3.1 The Extreme Value Theorem

IN THE FOUR PROBLEMS BELOW, MATCH THE TABLE WITH THE GRAPH.

676.

x	$f'(x)$
a	0
b	0
c	5

C

677.

x	$f'(x)$
a	0
b	0
c	-5

B

678.

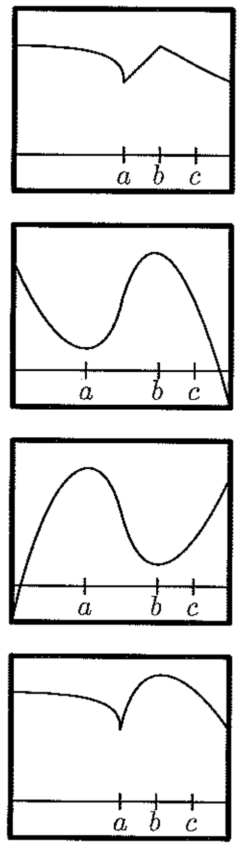
x	$f'(x)$
a	does not exist
b	0
c	-2

D

679.

x	$f'(x)$
a	does not exist
b	does not exist
c	-1.7

A



A) B) C) D)

680. Let $f(x) = (x-2)^{2/3}$.

a) Does $f'(2)$ exist? NO

b) Show that the only local extreme value of f occurs at $x = 2$.

c) Does the result in part (b) contradict the Extreme Value Theorem?

d) Repeat parts (a) and (b) for $f(x) = (x-k)^{2/3}$, replacing 2 with k.

$f'(x) = \frac{2}{3}(x-2)^{-1/3} = \frac{2}{3\sqrt[3]{x-2}}$

x	$f'(x)$
1	$-$
3	$+$

abs. min at $x=2$

abs. min at $x=k$

681. Let $f(x) = |x^3 - 9x|$.

$P(x) = x(x+3)(x-3)$

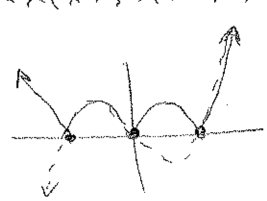
NO a) Does $f'(0)$ exist?

NO b) Does $f'(3)$ exist?

NO c) Does $f'(-3)$ exist?

d) Determine all extrema of f.

$(\pm 3, 0), (0,0)$
$(\pm\sqrt{3}, 6\sqrt{3})$

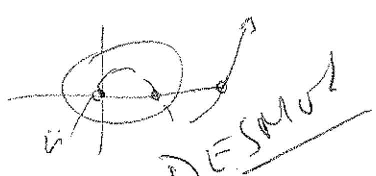


$f(x) = |x^3 - 9x| = \begin{cases} -(x^3-9x) & x < -3 \\ \boxed{x^3 - 9x} & -3 \le x < 0 \end{cases}$

682. The function $V(x) = x(10-2x)(16-2x)$ models the volume of a box. What is the domain of this function? What are the extreme values of V?

D: $x \mid 0 < x < 5$

extreme $x = 0$
$x = 144$



$f'(x) = 3x^2 - 9 = 0$
$3(x^2 - 3) = 0$
$x = \pm\sqrt{3}$

$f(\sqrt{3}) = |\sqrt{27} - 9\sqrt{3}|$
$= |3\sqrt{3} - 9\sqrt{3}| = |-6\sqrt{3}| = 6\sqrt{3}$

3.3 The First and Second Derivative Tests

FOR THE FOLLOWING, FIND: A) THE DOMAIN OF EACH FUNCTION, B) THE x-COORDINATE OF THE LOCAL EXTREMA, AND C) THE INTERVALS WHERE THE FUNCTION IS INCREASING AND/OR DECREASING.

701. $f(x) = \frac{1}{3}x^3 + \frac{5}{2}x^2 + 6x - 1$ ↑ $x<-3$, $x>2$ ↓ $-3<x<2$

702. $g(x) = x^3 - 5x^2 - 8x$ ↑ $x<-2/3$ $x>4$ ↓ $-2/3<x<4$

703. $h(x) = x + \dfrac{4}{x}$ ↑ $x<-2$, $x>2$ ↓ $-2<x<2$, $x \neq 0$

704. $p(x) = \sqrt[3]{x} + \dfrac{1}{\sqrt[3]{x}}$ ↑ $x<-1$ a $x>1$ ↓ $-1<x<1$, $x \neq 0$

705. $h(x) = (2-x)^2(x+3)^3$ ↑ $x<0$, $x \neq -3$, $x>2$ ↓ $0<x<2$

706. $m(x) = 3x\sqrt{5-x}$ ↑ $x<10/3$ ↓ $10/3<x<5$

707. $f(x) = x^{2/3}(x-5)^{-1/3}$ ↑ $x<0$, $x>10$ ↓ $0<x<10$, $x \neq 5$

708. $h(x) = \frac{1}{7}x^{7/3} - x^{4/3}$

709. Find the values of a and b so that the function $f(x) = \frac{1}{3}x^3 + ax^2 + bx$ will have a relative extreme at $(3, 1)$. $a = -19/9$ $b = 11/3$

710. Find the values of a, b, c, and d so that the function $f(x) = ax^3 + bx^2 + cx + d$ will have relative extrema at $(-1, 1)$ and $(-2, 4)$. $a = 6$, $b = 27$, $c = 36$, $d = 16$

IN THE FOLLOWING PROBLEMS, FIND A) THE COORDINATES OF INFLECTION POINTS AND B) THE INTERVALS WHERE THE FUNCTION IS CONCAVE UP AND/OR CONCAVE DOWN.

711. $g(x) = x^3 - 5x$ $(0,0)$ ccup $x>0$, ccd $x<0$

712. $h(x) = 2x^3 - 3x^2 - 8x + 1$ $(1/2,$

713. $h(x) = (3x+2)^3$

714. $p(x) = \dfrac{3}{x^2+4}$ $\left(\pm\frac{2}{\sqrt{3}}, \frac{9}{16}\right)$ ccup $x< -2/\sqrt{3}$ & $x>\frac{2}{\sqrt{3}}$; ccdn $-\frac{2}{\sqrt{3}}<x<\frac{2}{\sqrt{3}}$

715. $f(x) = \begin{cases} x^2 - 3 & x > 3 \\ 15 - x^2 & x \leq 3 \end{cases}$ $(3,6)$ ccup $x>3$, ccdn $x<3$

716. $p(x) = \begin{cases} 2x^2 & x \geq 0 \\ -2x^2 & x < 0 \end{cases}$

717. Determine the values of a and b so that the function $p(x) = ax^4 + bx^3$ will have a point of inflection at $(-1, 3)$. $a = -3$, $b = -6$

718. Determine the values of a, b, and c so that the function $p(x) = ax^3 + bx^2 + cx$ will have an inflection point at $(-1, 3)$ and the slope of the tangent at $(-1, 3)$ will be -2.

$a = -1$, $b = -3$, $c = -5$

The calculus is the greatest aid we have to the application of physical truth in the broadest sense of the word. —*W. F. Osgood*