aa)}{A^2+B^2}\right)^2+\left(\frac{B(C-Aa-Bb)}{A^2+B^2}\right)^2}$$
$$=\sqrt{\frac{A^2(C-Aa-Bb)^2}{(A^2+B^2)^2}+\frac{B^2(C-Aa-Bb)^2}{(A^2+B^2)^2}}$$
$$=\sqrt{\frac{(A^2+B^2)(C-Aa-Bb)^2}{(A^2+B^2)^2}}$$
$$=\sqrt{\frac{(C-Aa-Bb)^2}{A^2+B^2}}$$
$$=\frac{|C-Aa-Bb|}{\sqrt{A^2+B^2}}$$
$$=\frac{|Aa+Bb-C|}{\sqrt{A^2+B^2}}$$

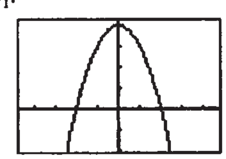
Section 1.2 Functions and Graphs (pp. 12–21)

Exploration 1 Composing Functions

1. $y_3=g\circ f$, $y_4=f\circ g$

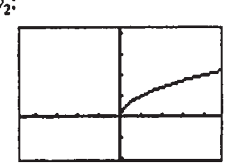
2. Domain of y_3: [−2, 2] Range of y_3: [0, 2]

y_1:



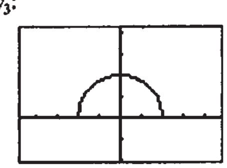
[−4.7, 4.7] by [−2, 4.2]

y_2:



[−4.7, 4.7] by [−2, 4.2]

y_3:



[−4.7, 4.7] by [−2, 4.2]