13. Consider the rational function $y=\frac{3 x-1}{x+2}$.
a) Graph the function.
b) State the domain, range, and any intercepts of the graph of the function.
c) Determine the root(s) of the equation $0=\frac{3 x-1}{x+2}$.
d) How are the answer(s) to part c) related to part of the answer to part b)?
14. Predict the locations of any vertical asymptotes, points of discontinuity, and intercepts for each function, giving a reason for each feature. Then, graph the function to verify your predictions.
a) $f(x)=\frac{x-4}{x^{2}-2 x-8}$
b) $f(x)=\frac{x^{2}+x-6}{x^{2}+2 x-3}$
c) $f(x)=\frac{x^{2}-5 x}{x^{2}-2 x-3}$
15. For the graphs of $f(x)$ and $g(x)$, determine the equation and graph of each combined function.

Then, state its domain and range.

a) $y=(f+g)(x)$
b) $y=(f-g)(x)$
c) $y=\left(\frac{f}{g}\right)(x)$
d) $y=(f \cdot g)(x)$
16. If $f(x)=x-3$ and $g(x)=\sqrt{x-1}$, determine each combined function, $h(x)$, and state its domain.
a) $h(x)=f(x)+g(x)$
b) $h(x)=f(x)-g(x)$
c) $h(x)=\left(\frac{f}{g}\right)(x)$
d) $h(x)=(f \cdot g)(x)$
17. For $f(x)=x^{2}-3$ and $g(x)=|x|$, determine the following.
a) $f(g(2))$
b) $(f \circ g)(-2)$
c) $f(g(x))$
d) $(g \circ f)(x)$
18. If $h(x)=f(g(x))$, determine $f(x)$ and $g(x)$ for each of the following to be true.
a) $h(x)=2^{3 x+2}$
b) $h(x)=\sqrt{\sin x+2}$
19. Solve for $n$.
a) $\frac{n!}{(n-2)!}=420$
b) ${ }_{n} C_{2}=78$
c) ${ }_{n} C_{n-2}=45$
20. Liz arranged the letters ABCD without repeating the letters.
a) How many arrangements are possible?
b) If the letters may be repeated, how many more four-letter arrangements are possible?
c) Compared to your answer in part a), are there more ways to arrange four letters if two are the same, for example, ABCC? Explain.
21. A student council decides to form a sub-committee of five council members. There are four boys and five girls on council.
a) How many different ways can the sub-committee be selected with exactly three girls?
b) How many different ways can the sub-committee be selected with at least three girls?
22. One term of $(3 x+a)^{7}$ is $81648 x^{5}$. Determine the possible value(s) of $a$.

## Answers

## Chapter 1 Function Transformations

### 1.1 Horizontal and Vertical Translations, pages 12 to 15

1. a) $h=0, k=5 \quad$ b) $h=0, k=-4 \quad$ c) $h=-1, k=0$
d) $h=7, k=-3 \quad$ e) $h=-2, k=4$
2. a)
a) $\mathrm{A}^{\prime}(-4,1), \mathrm{B}^{\prime}(-3,4)$
b) $\mathrm{A}^{\prime}(-2,-2)$, $C^{\prime}(-1,4), D^{\prime}(1,2)$, $\mathrm{E}^{\prime}(2,2)$

c) $\mathrm{A}^{\prime}(-8,-2), \mathrm{B}^{\prime}(-7,1)$, $\mathrm{C}^{\prime}(-5,1), \mathrm{D}^{\prime}(-3,-1)$, $\mathrm{E}^{\prime}(-2,-1)$

3. a) $(x, y) \rightarrow(x-10, y)$
c) $(x, y) \rightarrow(x+7, y+4)$ d)
4. a)

d) $\mathrm{A}^{\prime}(-4,-4), \mathrm{B}^{\prime}(-3,-1)$, $\mathrm{C}^{\prime}(-1,-1), \mathrm{D}^{\prime}(1,-3)$, $\mathrm{E}^{\prime}(2,-3)$

$(x, y) \rightarrow(x, y-6)$
$(x, y) \rightarrow(x+1, y+3)$ a vertical translation of 3 units down and a horizontal translation of 4 units left;
$(x, y) \rightarrow(x-4, y-3)$
b)

c)

d)
 a vertical translation of 4 units down and a horizontal translation of 2 units right;
$(x, y) \rightarrow(x+2, y-4)$
a vertical translation of 5 units up and a horizontal translation of 2 units right;
$(x, y) \rightarrow(x+2, y+5)$
a vertical translation of 2 units up and a horizontal translation of 3 units left;
$(x, y) \rightarrow(x-3, y+2)$
5. a) $h=-5, k=4 ; y-4=f(x+5)$
b) $h=8, k=6 ; y-6=f(x-8)$
c) $h=10, k=-8 ; y+8=f(x-10)$
d) $h=-7, k=-12 ; y+12=f(x+7)$
6. It has been translated 3 units up.
7. It has been translated 1 unit right.
8. 

| Translation | Transformed <br> Function | Transformation of <br> Points |
| :--- | :---: | :---: |
| vertical | $y=f(x)+5$ | $(x, y) \rightarrow(x, y+5)$ |
| horizontal | $y=f(x+7)$ | $(x, y) \rightarrow(x-7, y)$ |
| horizontal | $y=f(x-3)$ | $(x, y) \rightarrow(x+3, y)$ |
| vertical | $y=f(x)-6$ | $(x, y) \rightarrow(x, y-6)$ |
| horizontal and vertical | $y+9=f(x+4)$ | $(x, y) \rightarrow(x-4, y-9)$ |
| horizontal and vertical | $y=f(x-4)-6$ | $(x, y) \rightarrow(x+4, y-6)$ |
| horizontal and vertical | $y=f(x+2)+3$ | $(x, y) \rightarrow(x-2, y+3)$ |
| horizontal and vertical | $y=f(x-h)+k$ | $(x, y) \rightarrow(x+h, y+k)$ |

9. a) $y=(x+4)^{2}+5 \quad$ b) $\{x \mid x \in R\},\{y \mid y \geq 5, y \in R\}$
c) To determine the image function's domain and range, add the horizontal and vertical translations to the domain and range of the base function. Since the domain is the set of real numbers, nothing changes, but the range does change.
10. a) $g(x)=|x-9|+5$
b) The new graph is a vertical and horizontal translation of the original by 5 units up and 9 units right.
c) Example: $(0,0),(1,1),(2,2) \rightarrow(9,5),(10,6),(11,7)$
d) Example: $(0,0),(1,1),(2,2) \rightarrow(9,5),(10,6),(11,7)$
e) The coordinates of the image points from parts c) and d) are the same. The order that the translations are made does not matter.
11. a) $y=f(x-3)$
b) $y+5=f(x-6)$
12. a) Example: It takes her 2 h to cycle to the lake, 25 km away. She rests at the lake for 2 h and then returns home in 3 h .
b) This translation shows what would happen if she left the house at a later time.
c) $y=f(x-3)$
13. a) Example: Translated 8 units right.
b) Example: $y=f(x-8), y=f(x-4)+3.5$, $y=f(x+4)+3.5$
14. a) Example: A repeating X by using two linear equations $y= \pm x$.
b) Example: $y=f(x-3)$. The translation is horizontal by 3 units right.
15. a) The transformed function starts with a higher number of trout in 1970. $y=f(t)+2$
b) The transformed function starts in 1974 instead of 1971. $y=f(t-3)$
16. The first case, $n=f(A)+10$, represents the number of gallons he needs for a given area plus 10 more gallons. The second case, $n=f(A+10)$, represents how many gallons he needs to cover an area $A$ less 10 units of area.
17. a) $y=(x-7)(x-1)$ or $y=(x-4)^{2}-9$
b) Horizontal translation of 4 units right and vertical translation of 9 units down.
c) $y$-intercept 7
18. a) The original function is 4 units lower.
b) The original function is 2 units to the right.
c) The original function is 3 units lower and 5 units left.
d) The original function is 4 units higher and 3 units right.
19. a) The new graph will be translated 2 units right and 3 units down.
b)


C1 a) $y=f(x) \rightarrow y=f(x-h) \rightarrow y=f(x-h)+k$. Looking at the problem in small steps, it is easy to see that it does not matter which way the translations are done since they do not affect the other translation.
b) The domain is shifted by $h$ and the range is shifted by $k$.
C2 a) $f(x)=(x+1)^{2}$; horizontal translation of 1 unit left
b) $g(x)=(x-2)^{2}-1$; horizontal translation of 2 units right and 1 unit down
C3 The roots are 2 and 9 .
C4 The 4 can be taken as $h$ or $k$ in this problem. If it is $h$ then it is -4 , which makes it in the left direction.
1.2 Reflections and Stretches, pages 28 to 31

1. a)

| $x$ | $f(x)=2 x+1$ | $g(x)=-f(x)$ | $h(x)=f(-x)$ |
| ---: | :---: | :---: | :---: |
| -4 | -7 | 7 | 9 |
| -2 | -3 | 3 | 5 |
| 0 | 1 | -1 | 1 |
| 2 | 5 | -5 | -3 |
| 4 | 9 | -9 | -7 |

b)

c) The $y$-coordinates of $g(x)$ have changed sign. The invariant point is $(-0.5,0)$. The $x$-coordinates of $h(x)$ have changed sign. The invariant point is $(0,1)$.
d) The graph of $g(x)$ is the reflection of the graph of $f(x)$ in the $x$-axis, while the graph of $h(x)$ is the reflection of the graph of $f(x)$ in the $y$-axis.
2. a)

| $x$ | $f(x)=x^{2}$ | $g(x)=3 f(x)$ | $h(x)=\frac{1}{3} f(x)$ |
| ---: | :---: | :---: | :---: |
| -6 | 36 | 108 | 12 |
| -3 | 9 | 27 | 3 |
| 0 | 0 | 0 | 0 |
| 3 | 9 | 27 | 3 |
| 6 | 36 | 108 | 12 |

b)

c) The $y$-coordinates of $g(x)$ are three times larger. The invariant point is $(0,0)$. The $y$-coordinates of $h(x)$ are three times smaller. The invariant point is $(0,0)$.
d) The graph of $g(x)$ is a vertical stretch by a factor of 3 of the graph of $f(x)$, while the graph of $h(x)$ is a vertical stretch by a factor of $\frac{1}{3}$ of the graph of $f(x)$.
3. a)

$g(x)=-3 x$
$f(x):$ domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$ $g(x)$ : domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$
b)

c)

4. a)

b)

c)

$h(x)=-x^{2}-1$
$g(x)$ : domain $\{x \mid x \in R\}$, range $\{y \mid y \geq 1, y \in R\}$ $h(x)$ : domain $\{x \mid x \in R\}$, range $\{y \mid y \leq-1, y \in R\}$
$k(x)=-\frac{1}{x}$
$h(x)$ : domain
$\{x \mid x \neq 0, x \in R\}$,
range $\{y \mid y \neq 0, y \in R\}$
$k(x)$ : domain
$\{x \mid x \neq 0, x \in \mathrm{R}\}$,
range $\{y \mid y \neq 0, y \in R\}$
$g(x)=-3 x$
$f(x)$ : domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in R\}$
$g(x):$ domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$
$h(x)=x^{2}+1$
$g(x)$ : domain $\{x \mid x \in R\}$,
range $\{y \mid y \geq 1, y \in R\}$ $h(x)$ : domain $\{x \mid x \in R\}$, range $\{y \mid y \geq 1, y \in R\}$
$k(x)=-\frac{1}{x}$
$h(x)$ : domain
$\{x \mid x \neq 0, x \in \mathrm{R}\}$,
range $\{y \mid y \neq 0, y \in R\}$
$k(x)$ : domain
$\{x \mid x \neq 0, x \in R\}$,
range $\{y \mid y \neq 0, y \in R\}$

Answers • MHR 555
5. a) The graph of $y=4 f(x)$ is a vertical stretch by a factor of 4 of the graph of $y=f(x) .(x, y) \rightarrow(x, 4 y)$
b) The graph of $y=f(3 x)$ is a horizontal stretch by a factor of $\frac{1}{3}$ of the graph of $y=f(x) .(x, y) \rightarrow\left(\frac{x}{3}, y\right)$
c) The graph of $y=-f(x)$ is a reflection in the $x$-axis of the graph of $y=f(x) .(x, y) \rightarrow(x,-y)$
d) The graph of $y=f(-x)$ is a reflection in the $y$-axis of the graph of $y=f(x) .(x, y) \rightarrow(-x, y)$
6. a) domain $\{x \mid-6 \leq x \leq 6, x \in \mathrm{R}\}$, range $\{y \mid-8 \leq y \leq 8, y \in R\}$
b) The vertical stretch affects the range by increasing it by the stretch factor of 2 .
7. a) The graph of $g(x)$ is a vertical stretch by a factor of 4 of the graph of $f(x) . y=4 f(x)$
b) The graph of $g(x)$ is a reflection in the $x$-axis of the graph of $f(x) \cdot y=-f(x)$
c) The graph of $g(x)$ is a horizontal stretch by a factor of $\frac{1}{3}$ of the graph of $f(x) . y=f(3 x)$
d) The graph of $g(x)$ is a reflection in the $y$-axis of the graph of $f(x) . y=f(-x)$
8.

9. a) horizontally stretched by a factor of $\frac{1}{4}$
b) horizontally stretched by a factor of 4
c) vertically stretched by a factor of $\frac{1}{2}$
d) vertically stretched by a factor of 4
e) horizontally stretched by a factor of $\frac{1}{3}$ and reflected in the $y$-axis
f) vertically stretched by a factor of 3 and reflected in the $x$-axis
10. a)

c) They are both incorrect. It does not matter in which order you proceed.
11. a)

b) Both the functions are reflections of the base function in the $t$-axis. The object falling on Earth is stretched vertically more than the object falling on the moon.
12. Example: When the graph of $y=f(x)$ is transformed to the graph of $y=f(b x)$, it undergoes a horizontal stretch about the $y$-axis by a factor of $\frac{1}{|b|}$ and only the $x$-coordinates are affected. When the graph of $y=f(x)$ is transformed to the graph of $y=a f(x)$, it undergoes a vertical stretch about the $x$-axis by a factor of $|a|$ and only the $y$-coordinates are affected.
13. a)

b) As the drag factor decreases, the length of the skid mark increases for the same speed.

14. a) $x=-4, x=3$
b) $x=4, x=-3$
c) $x=-8, x=6$
d) $x=-2, x=1.5$
15. a) I

I b) III
c) IV
d) IV
16. a)

b)


C1 Example: When the input values for $g(x)$ are $b$ times the input values for $f(x)$, the scale factor must be $\frac{1}{b}$ for the same output values. $g(x)=f\left(\frac{1}{b}(b x)\right)=f(x)^{\text {b }}$
C2 Examples:
a) a vertical stretch or a reflection in the $x$-axis
b) a horizontal stretch or a reflection in the $y$-axis

C3

C3 \begin{tabular}{|c|c|l|}
\hline $\boldsymbol{f}(\boldsymbol{x})$ \& $\boldsymbol{g}(\boldsymbol{x})$ \& \multicolumn{1}{|c|}{ Transformation } <br>
\hline$(5,6)$ \& $(5,-6)$ \& reflection in the $x$-axis <br>
\hline$(4,8)$ \& $(-4,8)$ \& reflection in the $y$-axis <br>
\hline$(2,3)$ \& $(2,12)$ \& vertical stretch by a factor of 4 <br>

\hline$(4,-12)$ \& $(2,-6)$ \& | horizontal stretch by a factor of $\frac{1}{2}$ |
| :--- |
| and vertical stretch by a factor of $\frac{1}{2}$ | <br>

\hline
\end{tabular}



C5 a) $t_{n}=4 n-14$
b) $t_{n}=-4 n+14$
c) They are reflections of each other in the $x$-axis.

### 1.3 Combining Transformations, pages 38 to 43

1. a) $y=-f\left(\frac{1}{2} x\right)$ or $y=-\frac{1}{4} x^{2}$
b) $y=\frac{1}{4} f(-4 x)$ or $y=4 x^{2}$
2. The function $f(x)$ is transformed to the function $g(x)$ by a horizontal stretch about the $y$-axis by a factor of $\frac{1}{4}$. It is vertically stretched about the $x$-axis by a factor of 3 . It is reflected in the $x$-axis, and then translated 4 units right and 10 units down.
3. 

| Function |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $y-4=f(x-5)$ | none | none | none | 4 | 5 |
| $y+5=2 f(3 x)$ | none | 2 | $\frac{1}{3}$ | -5 | none |
| $y=\frac{1}{2} f\left(\frac{1}{2}(x-4)\right)$ | none | $\frac{1}{2}$ | 2 | none | 4 |
| $y+2=-3 f(2(x+2))$ | $x$-axis | 3 | $\frac{1}{2}$ | -2 | -2 |

4. a) $y=f(-(x+2))-2$
b) $y=f(2(x+1))-4$
5. a)

b)

6. a) $(-8,12)$
b) $(-4,72)$
c) $(-6,-32)$
d) $(9,-32)$
e) $(-12,-9)$
7. a) vertical stretch by a factor of 2 and translation of 3 units right and 4 units up;
$(x, y) \rightarrow(x+3,2 y+4)$
b) horizontal stretch by a factor of $\frac{1}{3}$, reflection in the $x$-axis, and translation of 2 units down;
$(x, y) \rightarrow\left(\frac{1}{3} x,-y-2\right)$
c) reflection in the $y$-axis, reflection in the $x$-axis, vertical stretch by a factor of $\frac{1}{4}$, and translation of 2 units left; $(x, y) \rightarrow\left(-x-2,-\frac{1}{4} y\right)$
d) horizontal stretch by a factor of $\frac{1}{4}$, reflection in the $x$-axis, and translation of 2 units right and 3 units up; $(x, y) \rightarrow\left(\frac{1}{4} x+2,-y+3\right)$
e) reflection in the $y$-axis, horizontal stretch by a factor of $\frac{4}{3}$, reflection in the $x$-axis, and vertical stretch by a factor of $\frac{2}{3} ;(x, y) \rightarrow\left(-\frac{4}{3} x,-\frac{2}{3} y\right)$
f) reflection in the $y$-axis, horizontal stretch by a factor of $\frac{1}{2}$, vertical stretch by a factor of $\frac{1}{3}$, and translation of 6 units right and 2 units up;
$(x, y) \rightarrow\left(-\frac{1}{2} x+6, \frac{1}{3} y+2\right)$
8. a) $y+5=-3 f(x+4) \quad$ b) $y-2=-\frac{3}{4} f(-3(x-6))$
9. a)

b)

c)

d)

e)

f)

10. a) $y=-3 f(x-8)+10 \quad$ b) $y=-2 f(x-3)+2$
c) $y=-\frac{1}{2} f(-2(x+4))+7$
11. a)

b)

c)

12. a) $A^{\prime}(-11,-2), B^{\prime}(-7,6), C^{\prime}(-3,4), D^{\prime}(-1,5), E^{\prime}(3,-2)$
b) $y=-f\left(\frac{1}{2}(x+3)\right)+4$
13. a) The graphs are in two locations because the transformations performed to obtain Graph 2 do not match those in $y=|2 x-6|+2$. Gil forgot to factor out the coefficient of the $x$-term, 2 , from -6 . The horizontal translation should have been 3 units right, not 6 units.
b) He should have rewritten the function as $y=|2(x-3)|+2$.
14. a)

b) $y=-\left(\frac{1}{2}(x+6)\right)^{2}+6$
15. a) $(-a, 0),(0,-b)$
b) $(2 a, 0),(0,2 b)$
c) and d) There is not enough information to determine the locations of the new intercepts. When a transformation involves translations, the locations of the new intercepts will vary with different base functions.
16. a) $A=-2 x^{3}+18 x$
c) For $(2,5)$, the area of the rectangle in part a) is 20 square units.
$A=-2 x^{3}+18 x$
b) $A=-\frac{1}{8} x^{3}+18 x$

For ( 8,5 ), the area of the rectangle in part b) is 80 square units.
$A=-\frac{1}{8} x^{3}+18 x$
$A=-\frac{1}{8}(8)^{3}+18(8)$
$A=-2(2)^{3}+18(2)$
$A=20$
$A=80$
17. $y=36(x-2)^{2}+6(x-2)-2$
18. Example: vertical stretches and horizontal stretches followed by reflections
C1 Step 1 They are reflections in the axes.
1: $y=x+3,2: y=-x-3,3: y=x-3$
Step 2 They are vertical translations coupled with
reflections. 1: $y=x^{2}+1,2: y=x^{2}-1,3: y=-x^{2}, 4$ :
$y=-x^{2}-1$
C2 a) The cost of making $b+12$ bracelets, and it is a horizontal translation.
b) The cost of making $b$ bracelets plus 12 more dollars, and it is a vertical translation.
c) Triple the cost of making $b$ bracelets, and it is a vertical stretch.
d) The cost of making $\frac{b}{2}$ bracelets, and it is a horizontal stretch.
C3 $y=2(x-3)^{2}+1$; a vertical stretch by a factor of 2 and a translation of 3 units right and 1 unit up
C4 a) H is repeated; J is transposed; K is repeated and transposed
b) H is in retrograde; J is inverted; K is in retrograde and inverted
c) H is inverted, repeated, and transposed; J is in retrograde inversion and repeated; K is in retrograde and transposed
1.4 Inverse of a Relation, pages 51 to 55

1. a)

b)

2. a)

b)

3. a) The graph is a function but the inverse will be a relation.
b) The graph and its inverse are functions.
c) The graph and its inverse are relations.
4. Examples:
a) $\{x \mid x \geq 0, x \in \mathrm{R}\}$ or $\{x \mid x \leq 0, x \in \mathrm{R}\}$
b) $\{x \mid x \geq-2, x \in \mathrm{R}\}$ or $\{x \mid x \leq-2, x \in \mathrm{R}\}$
c) $\{x \mid x \geq 4, x \in \mathrm{R}\}$ or $\{x \mid x \leq 4, x \in \mathrm{R}\}$
d) $\{x \mid x \geq-4, x \in R\}$ or $\{x \mid x \leq-4, x \in R\}$
5. a) $f^{-1}(x)=\frac{1}{7} x$
b) $f^{-1}(x)=-\frac{1}{3}(x-4)$
c) $f^{-1}(x)=3 x-4$
d) $f^{-1}(x)=3 x+15$
e) $f^{-1}(x)=-\frac{1}{2}(x-5)$
f) $f^{-1}(x)=2 x-6$
6. a)
b) C
c)
d) A
e) D
7. a)

function: domain $\{-2,-1,0,1,2\}$,
range $\{-2,1,4,7,10\}$
inverse: domain $\{-2,1,4,7,10\}$,
range $\{-2,-1,0,1,2\}$
b)

function: domain $\{-6,-4,-1,2,5\}$, range $\{2,3,4,5\}$ inverse: domain $\{2,3,4,5\}$, range $\{-6,-4,-1,2,5\}$
8. a)


The inverse is a function; it passes the vertical line test.
b)


The inverse is not a function; it does not pass the vertical line test.
c)


The inverse is not a function; it does not pass the vertical line test.
9. a) $f^{-1}(x)=\frac{1}{3}(x-2)$

$f(x)$ :
domain $\{x \mid x \in R\}$,
range $\{y \mid y \in R\}$
$f^{-1}(x):$
domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$
b) $f^{-1}(x)=\frac{1}{2}(-x+4)$

$f(x)$ :
domain $\{x \mid x \in \mathrm{R}\}$,
range $\{y \mid y \in R\}$
$f^{-1}(x)$ :
domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in R\}$
c)

$f(x)$ : domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$ $f^{-1}(x)$ : domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$
d) $f^{-1}(x)=-\sqrt{x-2}$

$f(x)$ : domain
$\{x \mid x \leq 0, x \in R\}$, range
$\{y \mid y \geq 2, y \in R\}$ $f^{-1}(x)$ : domain
$\{x \mid x \geq 2, x \in R\}$,
range
$\{y \mid y \leq 0, y \in R\}$
e) $f^{-1}(x)=\sqrt{2-x}$

$f(x)$ : domain
$\{x \mid x \geq 0, x \in \mathrm{R}\}$,
range
$\{y \mid y \leq 2, y \in R\}$
$f^{-1}(x)$ : domain
$\{x \mid x \leq 2, x \in R\}$,
range
$\{y \mid y \geq 0, y \in R\}$
10. a) i) $f(x)=(x+4)^{2}-4$, inverse of $f(x)= \pm \sqrt{x+4}-4$
ii)

b) i) $y=(x-2)^{2}-2, y= \pm \sqrt{x+2}+2$
ii)

11. Yes, the graphs are reflections of each other in the line $y=x$.
12. a) $y= \pm \sqrt{x-3} \quad$ restricted domain $\{x \mid x \geq 0, x \in R\}$


b) $y= \pm \sqrt{2 x} \quad$ restricted domain $\{x \mid x \geq 0, x \in R\}$


c) $y= \pm \sqrt{-\frac{1}{2} x} \quad$ restricted domain $\{x \mid x \geq 0, x \in R\}$

d) $y= \pm \sqrt{x}-1$ restricted domain $\{x \mid x \geq-1, x \in R\}$

e) $y= \pm \sqrt{-x}+3$ restricted domain $\{x \mid x \geq 3, x \in R\}$

f) $y= \pm \sqrt{x+2}+1$

restricted domain $\{x \mid x \geq 1, x \in R\}$

13. a) inverses b) inverses c) not inverses d) inverses e) not inverses
14. Examples:
a) $x \geq 0$ or $x \leq 0$
b) $x \geq 0$ or $x \leq 0$
c) $x \geq 3$ or $x \leq 3$
d) $x \geq-2$ or $x \leq-2$
15. a) $\frac{3}{2}$
b) 0
c) $\begin{array}{ll}\frac{5}{2} & \text { d) } \frac{1}{2}\end{array}$
16. a) approximately $32.22{ }^{\circ} \mathrm{C}$
b) $y=\frac{9}{5} x+32$; $x$ represents temperatures in degrees Celsius and y represents temperatures in degrees Fahrenheit
c) $89.6{ }^{\circ} \mathrm{F}$
d)


The temperature is the same in both scales $\left(-40^{\circ} \mathrm{C}=-40^{\circ} \mathrm{F}\right)$.
17. a) male height $=171.02 \mathrm{~cm}$, female height $=166.44 \mathrm{~cm}$ b) i) male femur $=52.75 \mathrm{~cm}$

$$
\text { ii) female femur }=49.04 \mathrm{~cm}
$$

18. a) 5
b) $y=2.55 x+36.5 ; y$ is finger circumference and $x$ is ring size
c) $51.8 \mathrm{~mm}, 54.35 \mathrm{~mm}, 59.45 \mathrm{~mm}$
19. Examples:

b) $\quad$ i) $4 \leq x \leq 8$

20. a) 17
b) $\sqrt{3}$

ii) $-10 \leq x \leq-6$

c) 10
21. a) $(6,10) \quad$ b) $(8,23) \quad$ c) $(-8,-9)$

C1 a) Subtract 12 and divide by 6 .
b) Add 1, take the positive and negative square root, subtract 3.
C2 a)

b) Example: The graph of the original linear function is perpendicular to $y=x$, thus after a reflection the graph of the inverse is the same.
c) They are perpendicular to the line.

C3 Example: If the original function passes the vertical line test, then it is a function. If the original function passes the horizontal line test, then the inverse is a function.

## C4 Step 1

$f(x):(1,2),(4,3),(-8,-1)$, and $\left(a, \frac{a+5}{3}\right)$;
$g(x):(2,1),(3,4),(-1,-8)$, and $\left(\frac{a+5}{3}, a\right)$
The output values for $g(x)$ are the same as the input values for $f(x)$.
Example: Since the functions are inverses of each other, giving one of them a value and then taking the inverse will always return the initial value. A good way to determine if functions are inverses is to see if this effect takes place.
Step 2 The order in which you apply the functions does not change the final result.
Step 4 The statement is saying that if you have a function that when given $a$ outputs $b$ and another that when given $b$ outputs $a$, then the functions are inverses of each other.

Chapter 1 Review, pages 56 to 57

1. a)

b)

c)

2. Translation of 4 units left and 5 units down:

$$
y+5=|x+4|
$$

3. range $\{y \mid 2 \leq y \leq 9, y \in R\}$
4. No, it should be $(a+5, b-4)$.
5. a) $x$-axis, $(3,-5) \quad$ b) $y$-axis, $(-3,5)$
6. a)

$f(-x)$ : domain $\{x \mid x \in R\}$, range $\{y \mid y \geq 1, y \in R\}$ $(0,10)$
b)

$-f(x)$ : domain
$\{x \mid-1 \leq x \leq 5, x \in R\}$, range $\{y \mid 0 \leq y \leq 3, y \in R\}$ $(5,0),(-1,0)$
7. a)

b) If the coefficient is greater than 1 , then the function moves closer to the $y$-axis. The opposite is true for when the coefficient is between 0 and 1.
8. a) In this case, it could be either. It could be a vertical stretch by a factor of $\frac{1}{2}$ or a horizontal stretch by a factor of $\sqrt{2}$.
b) Example: $g(x)=\frac{1}{2} f(x)$
9. a)

b)

10. They are both horizontal stretches by a factor of $\frac{1}{4}$. The difference is in the horizontal translation, the first being 1 unit left and the second being $\frac{1}{4}$ unit left.
11. $g(x)=f(2(x-5))-2$
12. a)

b)


13. a)

b) $y=x,\left(-\frac{1}{2},-\frac{1}{2}\right)$
c) $f(x)$ : domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$ $f(y)$ : domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$
14. 

| $y=f(x)$ |  | $y=f^{-1}(x)$ |  |
| ---: | ---: | ---: | ---: |
| $x$ | $y$ | $x$ | $y$ |
| -3 | 7 | 7 | -3 |
| 2 | 4 | 4 | 2 |
| 10 | -12 | -12 | 10 |

15. a)


The relation and its inverse are functions.
b)


The relation is a function. The inverse is not a function.
16. $y=\sqrt{x-1}+3$, restricted domain $\{x \mid x \geq 3, x \in R\}$
17. a) not inverses
b) inverses

Chapter 1 Practice Test, pages 58 to 59

1. D 2. D 3. B 4. B 5. B 6. C 7. C
2. domain $\{x \mid-5 \leq x \leq 2, x \in \mathrm{R}\}$
3. 


10. a)

b) To transform it point by point, switch the position of the $x$ - and the $y$-coordinate.
c) $(-1,-1)$
11. $y=\frac{1}{5}(x-2)$
12. $y=3 f\left(-\frac{1}{2}(x-2)\right)$
13. a) It is a translation of 2 units left and 7 units down.
b) $g(x)=|x+2|-7 \quad$ c) $(-2,-7)$
d) No. Invariant points are points that remain unchanged after a transformation.
14. a) $f(x)=x^{2}$
b) $g(x)=\frac{1}{4} f(x)$; a vertical stretch by a factor of $\frac{1}{4}$
c) $g(x)=f\left(\frac{1}{2} x\right)$; a horizontal stretch by a factor of 2
d) $\frac{1}{4} f(x)=\frac{1}{4} x^{2} ; f\left(\frac{1}{2} x\right)=\left(\frac{1}{2} x\right)^{2}=\frac{1}{4} x^{2}$
15. a) Using the horizontal line test, if a horizontal line passes through the function more than once the inverse is not a function.
b) $y= \pm \sqrt{-x-5}-3$
c) Example: restricted domain $\{x \mid x \geq-3, x \in R\}$

## Chapter 2 Radical Functions

2.1 Radical Functions and Transformations, pages 72 to 77

1. a)

domain $\{x \mid x \geq 1, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$
b)

domain $\{x \mid x \geq-6, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$
c)

domain $\{x \mid x \leq 3, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$
d)

domain $\left\{x \left\lvert\, x \leq-\frac{5}{2}\right., x \in R\right\}$,
range $\{y \mid y \geq 0, y \in R\}$
2. a) $\quad a=7 \rightarrow$ vertical stretch by a factor of 7 $h=9 \rightarrow$ horizontal translation 9 units right domain $\{x \mid x \geq 9, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$
b) $b=-1 \rightarrow$ reflected in $y$-axis
$k=8 \rightarrow$ vertical translation up 8 units
domain $\{x \mid x \leq 0, x \in R\}$, range $\{y \mid y \geq 8, y \in R\}$
c) $\quad a=-1 \rightarrow$ reflected in $x$-axis
$b=\frac{1}{5} \rightarrow$ horizontal stretch factor of 5
domain $\{x \mid x \geq 0, x \in R\}$, range $\{y \mid y \leq 0, y \in R\}$
d) $a=\frac{1}{3} \rightarrow$ vertical stretch factor of $\frac{1}{3}$
$h=-6 \rightarrow$ horizontal translation 6 units left
$k=-4 \rightarrow$ vertical translation 4 units down
domain $\{x \mid x \geq-6, x \in \mathrm{R}\}$,
range $\{y \mid y \geq-4, y \in R\}$
3. a) $B$
b) A
c) D
d) C
4. a) $y=4 \sqrt{x+6}$
b) $y=\sqrt{8 x}-5$
c) $y=\sqrt{-(x-4)}+11$ or $y=\sqrt{-x+4}+11$
d) $y=-0.25 \sqrt{0.1 x}$ or $y=-\frac{1}{4} \sqrt{\frac{1}{10} x}$
5. a)

domain
$\{x \mid x \leq 0, x \in R\}$,
range
$\{y \mid y \geq-3, y \in R\}$
b)

domain
$\{x \mid x \geq-1, x \in R\}$,
range
$\{y \mid y \geq 0, y \in R\}$
c)

d)

e)

f)

domain
$\{x \mid x \geq 0, x \in R\}$, range $\{y \mid y \geq 4, y \in R\}$
domain
$\{x \mid x \leq-2, x \in \mathrm{R}\}$ range
$\{y \mid y \geq-1, y \in R\}$
6. a) $\quad a=\frac{1}{4} \rightarrow$ vertical stretch factor of $\frac{1}{4}$
$b=5 \rightarrow$ horizontal stretch factor of $\frac{1}{5}$
b) $y=\frac{\sqrt{5}}{4} \sqrt{x}, y=\sqrt{\frac{5}{16} x}$
c) $\quad a=\frac{\sqrt{5}}{4} \rightarrow$ vertical stretch factor of $\frac{\sqrt{5}}{4}$
$b=\frac{5}{16} \rightarrow$ horizontal stretch factor of $\frac{16}{5}$
d)



All graphs are the same.
7. a) $r(A)=\sqrt{\frac{A}{\pi}}$
b)

| $\boldsymbol{A}$ | $\boldsymbol{r}$ |
| :---: | :---: |
| 0 | 0 |
| 1 | 0.6 |
| 2 | 0.8 |
| 3 | 1.0 |
| 4 | 1.1 |


8. a) $b=1.50 \rightarrow$ horizontal stretch factor of $\frac{1}{1.50}$ or $\frac{2}{3}$
b) $d \approx 1.22 \sqrt{h}$ Example: I prefer the original function because the values are exact.
c) approximately 5.5 miles
9. a) domain $\{x \mid x \geq 0, x \in R\}$, range $\{y \mid y \geq-13, y \in R\}$
b) $h=0 \rightarrow$ no horizontal translation
$k=13 \rightarrow$ vertical translation down 13 units
10. a) $y=-\sqrt{x+3}+4$
b) $y=\frac{1}{2} \sqrt{x+5}-3$
c) $y=2 \sqrt{-(x-5)}-1$ or $y=2 \sqrt{-x+5}-1$
d) $y=-4 \sqrt{-(x-4)}+5$ or $y=-4 \sqrt{-x+4}+5$
11. Examples:
a) $y-1=\sqrt{x-6}$ or $y=\sqrt{x-6}+1$
b) $y=-\sqrt{x+7}-9$
c) $y=2 \sqrt{-x+4}-3$
d) $y=-\sqrt{-(x+5)}+8$
12. a) $a=760 \rightarrow$ vertical stretch factor of 760
$k=2000 \rightarrow$ vertical translation up 2000
b)

c) domain
$\{n \mid n \geq 0, n \in \mathrm{R}\}$
range
$\{Y \mid Y \geq 2000, Y \in \mathrm{R}\}$
d) The minimum yield is $2000 \mathrm{~kg} /$ hectare. Example: The domain and range imply that the more nitrogen added, the greater the yield without end. This is not realistic.
13. a) domain $\{d \mid-100 \leq d \leq 0, d \in R\}$ range $\{P \mid 0 \leq P \leq 20, P \in \mathrm{R}\}$ The domain is negative indicating days remaining, and the maximum value of $P$ is 20 million.
b) $\quad a=-2 \rightarrow$ reflected in $d$-axis, vertical stretch factor of $2 ; b=-1 \rightarrow$ reflected in $P$-axis; $k=20 \rightarrow$ vertical translation up 20 units.
c)


Since $d$ is negative, then $d$ represents the number of days remaining before release and the function has a maximum of 20 million pre-orders.
d) 9.05 million or 9045549 pre-orders.
14. a) Polling errors reduce as the election approaches.
b) $y=0.49 \sqrt{-x}$ There are no translations since the graph starts on the origin. The graph is reflected in the $y$-axis then $b=-1$. Develop the equation by using the point $(-150,6)$ and substituting in the equation $y=a \sqrt{x}$, solving for $a$, then $a=0.49$.
c) $\quad a=0.49 \rightarrow$ vertical stretch factor of 0.49 $b=-1 \rightarrow$ reflected in the $y$-axis
15. $y \approx 2.07 \sqrt{-x}$
16. Examples
a) $y=-2 \sqrt{x-2}+5$
b) $y=\frac{2}{3} \sqrt{3-x}-2$
17. a) China, India, and USA (The larger the country the more unfair the "one nation - one vote" system becomes.) Tuvalu, Nauru, Vatican City (The smaller the nation the more unfair the "one person - one vote" system becomes.)
b)

| Nation | Percentage | d) | Nation | Percentage |
| :--- | :--- | :--- | :--- | :---: |
| China | $18.6 \%$ |  | China | $4.82 \%$ |
| India | $17.1 \%$ |  |  |  |
| US | $4.5 \%$ | India | $4.62 \%$ |  |
| Canada | $0.48 \%$ | US | $2.36 \%$ |  |
| Tuvalu | $0.000151 \%$ |  | Canada | $0.77 \%$ |
| Nauru | $0.000137 \%$ | Tuvalu | $0.014 \%$ |  |
| Vatican City | 0.000 | Nauru | $0.013 \%$ |  |

c) $V(x)=\frac{1}{1000} \sqrt{x}$
e) The Penrose system gives larger nations votes based on population but also provides an opportunity for smaller nations to provide influence.
18. Answers will vary.
19. a)

b) i) $g^{-1}(x)=x^{2}+5, x \leq 0$
ii) $h^{-1}(x)=-(x-3)^{2}, x \geq 3$
iii) $j^{-1}(x)=\frac{1}{2}(x+6)^{2}+\frac{7}{2}, x \geq-6$
20. Vertical stretch by a factor of $\frac{16}{25}$. Horizontal stretch by a factor of $\frac{7}{72}$. Reflect in both the $x$ and $y$ axes. Horizontal translation of 3 units left. Vertical translation of 4 units down.
C1 The parameters $b$ and $h$ affect the domain. For example, $y=\sqrt{x}$ has domain $x \geq 0$ but $y=\sqrt{2(x-3)}$ has domain $x \geq 3$. The parameters $a$ and $k$ affect the range. For example, $y=\sqrt{x}$ has range $y \geq 0$ but $y=\sqrt{x}-4$ has range $y \geq-4$.

C2 Yes. For example, $y=\sqrt{9 x}$ can be simplified to $y=3 \sqrt{x}$.
c3 The processes are similar because the parameters $a$, $b, h$, and $k$ have the same effect on radical functions and quadratic functions. The processes are different because the base functions are different: one is the shape of a parabola and the other is the shape of half of a parabola.
C4 Step $1 \sqrt{2}$; Step $2 \sqrt{3}$
Step $4 \longdiv { T r i a n g l e }$ Num

| Triangle Number, $\boldsymbol{n}$ | Length of Hypotenuse, $\boldsymbol{L}$ |
| :---: | :---: |
| First | $\sqrt{2}=1.414 \ldots$ |
| Second | $\sqrt{3}=1.732 \ldots$ |
| Third | $\sqrt{4}=2$ |

Step $5 L=\sqrt{n+1}$ Yes, the equation involves a horizontal translation of 1 unit left.

### 2.2 Square Root of a Function, pages 86 to 89

1. 

| $\boldsymbol{f}(\boldsymbol{x})$ | $\sqrt{\boldsymbol{f ( x )}}$ |
| :---: | :---: |
| 36 | 6 |
| 0.09 | 0.3 |
| 1 | 1 |
| -9 | undefined |
| 2.56 | 1.6 |
| 0 | 0 |

2. a) $(4,3.46) \quad$ b) $(-2,0.63)$
c) does not exist d) $(0.09,1)$
e) $(-5,0)$
f) $(m, \sqrt{n})$
3. a) C b) D c) $\mathrm{A} \quad$ d) B
4. a)

b) When $4-x<0$ then $\sqrt{4-x}$ is undefined; when $0<4-x<1$ then $\sqrt{4-x}>4-x$; when $4-x>1$ then $4-x>\sqrt{4-x} ; 4-x=\sqrt{4-x}$ when $y=0$ and $y=1$
c) The function $f(x)=\sqrt{4-x}$ is undefined when $4-x<0$, therefore the domain is
$\{x \mid x \leq 4, x \in R\}$ whereas the function
$f(x)=4-x$ has a domain of $\{x \mid x \in R\}$.
Since $\sqrt{f(x)}$ is undefined when $f(x)<0$, the range of $\sqrt{f(x)}$ is $\{f(x) \mid f(x) \geq 0, f(x) \in R\}$, whereas the range of $f(x)=4-x$ is $\{f(x) \mid f(x) \in \mathrm{R}\}$.
5. a)


For $y=x-2$, domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$; for $y=\sqrt{x-2}$, domain $\{x \mid x \geq 2, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$.
The domains differ since $\sqrt{x-2}$ is undefined when $x<2$. The range of $y=\sqrt{x-2}$ is $y \geq 0$, when $x-2 \geq 0$.
b)


For $y=2 x+6$, domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$. For $y=\sqrt{2 x+6}$, domain $\{x \mid x \geq-3, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in R\}$. $y=\sqrt{2 x+6}$ is undefined when $2 x+6<0$, therefore $x \geq-3$ and $y \geq 0$.
c)


For $y=-x+9$, domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$; for $y=\sqrt{-x+9}$, domain $\{x \mid x \leq 9, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$. $y=\sqrt{-x+9}$ is undefined when $-x+9<0$, therefore $x \leq 9$ and $y \geq 0$.
d)


For $y=-0.1 x-5$, domain $\{x \mid x \in R\}$,
range $\{y \mid y \in R\}$; for $y=\sqrt{-0.1 x-5}$,
domain $\{x \mid x \leq-50, x \in R\}$,
range $\{y \mid y \geq 0, y \in R\} . y=\sqrt{-0.1 x-5}$ is undefined when $-0.1 x-5<0$, therefore $x \leq-50$ and $y \geq 0$.
6. a) For $y=x^{2}-9$, domain $\{x \mid x \in R\}$,
range $\{y \mid y \geq-9, y \in R\}$.
For $y=\sqrt{x^{2}-9}$,
domain $\{x \mid x \leq-3$ and $x \geq 3, x \in R\}$, range $\{y \mid y \geq 0, y \in R\} . y=\sqrt{x^{2}-9}$ is undefined when $x^{2}-9<0$, therefore $x \leq-3$ and $x \geq 3$ and $y \geq 0$.
b) For $y=2-x^{2}$, domain $\{x \mid x \in R\}$,
range $\{y \mid y \leq 2, y \in R\}$. For $y=\sqrt{2-x^{2}}$,
domain $\{x \mid-\sqrt{2} \leq x \leq \sqrt{2}, x \in R\}$,
range $\{y \mid 0 \leq y \leq \sqrt{2}, y \in R\} . y=\sqrt{2-x^{2}}$ is
undefined when $2-x^{2}<0$, therefore $x \leq \sqrt{2}$ and $x \geq-\sqrt{2}$ and $0 \leq y \leq \sqrt{2}$.
c) For $y=x^{2}+6$, domain $\{x \mid x \in R\}$, range $\{y \mid y \geq 6, y \in R\}$.
For $y=\sqrt{x^{2}+6}$, domain $\{x \mid x \in R\}$,
range $\{y \mid y \geq \sqrt{6}, y \in R\} . y=\sqrt{x^{2}+6}$ is undefined when $x^{2}+6<0$, therefore $x \in \mathrm{R}$ and $y \geq \sqrt{6}$.
d) For $y=0.5 x^{2}+3$, domain $\{x \mid x \in R\}$, range $\{y \mid y \geq 3, y \in R\}$.
For $y=\sqrt{0.5 x^{2}+3}$, domain $\{x \mid x \in R\}$,
range $\{y \mid y \geq \sqrt{3}, y \in R\} . y=\sqrt{0.5 x^{2}+3}$ is undefined when $0.5 x^{2}+3<0$, therefore $x \in R$ and $y \geq \sqrt{3}$.
7. a) Since $y=\sqrt{x^{2}-25}$ is undefined when $x^{2}-25<0$, the domain changes from $\{x \mid x \in R\}$ to $\{x \mid x \leq-5$ and $x \geq 5, x \in \mathrm{R}\}$ and the range changes from $\{y \mid y \geq-25, y \in R\}$ to $\{y \mid y \geq 0, y \in R\}$.
b) Since $y=\sqrt{x^{2}+3}$ is undefined when $x^{2}+3<0$, the range changes from $\{y \mid y \geq 3, y \in R\}$ to $\{y \mid y \geq \sqrt{3}, y \in R\}$.
c) Since $y=\sqrt{32-2 x^{2}}$ is undefined when $32-2 x^{2}<0$, the domain changes from $\{x \mid x \in R\}$ to $\{x \mid-4 \leq x \leq 4, x \in R\}$ and the range changes from $\{y \mid y \leq 32, y \in R\}$ to $\{y \mid 0 \leq y \leq \sqrt{32}, y \in R\}$ or $\{y \mid 0 \leq y \leq 4 \sqrt{2}, y \in R\}$.
d) Since $y=\sqrt{5 x^{2}+50}$ is undefined when
$5 x^{2}+50<0$, the range changes from
$\{y \mid y \geq 50, y \in R\}$ to $\{y \mid y \geq \sqrt{50}, y \in R\}$ or $\{y \mid y \geq 5 \sqrt{2}, y \in R\}$.
8. a)

b)

c)

9. a) and b)

ii)

iii)

iv)


For $y=x^{2}+4$,
domain
$\{x \mid x \in R\}$, range
$\{y \mid y \geq 4, y \in R\}$

For $y=x^{2}-4$,
domain
$\{x \mid x \in R\}$,
range
$\{y \mid y \geq-4, y \in R\}$

For $y=-x^{2}+4$, domain
$\{x \mid x \in R\}$,
range
$\{y \mid y \leq 4, y \in R\}$

For $y=-x^{2}-4$,
domain
$\{x \mid x \in R\}$,
range
$\{y \mid y \leq-4, y \in R\}$.
c) The graph of $y=\sqrt{j(x)}$ does not exist because all of the points on the graph $y=j(x)$ are below the $x$-axis. Since all values of $j(x)<0$, then $\sqrt{j(x)}$ is undefined and produces no graph in the real number system.
d) The domains of the square root of a function are the same as the domains of the function when the value of the function $\geq 0$. The domains of the square root of a function do not exist when the value of the function $<0$. The ranges of the square root of a function are the square root of the range of the original function, except when the value of the function $<0$ then the range is undefined.
10. a) For $y=x^{2}-4$, domain $\{x \mid x \in R\}$, range $\{y \mid y \geq-4, y \in R\}$; for $y=\sqrt{x^{2}-4}$, domain $\{x \mid x \leq-2$ and $x \geq 2, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$.
b) The value of $y$ in the interval $(-2,2)$ is negative therefore the domain of $y=\sqrt{x^{2}-4}$ is undefined and has no values in the interval $(-2,2)$.
11. a)


I sketched the graph by locating key points, including invariant points, and determining the image points on the graph of the square root of the function.
b) For $y=f(x)$, domain $\{x \mid x \in R\}$, range $\{y \mid y \geq-1, y \in R\}$; for $y=\sqrt{f(x)}$,
domain $\{x \mid x \leq-0.4$ and $x \geq 2.4, x \in \mathrm{R}\}$,
range $\{y \mid y \geq 0, y \in R\}$
The domain of $y=\sqrt{f(x)}$, consists of all values in the domain of $f(x)$ for which $f(x) \geq 0$, and the range of $y=\sqrt{f(x)}$, consists of the square roots of all values in the range of $f(x)$ for which $f(x)$ is defined.
12. a) $d=\sqrt{h^{2}+12756 h}$
b) domain $\{h \mid h \geq 0, h \in \mathrm{R}\}$, range $\{d \mid d \geq 0, d \in \mathrm{R}\}$
c) Find the point of intersection between the graph of the function and $h=800$. The distance will be expressed as the $d$ value of the ordered pair $(h, d)$. In this case, $d$ is approximately equal to 3293 .
d) Yes, if $h$ could be any real number then the domain is $\{h \mid h \leq-12756$ or $h \geq 0, h \in R\}$ and the range would remain the same- since all square root values must be greater than or equal to 0 .
13. a) No, since $\sqrt{a}, a<0$ is undefined, then $y=\sqrt{f(x)}$ will be undefined when $f(x)<0$, but $f(x)$ represents values of the range not the domain as Chris stated.
b) If the range consists of negative values, then you know that the graph represents $y=f(x)$ and not $y=\sqrt{f(x)}$.
14. a) $v=\sqrt{3.24-h^{2}}$
b) domain $\{h \mid 0 \leq h \leq 1.8, h \in R\}$, range $\{v \mid 0 \leq v \leq 1.8, v \in R\}$ since both $h$ and $v$ represent distances.
c) approximately 1.61 m
15. Step 1


Step 2 The parameter $a$ determines the minimum value of the domain $(-a)$ and the maximum value of the domain (a); therefore the domain is
$\{x \mid-a \leq x \leq a, x \in R\}$. The parameter $a$ also determines the maximum value of the range, where the minimum value of the range is 0 ; therefore the range is $\{y \mid 0 \leq y \leq a, y \in R\}$.
Step 3 Example: $y=\sqrt{3^{2}-x^{2}}$ the reflection of the graph in the $x$-axis is the equation $y=-\sqrt{3^{2}-x^{2}}$.


The graph forms a circle.
16. a) $(-27,4 \sqrt{3})$
b) $(-6,12-2 \sqrt{3})$
c) $(26,6-4 \sqrt{3})$
17. a)

b)

c)

d)

18. Example: Sketch the graph in the following order:

1) $y=2 f(x)$ Stretch vertically by a factor of 2 .
2) $y=2 f(x-3)$
3) $y=\sqrt{2 f(x-3)}$ right.
Plot invariant points and sketch a smooth curve above the $x$-axis.
4) $y=-\sqrt{2 f(x-3)}$ Reflect $y=\sqrt{2 f(x-3)}$ in the $x$-axis.
19. a) $r=\sqrt{\frac{A}{6 \pi}} \quad$ b) $r=\sqrt{\frac{A}{\pi(1+\sqrt{37})}}$

C1 Example: Choose 4 to 5 key points on the graph of $y=f(x)$. Transform the points $(x, y) \rightarrow(x, \sqrt{y})$. Plot the new points and smooth out the graph. If you cannot get an idea of the general shape of the graph, choose more points to graph.
C2 The graph of $y=16-4 x$ is a linear function spanning from quadrant II to quadrant IV with an $x$-intercept of 4 and a $y$-intercept of 16 . The graph of $y=\sqrt{16-4 x}$ only exists when the graph of $y=16-4 x$ is on or above the $x$-axis. The $y$-intercept is at $\sqrt{16}=4$ while the $x$-intercept stays the same. $x$-values for $x \leq 4$ are the same for both functions and the $y$-values for $y=\sqrt{16-4 x}$ are the square root of $y$ values for $y=16-4 x$.
C3 No, it is not possible, because the graph of $y=f(x)$ may exist when $y<0$ but the graph of $y=\sqrt{f(x)}$ does not exist when $y<0$.
C4 a)

b) The graph of $y=(x-1)^{2}-4$ is a quadratic function with a vertex of $(1,-4), y$-intercept of -3 , and $x$-intercepts of -1 and 3 . It is above the $x$-axis when $x>3$ and $x<-1$.
The graph of $y=\sqrt{(x-1)^{2}-4}$ has the same $x$-intercepts but no $y$-intercept. The graph only exists when $x>3$ and $x<-1$.

### 2.3 Solving Radical Equations Graphically, pages 96 to 98

1. a) B
b) A
c) D
d) C
2. a) $x=9$
b)

c) The roots of the equation are the same as the $x$-intercept on the graph.
3. a) 24.714

b) -117.273


d) no solution

4. a) $x=5.08 \overline{3}$
5. a) $x=65, x \geq \frac{9}{2}$

c) $x=-3.95, x \geq-6.4$


b) $x=3, x \leq 12$

d) $x=-19.5, x \leq 12.5$

6. a) $x=\frac{7}{2}, x=-1$
b) $x=8, x=-2, x \leq-\frac{\sqrt{14}}{2}$ or $x \geq \frac{\sqrt{14}}{2}$
c) $x=1.8, x=-1,-\frac{\sqrt{13}}{2} \leq x \leq \frac{\sqrt{13}}{2}$
d) $x=0, x=2, \frac{-3 \sqrt{2}}{2} \leq x \leq \frac{3 \sqrt{2}}{2}$
7. a) $x \approx-2.725, x \leq 8$

c) $x=3, x \geq \frac{\sqrt{33}}{3}$
b) no real roots, $x \geq 7$

or $x \leq-\frac{\sqrt{33}}{3}$

8. a) $a \approx 13.10$


b) $a \approx-2.25$

c) no solution

|  |  |
| :---: | :---: |
|  |  |
|  | - |

d) $a \approx-2.25, a \approx 15.65$


9. a) $6+\sqrt{x+4}=2$

$$
\begin{aligned}
\sqrt{x+4} & =-4 \\
x+4 & =16 \\
x & =12
\end{aligned}
$$

Left Side $=6+\sqrt{12+4}$

$$
\begin{aligned}
& =6+\sqrt{16} \\
& =6+4
\end{aligned}
$$

$$
=10
$$

Right Side $=2$
Left Side $\neq$ Right Side
Since $10 \neq 2$, there is no solution.
b) Yes, if you isolate the radical expression
like $\sqrt{x+4}=-4$, if the radical is equal to a negative value then there is no solution.
10. Greg $\rightarrow N(t)=1.3 \sqrt{t}+4.2=1.3 \sqrt{6}+4.2$
$\approx 7.38$ million,
Yolanda $\rightarrow N(t)=1.3 \sqrt{t}+4.2=1.3 \sqrt{1.5}+4.2$
$\approx 5.79$ million
Greg is correct, it will take more than 6 years for the entire population to be affected.
11. approximately 99 cm
12. a) Yes
b) 3000 kg
13. No, $\sqrt{X^{2}}=9$ has two possible solutions $\pm 9$, whereas $(\sqrt{x})^{2}=9$ has only one solution +9 .
14. $x=\frac{3+\sqrt{5}}{2}$
15. a) $5 \mathrm{~m} / \mathrm{s}$
b) 75.2 kg
16. $c=-2$ or 1


If the function $y=\sqrt{-3(x+c)}+c$ passes through the point $(0.25,0.75)$, what is the value of $c$ ?
17. Lengths of sides are $55.3 \mathrm{~cm}, 60 \mathrm{~cm}$, and 110.6 cm or $30.7 \mathrm{~cm}, 60 \mathrm{~cm}$, and 61.4 cm .
C1 The $x$-intercepts of the graph of a function are the solutions to the corresponding equation. Example: A graph of the function $y=\sqrt{x-1}-2$ would show that the $x$-intercept has a value of 5 . The equation that corresponds to this function is $0=\sqrt{x-1}-2$ and the solution to the equation is 5 .
C2 a) $s=\sqrt{9.8 d}$ where $s$ represents speed in metres per second and $d$ represents depth in metres.
b) $s=\sqrt{9.8 d}$
$s=\sqrt{\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)(2500 \mathrm{~m})}$
$s=\sqrt{24500 \mathrm{~m}^{2} / \mathrm{s}^{2}}$
$s \approx 156.5 \mathrm{~m} / \mathrm{s}$
c) approximately 4081.6 m
d) Example: I prefer the algebraic method because it is faster and I do not have to adjust window settings.
C3 Radical equations only have a solution in the real number system if the graph of the corresponding function has an $x$-intercept. For example, $y=\sqrt{x}+4$ has no real solutions because there is no $x$-intercept.
C4 Extraneous roots occur when solving equations algebraically. Extraneous roots of a radical equation may occur anytime an expression is squared. For example, $x^{2}=1$ has two possible solutions, $x= \pm 1$. You can identify extraneous roots by graphing and by substituting into the original equation.

## Chapter 2 Review, pages 99 to 101

1. a)

domain $\{x \mid x \geq 0, x \in R\}$
range $\{y \mid y \geq 0, y \in R\}$ All values in the table lie on the smooth curve graph of $y=\sqrt{x}$.
b)

domain $\{x \mid x \leq 3, x \in \mathrm{R}\}$
range $\{y \mid y \geq 0, y \in R\}$ All points in the table lie on the graph of $y=\sqrt{3-x}$.
c)

domain $\{x \mid x \geq-3.5, x \in R\}$
range $\{y \mid y \geq 0, y \in R\}$ All points in the table lie on the graph of $y=\sqrt{2 x+7}$.
2. Use $y=a \sqrt{b(x-h)}+k$ to describe transformations.
a) $a=5 \rightarrow$ vertical stretch factor of 5
$h=-20 \rightarrow$ horizontal translation left 20 units; domain $\{x \mid x \geq-20, x \in R\}$; range $\{y \mid y \geq 0, y \in R\}$
b) $b=-2 \rightarrow$ horizontal stretch factor of $\frac{1}{2}$, then reflected on $y$-axis: $k=-8 \rightarrow$ vertical translation of 8 units down.
domain $\{x \mid x \leq 0, x \in R\}$; range $\{y \mid y \geq-8, y \in R\}$
c) $\quad a=-1 \rightarrow$ reflect in $x$-axis
$b=\frac{1}{6} \rightarrow$ horizontal stretch factor of 6
$h=11 \rightarrow$ horizontal translation right 11 units; domain $\{x \mid x \geq 11, x \in R\}$, range $\{y \mid y \leq 0, y \in R\}$.
3. a) $y=\sqrt{\frac{1}{10} x}+12$, domain $\{x \mid x \geq 0, x \in R\}$, range $\{y \mid y \geq 12, y \in R\}$
b) $y=-2.5 \sqrt{x+9}$
domain $\{x \mid x \geq-9, x \in R\}$, range $\{y \mid y \leq 0, y \in R\}$
c) $y=\frac{1}{20} \sqrt{-\frac{2}{5}(x-7)}-3$,
domain $\{x \mid x \leq 7, x \in R\}$, range $\{y \mid y \geq-3, y \in R\}$
4. a)

b)

domain
$\{x \mid x \leq 0, x \in R\}$,
range
$\{y \mid y \geq-4, y \in R\}$
c)

domain $\{x \mid x \geq-3, x \in R\}$, range $\{y \mid y \geq 1, y \in R\}$
5. The domain is affected by a horizontal translation of 4 units right and by no reflection on the $y$-axis. The domain will have values of $x$ greater than or equal to 4 , due to a translation of the graph 4 units right. The range is affected by vertical translation of 9 units up and a reflection on the $x$-axis. The range will be less than or equal to 9 , because the graph has been moved up 9 units and reflected on the $x$-axis, therefore the range is less than or equal to 9 , instead of greater than or equal to 9 .
6. a) Given the general equation $y=a \sqrt{b(x-h)}+k$ to describe transformations, $a=100$ indicates a vertical stretch by a factor of $100, k=500$ indicates a vertical translation up 500 units.
b)


Since the minimum value of the graph is 500, the minimum estimated sales will be 500 units.
c) domain $\{t \mid t \geq 0, t \in \mathrm{R}\}$ The domain means that time is positive in this situation.
range $\{S(t) \mid S(t) \geq 500, S(t) \in \mathrm{W}\}$. The range means that the minimum sales are 500 units.
d) about 1274 units
7. a) $y=\sqrt{\frac{1}{4}(x+3)}+2 \quad$ b) $y=-2 \sqrt{x+4}+3$
c) $y=4 \sqrt{-(x-6)}-4$
8. a) For $y=x-2$, domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$; for $y=\sqrt{x-2}$, domain $\{x \mid x \geq 2, x \in R\}$,
range $\{y \mid y \geq 0, y \in R\}$. The domain changes because the square root function has restrictions. The range changes because the function only exists on or above the $x$-axis.
b) For $y=10-x$, domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$; for $y=\sqrt{10-x}$, domain $\{x \mid x \leq 10, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$ The domain changes because the square root function has restrictions. The range changes because the function only exists on or above the $x$-axis.
c) For $y=4 x+11$, domain $\{x \mid x \in R\}$,
range $\{y \mid y \in R\}$; for $y=\sqrt{4 x+11}$,
domain $\left\{x \left\lvert\, x \geq-\frac{11}{4}\right., x \in R\right\}$,
range $\{y \mid y \geq 0, y \in R\}$. The domain changes because the square root function has restrictions. The range changes because the function only exists on or above the $x$-axis.
9. a)
 Plot invariant points at the intersection of the graph and lines $y=0$ and $y=1$. Plot any points ( $x, \sqrt{y}$ ) where the value of $y$ is a perfect square. Sketch a smooth curve through the invariant points and points satisfying $(x, \sqrt{y})$.
b) $y=\sqrt{f(x)}$ is positive when $f(x)>0$, $y=\sqrt{f(x)}$ does not exist when $f(x)<0$. $\sqrt{f(x)}>f(x)$ when $0<f(x)<1$ and $f(x)>\sqrt{f(x)}$ when $f(x)>1$
c) For $f(x)$ : domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$; for $\sqrt{f(x)}$, domain $\{x \mid x \geq-6, x \in \mathrm{R}\}$,
range $\{y \mid y \geq 0, y \in R\}$, since $\sqrt{f(x)}$ is undefined when $f(x)<0$.
10. a) $y=4-x^{2} \rightarrow$ domain $\{x \mid x \in R\}$,
range $\{y \mid y \leq 4, y \in R\}$ for $y=\sqrt{4-x^{2}} \rightarrow$ domain $\{x \mid-2 \leq x \leq 2, x \in R\}$, range $\{y \mid 0 \leq y \leq 2, y \in R\}$,
since $4-x^{2}>0$ only between -2 and 2 then the domain of $y=\sqrt{4-x^{2}}$ is $-2 \leq x \leq 2$. In the domain of $-2 \leq x \leq 2$ the maximum value of $y=4-x^{2}$ is 4 , so the maximum value of $y=\sqrt{4-x^{2}}$ is $\sqrt{4}=2$ then the range of the function $y=\sqrt{4-x^{2}}$ will be $0 \leq y \leq 2$.
b) $y=2 x^{2}+24 \rightarrow$ domain $\{x \mid x \in R\}$,
range $\{y \mid y \geq 24, y \in R\}$
for $y=\sqrt{2 x^{2}+24} \rightarrow$ domain $\{x \mid x \in R\}$,
range $\{y \mid y \geq \sqrt{24}, y \in R\}$. The domain does not change since the entire graph of $y=2 x^{2}+24$ is above the $x$-axis. The range changes since the entire graph moves up 24 units and the graph itself opens up, so the range becomes $y \geq \sqrt{24}$.
c) $y=x^{2}-6 x \rightarrow$ domain $\{x \mid x \in R\}$,
range $\{y \mid y \geq-9, y \in R\}$ for $y=\sqrt{x^{2}-6 x} \rightarrow$ domain $\{x \mid x \leq 0$ or $x \geq 6, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$, since $x^{2}-6 x<0$ between 0 and 6 , then the domain is undefined in the interval $(0,6)$ and exists when $x \leq 0$ or $x \geq 6$. The range changes because the function only exists above the $x$-axis.
11. a) $h(d)=\sqrt{625-d^{2}}$
b)

domain $\{d \mid-25 \leq d \leq 25, d \in \mathrm{R}\}$ range $\{h \mid 0 \leq h \leq 25, h \in R\}$
c) In this situation, the values of $h$ and $d$ must be positive to express a positive distance. Therefore the domain changes to $\{d \mid 0 \leq d \leq 25, d \in R\}$. Since the range of the original function $h(d)=\sqrt{625-d^{2}}$ is always positive then the range does not change.
12. a)
b)

c)

13. a) $x=46$
c) The root of the equation and the $x$ value of the $x$-intercept are the same.
b)

14. a) $x \approx 3.571$

c) $x \approx-7.616$ and $x \approx 7.616$

15. 4.13 m
16. a) $x=13.4$

c) $x \approx 8.781$

d) $x=-3$ and 1

17. a) Jaime found two possible answers which are determined by solving a quadratic equation.
b) Carly found only one intersection at $(5,5)$ or $x$-intercept $(5,0)$ determined by possibly graphing.
c) Atid found an extraneous root of $x=2$.
18. a) $130 \mathrm{~m}^{2}$ b) 6 m

Chapter 2 Practice Test, pages 102 to 103

1. B
2. A 3. A
3. C
4. D
5. B
6. $x \approx-16.62$

7. $y=4 \sqrt{x}$ or $y=\sqrt{16 x}$
8. For $y=7-x \rightarrow$ domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in R\}$. Since $y=\sqrt{7-x}$ is the square root of the $y$-values for the function $y=7-x$, then the domain and ranges of $y=\sqrt{7-x}$ will differ. Since $7-x<0$ when $x>7$, then the domain of $y=\sqrt{7-x}$ will be $\{x \mid x \leq 7, x \in R\}$ and since $\sqrt{7-x}$ indicates positive values only, then the range of $y=\sqrt{7-x}$ is $\{y \mid y \geq 0, y \in R\}$.
9. The domain of $y=f(x)$ is $\{x \mid x \in R\}$, and the range of $y=f(x)$ is $\{y \mid y \leq 8, y \in R\}$. The domain of $y=\sqrt{f(x)}$ is $\{x \mid-2 \leq x \leq 2, x \in R\}$ and the range of $y=\sqrt{f(x)}$ is $\{y \mid 0 \leq y \leq \sqrt{8}, y \in R\}$.
10. 





$$
x=-2, x=1
$$

12. $4+\sqrt{x+1}=x$
$\sqrt{x+1}=x-4$
$x+1=(x-4)^{2}$
$x+1=x^{2}-8 x+16$
$0=x^{2}-9 x+15$
$x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$
$=\frac{-(-9) \pm \sqrt{(-9)^{2}-4(1)(15)}}{2(1)}$
$\approx 2.2$ or 6.8
By checking, 2.2 is an extraneous root, therefore $x \approx 6.8$.

13. a) Given the general equation $y=a \sqrt{b(x-h)}+k$ to describe transformations, $b=255 \rightarrow$ indicating a horizontal stretch by a factor of $\frac{1}{255}$. To sketch the graph of $S=\sqrt{255 d}$, graph the function $S=\sqrt{d}$ and apply a horizontal stretch of $\frac{1}{255}$,
every point on the graph of $S=\sqrt{d}$ will become $\left(\frac{d}{255}, S\right)$.
b)
 $d \approx 39 \mathrm{~m}$ The skid mark of the vehicle will be approximately 39 m .
14. a) Given the general equation $y=a \sqrt{b(x-h)}+k$ to describe transformations, $a=-1 \rightarrow$ reflection of the graph in the $x$-axis, $b=2 \rightarrow$ horizontal stretch by a factor of $\frac{1}{2}, k=3 \rightarrow$ vertical translation up 3 units.
b)

c) domain $\{x \mid x \geq 0, x \in R\}$, range $\{y \mid y \leq 3, y \in R\}$.
d) The domain remains the same because there was no horizontal translation or reflection on the $y$-axis. But since the graph was reflected on the $x$-axis and moved up 3 units and then the range becomes $y \leq 3$.
e) The equation $5+\sqrt{2 x}=8$ can be rewritten as $0=-\sqrt{2 x}+3$. Therefore the $x$-intercept of the graph $y=-\sqrt{2 x}+3$ is the solution of the equation $5+\sqrt{2 x}=8$.
15. 



Step 1 Plot invariant points at the intersection of $y=f(x)$ and functions $y=0$ and $y=1$.
Step 2 Plot points at $\sqrt{\max \text { value }}$
and $\sqrt{\text { perfect square value of } y=f(x)}$
Step 3 Join all points with a smooth curve, remember that the graph of $y=\sqrt{f(x)}$ is above the original graph for the interval $0 \leq y \leq 1$. Note that for the interval where $f(x)<0$, the function $y=\sqrt{f(x)}$ is undefined and has no graph.
16. a) $y=(\sqrt{5}) \sqrt{-(x-5)}$
b) domain $\{x \mid 0 \leq x \leq 5, x \in R\}$, range $\{y \mid 0 \leq y \leq 5, y \in R\}$
Domain: $x$ cannot be negative nor greater than half the diameter of the base, or 5 . Range: $y$ cannot be negative nor greater than the height of the roof, or 5 .
c)


The height of the roof 2 m from the centre is about 4.58 m .

## Chapter 3 Polynomial Functions

### 3.1 Characteristics of Polynomial Functions, pages 114 to 117

1. a) No, this is a square root function.
b) Yes, this is a polynomial function of degree 1.
c) No, this is an exponential function.
d) Yes, this is a polynomial function of degree 4.
e) No, this function has a variable with a negative exponent.
f) Yes, this is a polynomial function of degree 3 .
